AnchorHash: A Scalable Consistent Hash

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***Abstract*—Consistent hashing (CH) is a central building block in many networking applications, from datacenter load-balancing to distributed storage. Unfortunately, state-of-the-art CH solutions cannot ensure full consistency under arbitrary changes and/or cannot scale while maintaining reasonable memory footprints and update times.**

We present AnchorHash, a scalable and fully-consistent hashing algorithm. AnchorHash achieves high key lookup rates, a low memory footprint, and low update times. We formally establish its strong theoretical guarantees, and present advanced implementations with a memory footprint of only a few bytes per resource. Moreover, extensive evaluations indicate that it outperforms state-of-the-art algorithms, and that it can scale on a single core to 100 million resources while still achieving a key lookup rate of more than 15 million keys per second.

I. Introduction

Background. Consistent hashing (CH) aims at mapping the identifiers (keys) of incoming objects into a dynamically-changing set of resources, while achieving (1) *minimal disruption*, *i.e.,* minimum mapping changes as resources are arbitrarily added or removed, and (2) *balance*, *i.e.,* even spreading of the keys across resources such that no resource is overloaded.

CH is a central building block in many networking applications, such as datacenter load balancing, distributed hash tables, and distributed storage [1]–[7]. For instance, it is used by L4 datacenter load-balancers to evenly forward incoming packets to servers, while maintaining the affinity of TCP connections to working servers as other servers are added or removed [1], [8], [9].

Related Work. Consistent hashing was first introduced in the context of caching using the Ring algorithm (also called Consistent Hashing) [10], [11]. Several variations of the traditional Ring algorithm have been suggested in the literature to improve balance, *e.g.,* [12], [13]. Unfortunately, such Ring-based solutions face significant scalability issues, since they require a significant memory footprint and an increasing key lookup complexity.

Another well-known CH algorithm is Highest-Random-Weight (HRW) [14], also designed with the goal of increasing cache hit rates. It was later applied in the design of a location service for wireless networks [15], as well as in data storage systems [16]. While HRW offers good balance and small memory footprint, its computational complexity is prohibitive.

To achieve high key-lookup rates, MaglevHash [1] and similar techniques (*e.g.,* [8], [9]) rely on large memory tables. However, they sacrifice full consistency, memory footprint and update times upon resource additions and removals.

	Consistency		Scalability		
	Min. dis.	**Balance**	**Lookup rate**	**Memory**	**Update time**
HRW [14]	✓	✓	×	✓	✓
Ring [10]	✓	✓✗	×	✓✗	✓✗
MaglevHash [1]	×	✓✗	✓	✓✗	×
AnchorHash	✓	✓	✓	✓	✓

TABLE I: Comparison of AnchorHash and common CH algorithms. Existing algorithms sacrifice full consistency and/or scalability, while AnchorHash aims at providing both.

Several additional algorithms are designed for special cases where resources cannot be removed or added arbitrarily, and therefore were out of scope for our evaluations. For example, Jump [17] assumes that resources can only be added or removed in a specific order. Two additional approaches that do not support resource additions are considered in [18]. The second approach shares some design features with our design but its implementation cannot scale due to a large memory footprint.

AnchorHash. In this paper, we present *AnchorHash*, a new computationally-light hashing technique that guarantees minimal disruption, balance, high lookup rates, low memory footprint, and fast update time after resource additions and removals. Table I shows how AnchorHash is the only algorithm that is able to achieve these goals at once.

We first introduce AnchorHash, which hashes the incoming key into successively smaller sets of resources until eventually obtaining its unique mapped resource. We show how AnchorHash stays consistent under bucket removals by keeping some history, and under bucket additions by relying on indirection (Sec. II).

Then, we formally prove that AnchorHash is consistent, *i.e.,* guarantees minimal disruption and balance. We further prove that the average number of hash computations a key lookup requires depends only on the fraction of randomly failed resources and not on the absolute number of resources. This allows for a very high key lookup rate at scale. We prove that even under extreme failure conditions, where 50% of resources are removed in an adversarial manner, a key lookup by AnchorHash still requires less than 2 hash computations on average together with a very low standard deviation (Sec. III).

Next, we focus on the AnchorHash implementation. Using several successive improvements in the data representation structures, we show that AnchorHash can be reduced to an $O(1)$ memory footprint per resource, at the cost of a slight increase in complexity (Sec. IV).

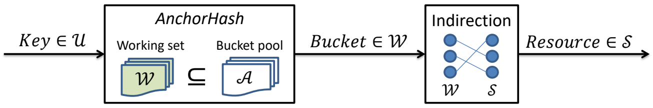


Fig. 1: AnchorHash uses indirection in order to compute the key-to-resource mapping. It first sets a bijective mapping between buckets and resources (right side), and then computes for each incoming key a key-to-bucket mapping (left side).

We then evaluate AnchorHash against HRW [14], Ring [10] and MaglevHash [1], using the criteria of Table I. AnchorHash and MaglevHash are the only algorithms that achieve fast key lookup rates at scale, but MaglevHash sacrifices its full consistency, and also requires a high memory footprint and a significant update time. In addition, we find that AnchorHash can scale on a single core to 100 million resources while achieving a key lookup rate of more than 15 million keys per second. We also show how AnchorHash can update its structures in a few tens of nanoseconds (Sec. V).

Upon publication of this work, we intend to release our code to benefit the research community.

II. AnchorHash

We wish to map incoming keys to resources. Let $\mathcal{U}$ denote the set of keys, and let $\mathcal{S}$ denote the current set of resources. For example, in the context of web caching, keys may correspond to cached URLs, and resources to cache servers. Likewise, in datacenter load-balancing, keys are packet 5-tuples and resources are servers.

Mapping keys to resources. As Fig. 1 illustrates, we use *indirection* by first mapping keys to *buckets*, then buckets to resources. Specifically, current existing resources (*i.e.,* members of $\mathcal{S}$) are assigned to buckets in a one-to-one correspondence. Buckets belong to a set denoted by $\mathcal{A}$, which we assume to be finite. Let $\mathcal{W} \subseteq \mathcal{A}$ denote the subset of buckets that are currently assigned to resources, which we call the *working set*. We refer to buckets in $\mathcal{W}$ as *working* buckets. Note that the set $\mathcal{A}$ of all possible buckets is fixed, while its subset $\mathcal{W}$ changes upon resource removals/additions. Thus the mapping can be decomposed into two parts:
(i) Keys to buckets. A key is first mapped to a bucket in $\mathcal{W}$.
(ii) Buckets to resources. The corresponding resource in $\mathcal{S}$ is deduced from the bucket using the indirection.

Resource removal and addition. We assume that resources can be added and removed arbitrarily. Upon a removal, the corresponding (bucket,resource) pair is removed from the indirection, and the bucket is removed from $\mathcal{W}$. When a resource is added, it is assigned a bucket in $\mathcal{A}\backslash\mathcal{W}$, the bucket is added to $\mathcal{W}$ and the pair (bucket,resource) is added to the indirection.

Note that a resource removal uniquely determines the bucket to remove from $\mathcal{W}$. However, when a resource is added, due to the indirection, any bucket in $\mathcal{A}\backslash\mathcal{W}$ can be added. This property is one of the building blocks we use to construct AnchorHash.

The rest of this section is devoted to the first part of mapping keys to buckets, since the second indirection-based part is straightforward. We henceforth refer to adding/removing a resource as adding/removing a bucket.

Goals. We seek a consistent hash algorithm that maps keys to buckets and satisfies the following joint objectives of *minimal disruption* and *balance*:

Definition 1 (Minimal disruption). *A hash algorithm achieves* minimal disruption *iff*
(i) *Upon the addition of a bucket $b \in \mathcal{A}\backslash\mathcal{W}$ to $\mathcal{W}$, keys either maintain their mapping or are remapped to b.*
(ii) *Upon the removal of a bucket $b \in \mathcal{W}$, keys that were not mapped to b keep their mapping. Keys that were mapped to b are remapped to members of $\mathcal{W}\backslash b$.*

Definition 2 (Balance). *Let $k \in \mathcal{U}$ be a key, chosen uniformly at random. A hash algorithm achieves* balance *iff k has an equal probability of being mapped to each bucket in $\mathcal{W}$.*

Definition 3 (Consistency). *We define a hash algorithm as* consistent *iff it achieves both* minimal disruption *and* balance*.*

AnchorHash uniformly hashes keys to bucket sets using standard hash functions. Accordingly, for our theoretical exposition, we make the following standard assumption.

Uniform hashing assumption. Fix $\mathcal{V} \subseteq \mathcal{U}$ and $\mathcal{B} \subseteq \mathcal{A}$ such that $|\mathcal{V}| \gg |\mathcal{B}|$. Denote by $H_{\mathcal{V}}$ a deterministic hash function that maps any key in $\mathcal{V}$ to a bucket in $\mathcal{B}$. We classically assume (*e.g.,* [19], [20]) that $H_{\mathcal{V}}$ divides the members of $\mathcal{V}$ equally among the members of $\mathcal{B}$, that is, if a key k is chosen uniformly at random from $\mathcal{V}$, then $H_{\mathcal{V}}(k)$ is a random variable with a uniform distribution over $\mathcal{B}$.

A. AnchorHash principles

We now explain how AnchorHash maps keys to buckets. We start with an initial working set, and then discuss how buckets are removed and added.

Initial mapping. Suppose we begin with a working set $\mathcal{W}$. We use the hash function $H_{\mathcal{W}}$ to map keys to $\mathcal{W}$. By the uniform hashing assumption, if a key is chosen uniformly at random, each member of $\mathcal{W}$ has an equal probability to be chosen, thus achieving balance (Def. 2).

Bucket removal. Now, suppose that we want to remove a bucket $b \in \mathcal{W}$. If we use the new hash function $H_{\mathcal{W}\backslash b}$ to map keys to buckets, keys that where mapped to members of $\mathcal{W} \setminus b$ by $H_{\mathcal{W}}$ might be remapped, and the minimal disruption property will not hold.

To address this issue, the key idea in AnchorHash is to keep using $H_{\mathcal{W}}(k)$ as long as $H_{\mathcal{W}}(k) \neq b$, and otherwise rehash the key to $\mathcal{W} \setminus b$ using $H_{\mathcal{W}\backslash b}(k)$. For instance, assume that the initial working set is $\mathcal{W} = \{0, \ldots, 6\}$. Then we are hashing any key k using $H_{\{0,\ldots,6\}}(k)$. Assume now that bucket 6 is removed. Then we continue to first hash any key k using $H_{\{0,\ldots,6\}}(k)$. If it hits a bucket in $\{0, \ldots, 5\}$, we are done. Otherwise, we rehash the key using $H_{\{0,\ldots,5\}}(k)$, with the result guaranteed to be a working bucket.

This approach preserves the consistency of the algorithm, as we later formally prove. First, only keys that were mapped to

	0	1	2	3	4	5	6
$\mathcal{W}$	✓	✓	✓	✓	✓	✓	✓
$\mathcal{W}_b$							

(a) Initial working set $\mathcal{W}$=$\{0,1,2,3,4,5,6\}$.

	0	1	2	3	4	5	6
$\mathcal{W}$	✓	✓	✓	✓	✓	✓	×
$\mathcal{W}_b$							{0,1,2,3,4,5}

(b) Removing bucket 6 with $\mathcal{W}_6$=$\{0,1,2,3,4,5\}$.

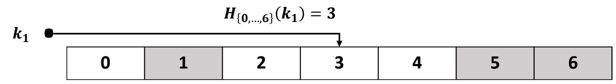
	0	1	2	3	4	5	6
$\mathcal{W}$	✓	✓	✓	✓	✓	×	×
$\mathcal{W}_b$						{0,1,2,3,4}	{0,1,2,3,4,5}

(c) Removing bucket 5 with $\mathcal{W}_5$=$\{0,1,2,3,4\}$.

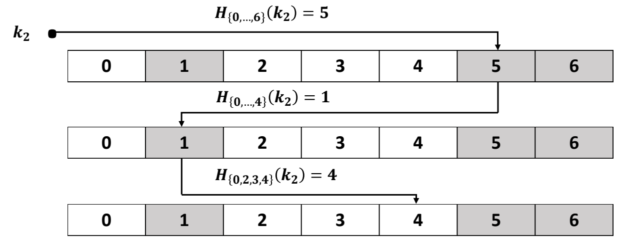
	0	1	2	3	4	5	6
$\mathcal{W}$	✓	×	✓	✓	✓	×	×
$\mathcal{W}_b$		{0,2,3,4}				{0,1,2,3,4}	{0,1,2,3,4,5}

(d) Removing bucket 1 with $\mathcal{W}_1$=$\{0,2,3,4\}$.

Fig. 2: Example with an initial working set $\mathcal{W} = \{0,1,2,3,4,5,6\}$ (Fig. 2(a)). Then, bucket 6, 5 and 1 are removed consecutively (in Figures 2(b), 2(c) and 2(d), respectively).



(a) k_1 is immediately hashed to bucket $3 \in \mathcal{W}$.



(b) k_2 is initially hashed to bucket 5. Then, since $5 \notin \mathcal{W}$, k_2 is rehashed to $1 \in \mathcal{W}_5$. Since $1 \notin \mathcal{W}$ as well, k_2 is rehashed again to $4 \in \mathcal{W}_1$. Since $4 \in \mathcal{W}$, the process terminates.

Fig. 3: Example of possible key lookups in the state presented in Fig. 2(d).

b are remapped, thus minimal disruption is achieved. Second, by the uniform hashing assumption, keys that did not initially hit b are spread uniformly over $\mathcal{W} \setminus b$, and the same is true for the keys that initially hit b and are rehashed. Therefore, balance is also achieved.

When several buckets are removed, we repeat this procedure iteratively until hitting a bucket in the working set. To simplify the notation, we denote by $\mathcal{W}_b$ the working set right after the removal of a bucket b.

Example. Fig. 2 illustrates this procedure with an initial working set $\mathcal{W}$=$\{0,1,2,3,4,5,6\}$ and buckets 6, 5 and 1 removed consecutively. Fig. 3(a) shows a simple example of a key that is immediately hashed to a bucket in the working set. Fig. 3(b) shows a more complex example in which the key is repeatedly hashed to decreasing subsets until reaching a bucket in the working set.

Bucket addition. Suppose that the last bucket that was removed was b, and the current working set is $\mathcal{W}$ (*i.e.,* $\mathcal{W}_b$=$\mathcal{W}$). Recall that AnchorHash may add any bucket not in $\mathcal{W}$ by virtue of the indirection. If we need to add a new bucket, we choose to add back bucket b. More generally, upon bucket addition, AnchorHash *always* adds the last removed bucket. We show in Sec. IV that this allows for an extremely efficient implementation. This is because by our iterative construction, adding the last removed bucket b simply brings us back to the state just before b's removal. Specifically, upon the addition of b, (1) the only remapped keys are remapped to b (these are the same keys that hit b and were rehashed after b was previously removed), and minimal disruption holds; and (2) since balance was achieved before b was removed, it is also achieved after it is added back. We prove these claims formally in Sec. III.

Example. Consider Fig. 2(d). If we add the last removed bucket 1, we simply return to the state illustrated in Fig. 2(c). At this point, if we add the last removed bucket 5, we simply return to the state illustrated in Fig. 2(b), and so on.

We maintain a LIFO queue (*i.e.,* stack) for the removed buckets, denoted by $\mathcal{R}$. For example, in the state illustrated in Fig. 2(d), $\mathcal{R}$=$\{6 \leftarrow 5 \leftarrow 1\}$.

Anchor. By construction, $|\mathcal{A}|$ is an upper bound on the number of buckets that we allow. Therefore, in practice, we simply set the value of $|\mathcal{A}|$ to a larger value than may be needed (*e.g.,* 2× the initial system size) and insert the unused buckets (*i.e.,* members of $\mathcal{A} \backslash \mathcal{W}$) into $\mathcal{R}$. Note that this initial order within $\mathcal{R}$ may be arbitrary. We later leverage this observation to optimize implementation. Since $\mathcal{A}$ serves as the starting point of the algorithm on which everything is defined on, we refer to it as the *Anchor*.

Example. Consider again Fig. 2(a). Assume that instead of beginning our operation with $\mathcal{W}$=$\{0,1,2,3,4,5,6\}$, we would like to start our system with only $\mathcal{W}$=$\{0,1,2,3,4\}$, but want to be prepared to increase $\mathcal{W}$ to include buckets 5 and 6 if needed. Then, we simply start our system with $\mathcal{A}$=$\{0,1,2,3,4,5,6\}$. Namely, we initially set $\mathcal{W}$=$\{0,1,2,3,4\}$ and $\mathcal{R}$=$\{6 \leftarrow 5\}$. This precise state is illustrated in Fig. 2(c).

B. AnchorHash algorithm

The pseudo-code for AnchorHash is given in Alg. 1.

Initialization. INITANCHOR($\mathcal{A},\mathcal{W}$) receives as an input the Anchor $\mathcal{A}$ and the initial working set of buckets $\mathcal{W}$. We fill the stack $\mathcal{R}$ with the initially unused buckets. For each such unused bucket b, we remember $\mathcal{W}_b$, *i.e.,* the working set just after its removal.

GetBucket. GETBUCKET(k) receives a key $k \in \mathcal{U}$ as an input and returns a working bucket $b \in \mathcal{W}$ as an output. Initially, we hash the key uniformly over the Anchor $\mathcal{A}$; then, if the calculated bucket b is not a member of $\mathcal{W}$, the key is rehashed into $\mathcal{W}_b$. This process continues until hitting a working bucket. Both theoretically in Sec. III and empirically in Sec. V we show that this procedure terminates quickly.

Algorithm 1 — AnchorHash

function INITANCHOR($\mathcal{A}, \mathcal{W}$)
    $\mathcal{R} \leftarrow \emptyset$
    for $b \in \mathcal{A} \backslash \mathcal{W}$ **do**
        $\mathcal{R}.push(b)$
        $\mathcal{W}_b \leftarrow \mathcal{A} \backslash \mathcal{R}$

---

function GETBUCKET(k)
    $b \leftarrow H_{\mathcal{A}}(k)$
    while $b \notin \mathcal{W}$ **do**
        $b \leftarrow H_{\mathcal{W}_b}(k)$
    return b

---

function ADDBUCKET( )
    $b \leftarrow \mathcal{R}.pop()$
    $delete\ \mathcal{W}_b$
    $\mathcal{W} \leftarrow \mathcal{W} \cup \{b\}$
    return b

---

function REMOVEBUCKET(b)
    $\mathcal{W} \leftarrow \mathcal{W} \backslash \{b\}$
    $\mathcal{W}_b \leftarrow \mathcal{W}$
    $\mathcal{R}.push(b)$

Algorithm 2 — AnchorHash Wrapper

function INITWRAPPER($\mathcal{A}, \mathcal{S}$)
    $M \leftarrow \emptyset, \mathcal{W} \leftarrow \emptyset$
    for $i \in (0, 1, \ldots, |\mathcal{S}| - 1)$ **do**
        $M \leftarrow M \cup \{(\mathcal{A}[i], \mathcal{S}[i])\}$
        $\mathcal{W} \leftarrow \mathcal{W} \cup \{\mathcal{A}[i]\}$
    INITANCHOR($\mathcal{A}, \mathcal{W}$)

---

function GETRESOURCE(k)
    $b \leftarrow$ GETBUCKET(k)
    $r \leftarrow M(b)$
    return r

---

function ADDRESOURCE(s)
    $b \leftarrow$ ADDBUCKET( )
    $M \leftarrow M \cup \{(b, s)\}$

---

function REMOVERESOURCE(s)
    $b \leftarrow M^{-1}(s)$
    $M \leftarrow M \backslash \{(b, s)\}$
    REMOVEBUCKET(b)

AddBucket. As mentioned, when adding a bucket, we add the the last removed bucket. Accordingly, ADDBUCKET() has *no input* and simply returns the added bucket. It pops the last removed bucket b from $\mathcal{R}$, deletes the no-longer-needed $\mathcal{W}_b$, adds b to $\mathcal{W}$ and returns b.

RemoveBucket. REMOVEBUCKET(b) receives as an input the bucket we want to remove, and has no return value. We simply remove b from $\mathcal{W}$, record the working set just after b's removal $\mathcal{W}_b$ and push b to the top of $\mathcal{R}$.

Indirection. For completeness, Alg. 2 presents the full key-to-resource mapping based on indirection (as presented in Fig. 1). It complements the key-to-bucket mapping of Alg. 1 with a standard bucket-to-resource bijection function M. For simplicity, we represent this bijection using a set of coupled pairs $(b, s) \in M$ such that $M(s) = b$ and $M^{-1}(b) = s$.

III. ANCHORHASH PROPERTIES

In this section we first prove that AnchorHash is consistent (*i.e.,* provides minimal disruption and balance), and then analyze its complexity.

Theorem 1 (Minimal disruption). *AnchorHash guarantees minimal disruption.*

Proof. **(i).** Assume a newly added bucket b. Consider function GETBUCKET(k). Then, before b's addition, each $k \in \mathcal{U}$ either encountered bucket b before terminating or not. After the addition of b, keys that did not encounter b are clearly not affected. Those who did now terminate at b.

(ii). Assume a newly removed bucket b. Consider again function GETBUCKET(k). Before b's removal, each $k \in \mathcal{U}$ either terminated at bucket b or did not encounter it at all. After the removal of b, keys that did not encounter b are clearly not affected. Those who did, now terminate at $H_{\mathcal{W}_b}(k)$. □

Theorem 2 (Balance). *AnchorHash achieves balance.*

Proof. Our proof is by induction on $|\mathcal{R}|$.

Basis: $|\mathcal{R}| = 0$**.** In this case, balance trivially holds since, according to the uniform hashing assumption, $H_{\mathcal{A}}$ divides $\mathcal{U}$ equally among the members of $\mathcal{A}$.

Induction hypothesis. Assume that balance holds for $|\mathcal{R}| = k$.

Inductive step. Consider a newly removed bucket b that results in $|\mathcal{R}| = k + 1$. By the induction hypothesis, $\mathcal{U}$ is divided equally among $\mathcal{W}$ just before b's removal. After b's removal, according to Theorem 1 (minimal disruption), only keys that were mapped to b are remapped. These are remapped using $H_{\mathcal{W}_b}$, which divides them equally among $\mathcal{W}_b$ according to the uniform hashing assumption. This results in a balanced partition of $\mathcal{U}$ among the members of $\mathcal{W}_b$. □

We now turn to providing a strong theoretical guarantee on the run-time complexity of GETBUCKET(k), which explains why AnchorHash is able to process keys at a high rate at scale.

Theorem 3 (Computational complexity). *Let* $|\mathcal{W}|=w$, $|\mathcal{A}|=a$. *Denote by τ the number of hash operations performed by* GETBUCKET(k) *for a randomly chosen key k. Then:*

1) The average of τ is smaller than $1 + \ln(\frac{a}{w})$.
2) The standard deviation of τ is smaller than $\sqrt{\ln(\frac{a}{w})}$.

Proof. Once GETBUCKET(k) is invoked, we repeatedly hash k into decreasing subsets of $\mathcal{A}$ until hitting a working bucket. Let $H_{\mathcal{A}}=b$. Then, conditioned on $b \in \mathcal{R}$, τ has the same distribution as 1 plus the number of hash calculations GETBUCKET(k) performs as if it was invoked with $\mathcal{W}_b$ instead of $\mathcal{A}$. We use this observation to derive a recursive formula and find a closed-form expression for the moment generating function (MGF) of τ. We then use it to find the first and second moments of τ.

Fix a sequence of removals $\mathcal{R} = \{r_{a-w} \leftarrow \ldots \leftarrow r_1\}$. Namely, r_{a-w} is the first removed bucket and r_1 is the last. For $0 \leq i < a - w$, consider GETBUCKET(k) when invoked with $\mathcal{W}_{r_{i+1}}$, and denote by τ_i the number of hash operations it performs, as well as by b_i the first bucket the key k hits (*i.e.,* $b_i = \mathcal{H}_{\mathcal{W}_{r_{i+1}}}(k)$). Let $\tau_{a-w} = \tau$. Define

$$\phi_i(s) = \mathbb{E}[e^{s\tau_i}]. \tag{1}$$

Then, by the law of total expectation,

$$\begin{aligned}\phi_i(s) &= \mathbb{P}(b_i{\in}\mathcal{W})\mathbb{E}[e^{s\tau_i}|b_i{\in}\mathcal{W}] + \sum_{j=1}^{i}\mathbb{P}(b_i{=}r_j)\mathbb{E}[e^{s\tau_i}|b_i{=}r_j] \\ &= \frac{w}{w+i}\mathbb{E}[e^{s\tau_i}|b_i{\in}\mathcal{W}] + \frac{1}{w+i}\sum_{j=1}^{i}\mathbb{E}[e^{s\tau_i}|b_i{=}r_j].\end{aligned} \tag{2}$$

First, conditioned on $b_i \in \mathcal{W}$, the process terminates after a single hash calculation, *i.e.,* $\tau_i = 1$. Thus

$$\mathbb{E}[e^{s\tau_i}|b_i{\in}\mathcal{W}] = e^s. \tag{3}$$

Second, recall that the distribution of τ_i conditioned on $b_i = r_j$ follows the same distribution as $1 + \tau_{j-1}$. Therefore

$$\begin{aligned}\mathbb{E}[e^{s\tau_i} \mid b_i = r_j] &= \mathbb{E}[e^{s(1+\tau_{j-1})}] \\ &= e^s\mathbb{E}[e^{s\tau_{j-1}}] = e^s\phi_{j-1}(s).\end{aligned} \tag{4}$$

Substituting (3) and (4) in (2) yields

$$\phi_i(s) = \frac{w \cdot e^s}{w+i} + \frac{e^s}{w+i}\cdot\sum_{j=1}^{i}\phi_{j-1}(s). \tag{5}$$

Now that we have a recursive formula for $\phi_i(s)$, we are able to calculate its closed-form expression. Rearranging (5) yields

$$\sum_{j=1}^{i}\phi_{j-1}(s) = \frac{w+i}{e^s}\cdot\phi_i(s) - w, \tag{6}$$

and similarly for $i-1$,

$$\sum_{j=1}^{i-1}\phi_{j-1}(s) = \frac{w+i-1}{e^s}\cdot\phi_{i-1}(s) - w, \tag{7}$$

where we use the convention $\sum_1^0 = 0$. Using (7) in (6) and rearranging, we obtain

$$\phi_i(s) = \phi_{i-1}(s)\cdot\frac{w+i-1+e^s}{w+i}. \tag{8}$$

Now, using (8) and the stopping condition $\phi_0(s) = e^s$, we obtain

$$\phi_i(s) = e^s\prod_{j=1}^{i}\Big(\frac{w+j-1+e^s}{w+j}\Big), \tag{9}$$

with the convention $\prod_1^0 = 1$. Taking the logarithm and then differentiating with respect to s yields

$$\frac{\phi_i'(s)}{\phi_i(s)} = 1 + \sum_{j=1}^{i}\Big(\frac{e^s}{e^s+w+j-1}\Big). \tag{10}$$

By (1), $\phi_i(0) = 1$. Hence, substituting $s = 0$ in (10) yields

$$\mathbb{E}[\tau_i] = \phi_i'(0) = 1 + \sum_{j=1}^{i}\frac{1}{w+j}, \tag{11}$$

and therefore

$$\begin{aligned}\mathbb{E}[\tau] = \phi_{a-w}'(0) &= 1 + \sum_{j=1}^{a-w}\frac{1}{w+j} \\ &\le 1 + \int_w^a \frac{1}{x}dx = 1 + \ln\Big(\frac{a}{w}\Big).\end{aligned} \tag{12}$$

Now, to obtain the bound on the standard deviation, we take the derivative with respect to s in (10) and obtain

$$\frac{\phi_i''(s)\phi_i(s) - (\phi_i'(s))^2}{(\phi_i(s))^2} = \sum_{j=1}^{i}\Big(\frac{e^s(e^s+w+j-1) - e^{2s}}{(e^s+w+j-1)^2}\Big).$$

Setting $i = a - w$, $s = 0$ and using $\phi_{a-w}(0) = 1$ yields

$$\begin{aligned}\mathrm{Var}(\tau) &= \phi_{a-w}''(0) - (\phi_{a-w}'(0))^2 \\ &= \sum_{j=1}^{a-w}\frac{w+j-1}{(w+j)^2} \le \sum_{j=1}^{a-w}\frac{1}{w+j} \le \ln(\frac{a}{w}).\end{aligned}$$

Thus the standard deviation is upper bounded by $\sqrt{\ln(\frac{a}{w})}$. Note that the bounds do not depend on the removal sequence we fixed. This concludes the proof. □

IV. AnchorHash implementation

Anchor representation. We use an integer array A of size a to represent the Anchor. Each bucket $b \in \{0, 1, \ldots, a-1\}$ is represented by $A[b]$ that either equals 0 if b is a working bucket (*i.e.,* $A[b] = 0$ if $b \in \mathcal{W}$), or else equals the size of the working set just after its removal (*i.e.,* $A[b] = |\mathcal{W}_b|$ if $b \in \mathcal{R}$).

Example. Considering again the example in Fig. 2(d), we have

b:	0	1	2	3	4	5	6
$A[b]$:	0	4	0	0	0	5	6

By examining this array we can determine that buckets 0, 2, 3, and 4 are working, and buckets $1, 5, 6$ are removed, with $|\mathcal{W}_1| = 4$, $|\mathcal{W}_5| = 5$ and $|\mathcal{W}_6| = 6$.

Hashing. Denote $h_b(k) \equiv \text{hash}(k) \bmod A[b]$. To implement hash$(k)$ efficiently, recent software-based solutions such as xxHash [21] and even hardware-supported hashing [22] can be used to accelerate performance.

Removed buckets. AnchorHash saves the removed buckets in a LIFO order for possible future bucket additions. Accordingly, we use an efficient implementation of a stack data structure R to hold the removed buckets.

Example. In the example of Fig. 2(d), R looks like:

6	5	1				⇆

Decreasing subsets. For each removed bucket b, we need an efficient way of representing $\mathcal{W}_b$ and calculating $H_{\mathcal{W}_b}(k)$. For clarity, we tackle this challenge in stages: we begin with a *naive* implementation, which we successively improve to implementations with a *partial* then *minimal* memory usage.

A. Naive implementation

A naive approach to representing $\{\mathcal{W}_b \,|\, b \in \mathcal{R}\}$ is using a key-value store, KV, that holds the pairs $\{(b, \mathcal{W}_b) \,|\, b \in \mathcal{R}\}$, where the *key* is a removed bucket b and the *value* is $\mathcal{W}_b$, stored in KV$[b]$ as an array. This way, implementing $H_{\mathcal{W}_b}(k)$ simply translates to $H_{\mathcal{W}_b}(k) \equiv \text{KV}[b][h_b(k)]$.

Unfortunately, albeit simple, this approach is not scalable since it requires to maintain an array of size $|\mathcal{W}_b|$ for each removed bucket b, thus incurs an overwhelming memory footprint of $\Theta(|\mathcal{A}| + |\mathcal{A}||\mathcal{R}|)$.

B. Reduced-memory implementation

Non-fixed points. Consider again the naive implementation. Recall that all of the theoretical properties of AnchorHash are *independent* of the exact bucket order within the sets $\{\mathcal{W}_b \,|\, b \in \mathcal{R}\}$. Also, for any two *consecutively* removed buckets b_1 and b_2, the sets $\mathcal{W}_{b_1}$ and $\mathcal{W}_{b_2}$ only differ by a single bucket.

We want to leverage these properties to reduce the memory footprint of AnchorHash and accelerate its performance. Accordingly, we seek to *minimize the number of non-fixed point entries* in the members of $\{\mathcal{W}_b \,|\, b \in \mathcal{R}\}$, which we define as entries that respect $\mathcal{W}_b[h] \neq h$. This way we do not need to remember the full arrays, but only the *difference* between the initial order of buckets and each member of $\{\mathcal{W}_b \,|\, b \in \mathcal{R}\}$, *i.e.,* the *non-fixed points*.

Example. Recall the example in Fig. 2(d). In this example, the naive approach holds three arrays: KV[6], KV[5], and KV[1]. Our goal is to minimize the number of non-fixed point entries between the initial order of buckets $\{0, 1, 2, \ldots, 6\}$ and the order of buckets in the members of $\{\mathcal{W}_b \,|\, b \in \{1, 5, 6\}\}$. For example, to obtain the desired order for $\mathcal{W}_1 = \{0, 2, 3, 4\}$ and minimize the difference with $\boxed{0}\boxed{1}\boxed{2}\boxed{3}$, we simply use $\boxed{0}\boxed{\mathbf{4}}\boxed{2}\boxed{3}$, *i.e.,* take bucket 4 which is the last element in KV[5], and put it instead of the removed bucket 1. This yields

$$\begin{aligned} \text{KV}[6]: &\quad \boxed{0}\boxed{1}\boxed{2}\boxed{3}\boxed{4}\boxed{5} \\ \text{KV}[5]: &\quad \boxed{0}\boxed{1}\boxed{2}\boxed{3}\boxed{4} \\ \text{KV}[1]: &\quad \boxed{0}\boxed{\mathbf{4}}\boxed{2}\boxed{3} \end{aligned} \tag{13}$$

Examining (13) reveals that instead of remembering all three arrays, we can just remember that KV[1][1]=4 (recall that A provides the length of each array). Namely, all other elements are simply fixed points. Each time we calculate $H_{\mathcal{W}_b}(k)$, it equals $h_b(k)$ without the need to access any data structure. The only exception is when an entering key hits bucket 1 and then hashes to 1 again (*i.e.,* calculating $H_{\mathcal{W}_1}(k)$ yields $h_1(k) = 1$). For this specific case, we need to remember that we hit bucket KV[1][1]=4 instead of 1.

Now, assume that in this state bucket 0 is removed. Similarly, the desired ordering for $\mathcal{W}_0$ is obtained by taking the last element in KV[1], which is bucket 3, and putting it instead of the removed bucket 0. This yields

$$\text{KV}[0]: \quad \boxed{\mathbf{3}}\boxed{\mathbf{4}}\boxed{2} \tag{14}$$

Again, we only need to store KV[0][0]=3 and KV[0][1]=4 since the location of the working bucket 2 is identical to its location in the initial ordering. To summarize, in this example we only need to remember 3 elements (the bold numbers in (13) and (14)) instead of the original $6 + 5 + 4 + 3 = 18$.

Individual KV entries. To leverage this solution with reduced memory requirements, we stop organizing the key-value store using arrays. Instead of keeping an entry KV$[b][h]$, we keep an entry KV$[(b, h)]$ where the pair (b, h) is the key. This can be efficiently implemented by simply concatenating b and h to form a single key. For example, in (14), instead of using KV[0][1]=4 with an array, we use KV$[(0, 1)]$=4.

To efficiently determine the desired order within $\mathcal{W}_b$ for a newly removed bucket b and the exact elements that we need to store, we maintain two additional arrays: (1) W, which always contains the *current* set of working buckets in their desired order, and (2) L, which stores for each bucket its *most recent* location in W. Both arrays are initialized identically $W[b] = L[b] = b \quad \forall b \in \{0, 1, \ldots, a-1\}$. For instance, after bucket 1 is removed (*i.e.,* last array in (13)), W and L obtain the following form:

$$\begin{array}{lccccccc} b\text{:} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ W[b]\text{:} & \boxed{0} & \boxed{4} & \boxed{2} & \boxed{3} & \boxed{4} & \boxed{5} & \boxed{6} \end{array} \quad \begin{array}{lccccccc} b\text{:} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ L[b]\text{:} & \boxed{0} & \boxed{1} & \boxed{2} & \boxed{3} & \boxed{1} & \boxed{5} & \boxed{6} \end{array}.$$

That is, bucket 4 replaced bucket 1 in W and the most recent location of bucket 4 updated to index 1. Note that the removals of buckets 5 and 6 did not require any updates in both W and L. With this example at hand, we now detail the update rules for W and L upon bucket removals and additions.

Removal. Assume a newly removed bucket b and let $N = |\mathcal{W}_b|$. Then in W, b is replaced by the last positioned working bucket (*i.e.,* $W[N]$), and its most recent location (*i.e.,* $L[b]$) is correspondingly updated in L. This yields,

$$W[L[b]] \leftarrow W[N], \;\; L[W[N]] \leftarrow L[b].$$

Now, we use the updated array W to determine which entries to store: for all $h \in \{0, 1, \ldots, |\mathcal{W}_b| - 1\}$ such that $W[h] \neq h$, we store KV$[(b, h)] = W[h]$.

Addition. Upon bucket addition, we need to *restore* the state prior to the last removal. To do so, we delete the corresponding entries in KV by the same rule we used to remember them. Then, we restore W and L to their previous state using:

$$L[W[N]] \leftarrow N, \;\; W[L[b]] \leftarrow b.$$

For example, given the state in (13), if we now add back bucket 1 then we simply restore W and L to their initial state since using the rules yields $L[4] \leftarrow 4$ and $W[1] \leftarrow 1$.

Complexity. In the worst case, each consecutive removed bucket may require one additional entry in addition to the entries required by the previously removed bucket. Accordingly, this method for resolving $H_{\mathcal{W}_b}(k)$ results in a memory footprint of $O(|\mathcal{R}|^2)$ and a total memory footprint of $O(|\mathcal{A}| + |\mathcal{R}|^2)$ for AnchorHash. Upon a bucket addition or removal, the complexity accounts for $O(|\mathcal{W}|)$.

Algorithm 3 — AnchorHash Implementation

function INITANCHOR(a, w)
  $A[b] \leftarrow 0$ for $b = 0, 1, \ldots, a{-}1$ ▷ $|\mathcal{W}_b| \leftarrow 0$ for $b \in \mathcal{A}$
  $R \leftarrow \emptyset$ ▷ Empty stack
  $N \leftarrow w$ ▷ Number of initially working buckets
  $K[b] \leftarrow L[b] \leftarrow W[b] \leftarrow b$ for $b = 0, 1, \ldots, a-1$
  for $b = a{-}1$ **downto** w **do** ▷ Remove initially unused buckets
    $R.push(b)$

---

function GETBUCKET(k)
  $b \leftarrow \text{hash}(k) \bmod a$
  while $A[b]{>}0$ **do** ▷ b is removed
    $h \leftarrow h_b(k)$
    while $A[h] \geq A[b]$ **do** ▷ $\mathcal{W}_b[h] \neq h$, b removed prior to h
      $h \leftarrow \text{K}[h]$ ▷ search for $W_b[h]$
    $b \leftarrow h$
  return b

---

function ADDBUCKET( )
  $b \leftarrow R.pop()$
  $A[b] \leftarrow 0$ ▷ $\mathcal{W} \leftarrow \mathcal{W} \cup \{b\}$, delete $\mathcal{W}_b$
  $L[W[N]] \leftarrow N$
  $W[L[b]] \leftarrow K[b] \leftarrow b$
  $N \leftarrow N + 1$
  return b

---

function REMOVEBUCKET(b)
  $R.push(b)$
  $N \leftarrow N - 1$
  $A[b] \leftarrow N$ ▷ $\mathcal{W}_b \leftarrow \mathcal{W} \backslash b$, $A[b] \leftarrow |\mathcal{W}_b|$
  $W[L[b]] \leftarrow K[b] \leftarrow W[N]$
  $L[W[N]] \leftarrow L[b]$

C. Minimal-memory implementation

While the previous implementation may be sufficient for systems with a small $|\mathcal{R}|$ value, we present our final implementation of AnchorHash that results in a remarkably low-memory footprint, negligible response time to changes and high key lookup rate. Specifically, we show how to efficiently calculate KV$[(b, h)]$ for all (b, h) pairs, using a single array that replaces the key-value store functionality.

Successors. To do so, for each removed bucket b, we are only storing its *successor*, *i.e.,* the bucket that replaced it in W. That is, we define an array K, such that its entry for each removed bucket b is K$[b]$=KV$[(b, L[b])]$. We initiate $K[b] = b \;\forall b \in \{0, 1, \ldots, a-1\}$, as initially a working bucket b appears at $W[b]$ (*i.e.,* replaces itself). For example, in (13) we just remember that bucket 4 replaced bucket 1 (*i.e.,* K[1]=4), and then in (14) that bucket 3 replaced bucket 0 (*i.e.,* K[0]=3). This yields,

$$\text{K} = \quad \boxed{\mathbf{3}}\boxed{\mathbf{4}}\boxed{2}\boxed{3}\boxed{4}\boxed{5}\boxed{6}. \tag{15}$$

We next show that we can use this information to reconstruct the individual KV entries used by the reduced-memory footprint implementation. Our key observation is that when trying to resolve KV$[(b, h)]$, we are actually searching for $W[h]$ just after b's removal. Therefore, we can trace through the history of $W[h]$, until we reach $\mathcal{W}_b[h]$. We start from h, which is the initial value of $W[h]$. When bucket h was removed, $W[h]$ was updated to its successor, *i.e.,* K$[h]$. Similarly, when K$[h]$ was removed, it was updated to its successor as well, *i.e.,* K[K$[h]$], and so on. Accordingly, we iteratively set $h \leftarrow$ K$[h]$, until we reach the first working bucket at $W[h]$ just after b's removal. We determine the stopping condition by looking at the sizes of $\mathcal{W}_b$ and $\mathcal{W}_h$: when $A[b] \geq A[h]$ we know that K$[h]$ was working when b was removed, and can terminate.

Example. Consider the example in (15). If in this state we further remove bucket 4, we obtain

$$\text{K} = \quad \boxed{3}\boxed{4}\boxed{2}\boxed{3}\boxed{\mathbf{2}}\boxed{5}\boxed{6}. \tag{16}$$

Namely, bucket 2 replaces bucket 4. Now, for example, assume an entering key hits bucket 4. We first examine $A[4]$. Since $A[4] = 3$, we know that bucket 4 is removed and rehash the key into the set $\{0, 1, 2\}$ (of size $A[4] = 3$). Assume that we obtained 1. We then examine $A[1]$ and see that it is removed as well with $A[4] = 2$. Now, we need to resolve KV$[(4, 1)]$, *i.e.,* to obtain the identity of $W[1]$ just after 4 was removed. To do so, we start at K$[1] = 4$. Since $A[4] \geq A[1]$, namely bucket 4 was not removed before bucket 1, we keep looking at K[K[1]] $=$ K$[4] = 2$. Since $A[2] < A[1]$ (in this example $A[2] = 0$) we establish KV$[(4, 1)] = 2$ and return bucket 2.

Complexity. Alg. 3 provides the pseudo-code for AnchorHash's final array-based implementation. The memory footprint for this solution is $O(|\mathcal{A}|)$ *independently of the system state*. *e.g.,* the number of removed buckets or their identity. The update time upon a bucket removal or addition accounts for $O(1)$ operations and is negligible for any $\mathcal{A}$ and $\mathcal{R}$. Note that $\Theta(|\mathcal{A}|)$ is required to save resource details (*e.g.,* server IP addresses).

While we already established bounds on the number of *hash operations*, we now provide an upper bound on the average number of *memory accesses* a key lookup requires when using our final minimal-memory implementation.

Theorem 4 (Memory accesses). *Assume* random *removals. Let $|\mathcal{W}| = w$ and $|\mathcal{A}| = a$. Denote by ξ the total number of memory accesses performed by* GETBUCKET(k) *for a randomly chosen key k when using the minimal-memory implementation. Then, the average of ξ is smaller than* $\left(1 + \ln\left(\frac{a}{w}\right)\right)^2$.

Proof Outline. Since this proof follows similar lines to the proof of Theorem 3 and due to space limits, we give a proof outline. We first prove that, given random failures, the average number of successors corresponding to each index in W is upper-bounded by $1 + \ln\left(\frac{a}{w}\right)$. Then, we show that given any key, the average number of memory accesses it requires is upper-bounded by the number of buckets it hits before terminating multiplied by our bound of $1 + \ln\left(\frac{a}{w}\right)$. We then use the law of total expectation together with Theorem 3 (that upper bounds the average number of buckets a key hits) to obtain the result. ☐

Finally, Table II summarizes the differences between the naive, reduced-memory, and the final minimal-memory implementations.

	Hash operations	Memory accesses	Memory	Update
Naive	$O(1+\log(\frac{a}{w}))$	$O(1+\log(\frac{a}{w}))$	$O(a+ar)$	$O(w)$
Reduced	$O(1+\log(\frac{a}{w}))$	$O(1+\log(\frac{a}{w}))$	$O(a+r^2)$	$O(w)$
Minimal	$O(1+\log(\frac{a}{w}))$	$O((1+\log(\frac{a}{w}))^2)$	$O(a)$	$O(1)$

TABLE II: AnchorHash implementation evolution: naive, then reduced-memory, then minimal-memory implementations. Successive implementations reduce the memory footprint to improve scalability, but the final implementation also slightly increases the guaranteed number of memory accesses.

V. Evaluation

Algorithms. In this section we test and compare AnchorHash to HRW, Ring, and MaglevHash, according to the evaluation metrics of Table I: consistency (*i.e.,* minimal disruption and balance), key lookup rate, memory footprint, and update time upon additions and removals.

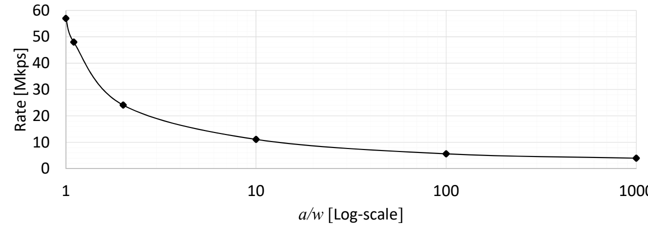
Testbed. All our experiments were conducted on a single core of a commodity machine with an Intel i7-7000 CPU at 3.6 GHz, 16 GB of RAM and an Ubuntu 16.04 LTS operating system. All algorithm implementations are in C++ and are optimized for run-time purposes. In our evaluation, each bucket has a 32-bit identifier (*i.e.,* up to 2^{32} buckets are supported), and we use 64-bit randomly-generated keys. For all algorithms we use the crc32 [22] hash function with two 64-bit inputs (key and seed) for uniform hashing.

Memory footprint. Before turning to empirical evaluation, we first discuss the memory footprint of the four approaches, as it has a significant impact on all other qualities such as key lookup rate and update time.

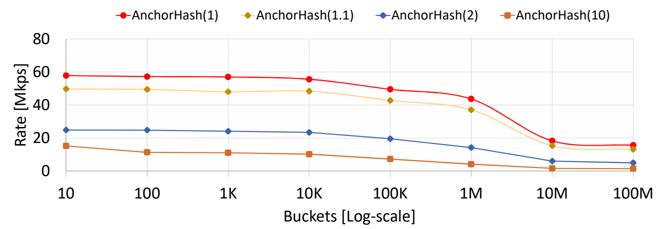
The memory footprint of Ring and MaglevHash depends on the theoretical hash-space balance guarantee these algorithms provide. For example, in MaglevHash, reaching a maximum of 1% hash space imbalance requires *at least* $\frac{1}{0.01} = 100$ copies for each resource. Throughout our evaluation, for MaglevHash and Ring we use 100 copies for each resource [1]. On the other hand, HRW and AnchorHash provide perfect hash-space balance and do not require copies to do so. That is, in our implementation, AnchorHash requires only 16 Bytes of memory per resource. This means that even for 10^6 resources, AnchorHash uses 16 MB of space, whereas MaglevHash requires at least 400 MB to achieve a reasonable balance for the same scenario.

Lookup rate. We test AnchorHash's key lookup rate for different Anchor sizes (up to 10^8) and different $\frac{a}{w}$ ratios (up to 10^3). For example, $w = 1{,}000$ and $\frac{a}{w} = 100$ means that only 1,000 resources are still active out of 100,000 (*i.e.,* a scenario with 99,000 random removals).

The results are depicted in Fig. 4. Fig. 4(a) shows the key lookup rate achieved by AnchorHash with 1,000 working buckets with respect to different $\frac{a}{w}$ ratios. Fig. 4(b) depicts AnchorHash rate with respect to the number of working buckets for different fixed $\frac{a}{w}$ ratios. Note that, even for a fixed $\frac{a}{w}$ ratio, the rate slightly decreases as the number of buckets increases. This is because of the increased percentage of L3 cache misses as follows from the increased memory footprint. Remarkably, even for a million buckets, AnchorHash achieves



(a) AnchorHash key lookup rate with 1,000 working buckets with respect to different a/w ratios.



(b) AnchorHash key lookup rate with respect to the number of working buckets for different fixed $\frac{a}{w}$ ratios. *AnchorHash(x)* stands for an AnchorHash instance with $\frac{a}{w} = x$.

Fig. 4: AnchorHash key lookup rate in millions of keys per second (Mkps).

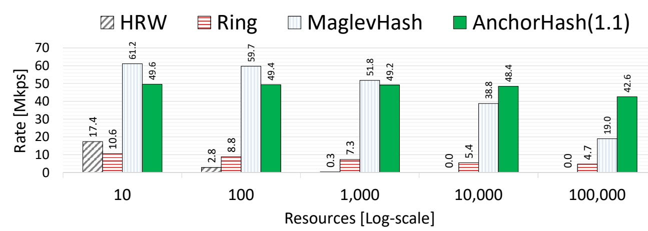


Fig. 5: Comparing the key lookup rates between HRW, Ring, MaglevHash and AnchorHash for different resource counts. Due to the significantly smaller memory footprint, AnchorHash maintains an extremely high rate even for 10^5 resources.

a rate of tens of millions of keys per second for reasonable and even extreme operating points (*e.g.,* half of the buckets have randomly removed).

Next, Fig. 5 compares the key lookup rates achieved by the four approaches. For AnchorHash, we depict a scenario with 10% random removals, which can hold as a lower bound for performance in most reasonable scenarios [1]. AnchorHash reaches an extremely high key lookup rate that is similar to MaglevHash, even though MaglevHash has abandoned full consistency to achieve this. Note that as the resource count increases, MaglevHash suffers from a more significant rate degradation due to increased L3 cache misses that stem from its much larger memory footprint.

We also tested the lookup rate of the four approaches using a backbone router CAIDA trace [23]. The results follow similar trends. Interestingly, all approaches run faster since the often reoccurring flow packets increase cache hit rate.

Balance. Essentially, there are three sources of imbalance, all reflected in an algorithm's load-balancing abilities: (1) hash space imbalance; (2) quality of the hash function; and (3) arriving keys. While the last two are implementation- and workload-dependent, the first is algorithm-dependent. Thus,

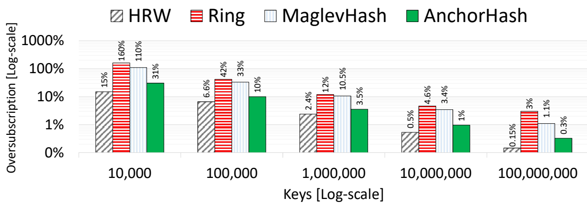


Fig. 6: Comparing worst-case oversubscription. Lower is better (better balance). All instances have 1,000 resources. For AnchorHash we have an Anchor of 1,100 buckets with 100 random removals.

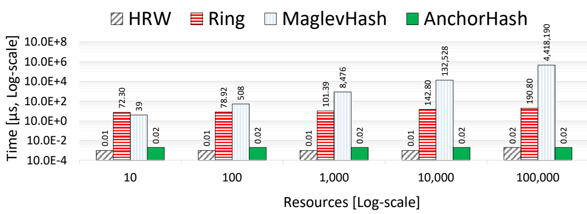


Fig. 7: Comparing update time for resource removals/additions. HRW and AnchorHash require only a few tens of nano-seconds independently of the size of the system. For Ring and especially MaglevHash, the update time increases with the size of the system. For example, for 10^5 resources, MaglevHash requires more than 4 seconds to repopulate its array.

in terms of balance, assuming uniform hashing, HRW and AnchorHash have an inherent advantage over MaglevHash and Ring. To demonstrate this, we tested the four approaches using the same hash function and a random stream of keys.

By standard practice [1] we measure the worst-case resource oversubscription in %. For instance, an oversubscription of 10% means that the most loaded resource has 10% more load than the average. All instances run with 1,000 resources. In AnchorHash, we use an Anchor of 1,100 buckets with 100 random removals. Ring and MaglevHash both run with 100 copies per resource. The results are depicted in Fig. 6. As expected, while both HRW and Ring approach zero as the number of keys increases, MaglevHash for example cannot go below 1.01 with 100 copies per resource.

Update time. We next test for the time it takes to update the data structure of each of the algorithms with a newly added or removed resource. The results are averaged over 100 trials, and depicted in Fig. 7. Both HRW and AnchorHash respond in nanosecond scale disregarding the size of the system. On the other hand, Ring and MaglevHash respond slower as the system size increases. For example, with 10^5 resources, MaglevHash requires more than 4 seconds to respond.

Minimal disruption. We also test the minimal-disruption property for all approaches. Following theory, HRW, Ring and AnchorHash achieve the minimal-disruption property in practice as well. Unfortunately, MaglevHash fails to achieve minimal disruption and therefore is not fully consistent. For example, in a scenario with 900 resources and 100 consecutive resource additions, we find that *at each resource addition*, MaglevHash wrongfully reassigns a near-constant fraction of $\approx$ 0.6% of the hash space, *i.e.,* $\approx$ 0.6% of the keys are needlessly remapped at each of the 100 resource additions. While such *flips* may be acceptable when used together with key tracking (*e.g.,* connection tracking in datacenter load-balancing), they may not be acceptable in other systems such as cache servers.

VI. Conclusion

In this paper we have introduced AnchorHash, a new consistent hashing technique that guarantees full consistency, high lookup rates, low memory footprint, and fast update time. We have provided implementation details and strong theoretical guarantees for AnchorHash. We then conducted extensive evaluations comparing to existing algorithms. Evaluation results indicate that AnchorHash is the first scalable and fully-consistent hashing technique. It is capable of handling millions of resources while maintaining high key lookup rate, low memory footprint, and negligible update time upon resource additions and removals.

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