Home Assignment 3 part 2

Stopping times

Submit until 25 May

Stopping times:

1. Let M, N be stopping times on a filtration $(\mathcal{F}_n)$. Show that $\mathcal{F}_N$ is a σ-field. Show that N is measurable on $\mathcal{F}_N$. Assuming $M \leq N$ a.s., show that $\mathcal{F}_M \subset \mathcal{F}_N$. Show that if $N \equiv n$ then $\mathcal{F}_N = \mathcal{F}_n$.
2. If M, N are stopping times, show that so are $M \vee N$, $M \wedge N$. Is $M + N$ a stopping time? (Give a proof or a counterexample).
3. Let X_n, Y_n be positive integrable and adapted to $\mathcal{F}_n$. Suppose $E(X_{n+1}|\mathcal{F}_n) \leq X_n + Y_n$, with $\sum_1^\infty Y_n < \infty$ a.s. Prove that X_n converges to a finite limit. Hint: Let $N = \inf\{k : \sum_i^k Y_m > M\}$, and stop your supermartingale at time N.