Polarization: Beneficial for Visibility Enhancement?

Tali Treibitz and Yoav Y. Schechner
Department of Electrical Engineering
Technion - Israel Inst. of Technology, Haifa 32000, Israel
ttali@tx.technion.ac.il , yoav@ee.technion.ac.il

Abstract

When imaging in scattering media there is a need to enhance visibility. Some approaches have used polarized images in this context with apparent success. These methods take advantage of the fact that the path radiance (airlight) is partially polarized. However, mounting a polarizer attenuates the signal associated with the object. This attenuation degrades the image quality. Thus, a question arises: is the use of a polarizer worth the mentioned loss? The ability to see objects is limited by noise. Therefore, in this work we analyze the change in signal to noise ratio (SNR) following the use of a polarizer or a dehazing process. Typically, methods use either one polarized image (with minimum path radiance) or two polarized images corresponding to extrema of the path radiance. We show that if the only goal is signal discrimination over noise (and not color or radiance recovery) in haze, the use of polarization in both approaches is unnecessary: polarization rarely improves the SNR over an average of unpolarized images acquired under the same acquisition time. Nevertheless, under a single frame constraint, the use of a single polarized image is beneficial.

1. Introduction

Images taken in scattering media such as haze or water have limited visibility. Scattered light from the medium (*airlight*[1]) veils the imaged scene, reducing contrast. Airlight increases with distance, while the object signal is attenuated. Therefore, visibility significantly decreases with the distance. Moreover, photon noise increases with the object distance [21], further worsening visibility. Overcoming this visibility problem is important in a multitude of applications such as surveillance, remote sensing and navigation in haze.

In the field of computer vision, several methods have been proposed to enhance visibility in scattering media. These works differ in the acquisition process and in the post processing method. Some studies have suggested using multiple frames of the scene, taken in different weather conditions [1, 13] or different states of a mounted polarizer [11, 15]. Polarization has been considered helpful since the airlight is partially polarized.[2] This fact is also exploited by photographers, who sometimes take a single frame with a mounted polarizer, to increase vividness. Some methods attempt dehazing from a single unpolarized image [2, 7, 12, 19]. The recovery there is based on smoothness constraints [2, 19], prior knowledge about the scene [7, 18] or on user input [12].

The emergence of such a variety of methods seeking visibility enhancement raises questions regarding their performance, such as: which method can yield the best enhancement for a given acquisition time? Are there fundamental recovery limits? In this paper we focus on methods that use a polarizer. We ask: does a polarizer really improve visibility of objects in haze? If so, under which conditions and for which ranges? How many polarization-filtered frames are needed? We note that an analysis about the tradeoffs between modulating camera optics and acquiring multiple frames has been done by [3], in a different context.

To answer these questions, we quantitatively analyze the change in signal to noise ratio (SNR) induced by using a polarization-filtered frame or two, compared to the SNR in unpolarized frames. The SNR change is quantitatively related to visibility changes in Ref. [21]. Here we show that if the same acquisition time is allocated to each method, polarization rarely helps. However, allowing only a single (polarized/unpolarized) frame, polarization can help.

2. Background and Models

In haze, the measured image $I^{\rm total}(\mathbf{x})$ at pixel $\mathbf{x} = (x, y)$ is composed of two components

$$I^{\rm total}(\mathbf{x}) = D(\mathbf{x}) + A(\mathbf{x}) \ , \tag{1}$$

as we now detail [15]. The *direct transmission* is

$$D(\mathbf{x}) = L^{\rm object}(\mathbf{x})T(\mathbf{x}) \ . \tag{2}$$

Here $L^{\rm object}$ is the object radiance as if it was taken in a clear medium. The atmospheric transmittance is

[1] In the underwater realm, the scattered light is termed *backscatter* [20] or *veiling light*. In general, the term is *path radiance*. In this paper we use the term airlight for simplicity.

[2] Polarization is also used when the object is assumed to be polarized [22] or by polarizing the veiling light [23].

$$T(\mathbf{x}) = e^{-b(\mathbf{x})} \quad , \tag{3}$$

which depends on the optical depth $b = \beta z(\mathbf{x})$. The optical depth is a function of the distance z between the object and the camera. The optical depth is proportional to the atmospheric attenuation coefficient $\beta \in (0, \infty)$.

The second component in Eq. (1) is the airlight [15], which increases with object distance

$$A(\mathbf{x}) = A_\infty[1 - T(\mathbf{x})] \ . \tag{4}$$

Here A_∞ is the value of airlight at a non-occluded horizon. It depends on the haze and the illumination conditions.

Polarization-based methods exploit the natural partial polarization of airlight [9]. Consider a scattering particle. Jointly with the line of sight, a ray from the light source to the scatterer defines a plane of incidence. The airlight is divided into two polarization components that are parallel and perpendicular to this plane, $A_\parallel$ and $A_\perp$, respectively, i.e., $A = A_\perp + A_\parallel$. The airlight degree of polarization (DOP) is then

$$p = (A_\perp - A_\parallel)/A \ . \tag{5}$$

The DOP p depends on the viewing and illumination directions and on the scattering particles. Typically, $A_\perp > A_\parallel$. Therefore, without loss of generality, in this paper we refer to $A_\parallel$ as having a minimal intensity of airlight and to $A_\perp$ as having a maximal intensity.

2.1. Imaging in Haze Via a Polarizer

Since airlight is partially polarized, imaging in haze through a polarizer decreases the acquired airlight and may thus improve the image. Throughout the paper, we refer to two images $I_\parallel$ and $I_\perp$ taken at two polarizer angles that correspond to $A_\parallel$ and $A_\perp$. Denote the exposure times for $I_\parallel$ and $I_\perp$ as $t_\parallel = \alpha_\parallel t$ and $t_\perp = \alpha_\perp t$, respectively. Here t is the exposure time when the scene is photographed by a single frame without a polarizing filter. The parameters $\alpha_\parallel$ and $\alpha_\perp$ represent the change of exposure time, relative to t. It is common to assume [15] that D is unpolarized. Therefore, in either $I_\parallel$ or $I_\perp$, an ideal polarizer attenuates D by half. Then, using Eq. (5), the components of a frame taken with a polarizer are[3]

$$\begin{aligned} D_\parallel = \alpha_\parallel D(\mathbf{x})/2 \quad &, \quad A_\parallel = \alpha_\parallel A(\mathbf{x})(1-p)/2 \\ D_\perp = \alpha_\perp D(\mathbf{x})/2 \quad &, \quad A_\perp = \alpha_\perp A(\mathbf{x})(1+p)/2 \ . \end{aligned} \tag{6}$$

The acquired images are then

$$\begin{aligned} I_\parallel = D_\parallel + A_\parallel = \alpha_\parallel \left[D(\mathbf{x}) + A(\mathbf{x})(1-p) \right]/2 \\ I_\perp = D_\perp + A_\perp = \alpha_\perp \left[D(\mathbf{x}) + A(\mathbf{x})(1+p) \right]/2 \ . \end{aligned} \tag{7}$$

[3]The calculations assume a linear camera response.

2.2. Dehazing Using a Polarized Image Pair

In polarization-based dehazing [15], the idea is that using $I_|$, $I_\perp$ and an estimate of p, the airlight can be estimated

$$\hat{A}(\mathbf{x}) = [I_\perp(\mathbf{x})/\alpha_\perp - I_\parallel(\mathbf{x})/\alpha_\parallel]/p \ . \tag{8}$$

Here, the raw images $I_\parallel$ and $I_\perp$ are normalized by $\alpha_\parallel$ and $\alpha_\perp$, respectively. This way, $\hat{A}(\mathbf{x})$ has the same graylevel scale as the unpolarized image I^{total} in Eq. (1). The estimated airlight can then be removed from the unpolarized image to yield an estimation of D

$$\hat{D}(\mathbf{x}) = I^{\text{total}}(\mathbf{x}) - \hat{A}(\mathbf{x}) \ . \tag{9}$$

Combining Eqs. (1,7-9) yields

$$\hat{D}(\mathbf{x}) = \frac{I_\parallel(\mathbf{x})}{\alpha_\parallel}\frac{1+p}{p} - \frac{I_\perp(\mathbf{x})}{\alpha_\perp}\frac{1-p}{p} \ . \tag{10}$$

Due to Eq. (9), the noise in $\hat{D}$ is coupled to noise in the airlight $\hat{A}$. Thus, previous works [6, 14] have suggested denoising the airlight before using it in Eq. (9), in order to improve the SNR in $\hat{D}$. These works rely on the assumption that the airlight is naturally smooth, and therefore can be denoised by smoothing without significant loss of information. Denote the denoised airlight as $\hat{A}^{\text{denoised}}$. Then, the direct component is estimated [6, 14] by

$$\hat{D}^{\text{smooth}A}(\mathbf{x}) = I(\mathbf{x}) - \hat{A}^{\text{denoised}}(\mathbf{x}) \ . \tag{11}$$

Using either $\hat{D}$ or $\hat{D}^{\text{smooth}A}$, dehazing compensates for the attenuation, e.g.,

$$\hat{L}^{\text{object}}(\mathbf{x}) = \hat{D}(\mathbf{x})/\hat{T}(\mathbf{x}) \ . \tag{12}$$

An accurate $\hat{T}$ (e.g., based on a known distance map) equally scales the noise levels in $\hat{L}^{\text{object}}(\mathbf{x})$ and in $\hat{D}(\mathbf{x})$. Thus, the SNR in $\hat{D}$ is an upper bound for the SNR of $\hat{L}^{\text{object}}$. Any limitation to the SNR in $\hat{D}$ is certain to apply to the SNR in $\hat{L}^{\text{object}}(\mathbf{x})$. Consequently, when referring to SNR in dehazing, we refer to $\hat{D}$.

2.3. The Noise Model

Photon noise is a fundamental quantum-mechanical effect. It cannot be overcome, regardless of the camera quality. Accounting for this noise component [4, 24], the overall noise variance[4] in the raw image data [17] is approximately

$$\sigma^2 = \kappa^2(t) + I(\mathbf{x})/g. \tag{13}$$

Here $\kappa^2(t)$ encompasses the variance of the signal-independent components of the gray-level noise, as detailed

[4]The affine relation in Eq. (13) does not hold [8, 10] for some cameras. However, this analysis is targeted at fundamental bounds, which are set by the use of high quality cameras, where Eq. (13) is typically followed.

in [4, 16, 24]. One of these components is *dark noise*, which depends on t.

The signal-dependent photon noise is characterized by g, the number of photo-generated electrons required to change a unit gray-level [16]. These parameters depend on the sensor and camera settings, such as ISO. For example, calibration of Nikon D100 at ISO 200 and $t = 1/200$sec yielded $\kappa^2 = 210$ and $\sigma^2(V) \approx 6000$, where V is the saturation graylevel of the camera. An important observation based on such values is that in bright areas, photon noise dominates over the signal independent noise, i.e.,

$$I/g \gg \kappa^2 \quad . \tag{14}$$

Eq. (14) is valid in many cameras [4, 16, 24]. In this work we concentrate on visibility limits, in very poor visibility domains. There, high airlight leads to high values of I. In these cases, Eq. (14) holds in Eq. (13). Therefore, from now on, the following expression is used for the noise variance:

$$\sigma^2 \approx I(\mathbf{x})/g \quad . \tag{15}$$

Here we understand the implication of A. Being non-negative, A increases I, sometimes significantly [21]. Thus, A increases the variance of the *random* noise (Eqs. 13,15), without increasing the desired signal. Visibility deteriorates under high noise levels [21]. Therefore, an increase in A harms the ability to distinguish objects.

3. The Airlight Dominance

As discussed in Sec. 2.3, the airlight increases the random noise *without* contributing to the signal. Thus, it is important to examine the magnitude of airlight relative to the direct component. From Eqs. (2,4),

$$\frac{A}{D} = \frac{A_\infty(1-T)}{L^{\text{object}}T} = \frac{A_\infty}{L^{\text{object}}}(e^b - 1) \quad . \tag{16}$$

Thus, as the optical depth increases, the airlight becomes more dominant [5]. The dominance depends on

$$\mu \equiv A_\infty / L^{\text{object}} \quad . \tag{17}$$

According to [5], $A/D = 1$ for objects that have reflectance similar to a medium gray card at $b = 0.2$. Using these relations in Eqs. (16,17) implies that for a medium gray object $\mu \approx 4.5$. From now on, we use this value as an anchor for assessing typical scenarios. When a polarizer is used, the airlight is less dominant than in Eq. (16),

$$A_\parallel / D_\parallel = \mu(e^b - 1)(1-p) \quad . \tag{18}$$

To gain an intuition on how polarization affects the dominance of A, Fig. 1 plots Eqs. (16,18) for $\mu = 4.5$ and two values of p. According to [9], in nature $p \leq 0.75$. Therefore, higher values of p are not plotted. We can see in Fig. 1 that even at large p values, the airlight A exceeds D at distances having $b > 0.75$.

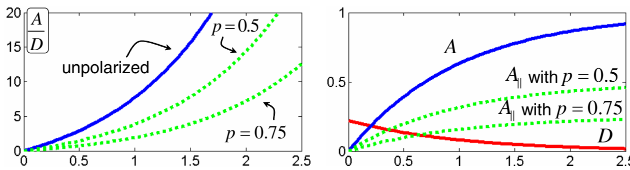

Figure 1. Plots of Eqs. (16,18). Excluding close ranges, airlight dominates over D even in polarized images where p is relatively high.

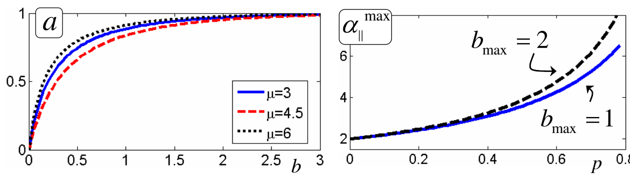

Figure 2. [Left] The value of a (Eq. 20) increases rapidly as a function of b. Here, $\mu = 4.5$ represents a medium gray object. [Right] The value of $\alpha_\parallel^{\max}$ as a function of p and $b_{\max}$ (Eq. 21).

For the analysis in the following sections, we define the relative airlight as

$$a(\mathbf{x}) \equiv A(\mathbf{x})/I(\mathbf{x}) \;\; , \;\; 0 \leq a \leq 1 \quad . \tag{19}$$

Combining Eqs. (1,16,19),

$$a(b, \mu) = 1 / \left[1 + \frac{1}{\mu(e^b - 1)}\right] \quad . \tag{20}$$

Fig. 2[Left] demonstrates that a saturates rapidly with b. For example, for $\mu = 4.5$, $a = 0.88$ already at $b = 1$. As demonstrated in Fig. 2[Left], μ has only little influence on a. Therefore, from now on we omit μ, for simplicity.

Automatic Exposure in Haze

When using automatic exposure time settings, the camera seeks well-exposed results. A mounted polarizer attenuates part of the light, and therefore the automatic exposure time is expected to increase. Since the SNR in the polarized image increases with the exposure time, we need to calculate what is the maximum possible increase in exposure in a polarized image. In App. A, we show that the relative exposure increase in $I_\parallel$ has an upper bound

$$\alpha_\parallel^{\max} = \frac{2}{1 - a(b_{\max})p} \quad , \tag{21}$$

where $b_{\max}$ is the maximal optical depth in the field of view. Fig. 2[Right] plots $\alpha_\parallel^{\max}$. For $p = 0$, $\alpha_\parallel^{\max} = 2$, since the polarizer simply blocks half the energy. But, for $p > 0$, Fig. 2 shows that $\alpha_\parallel^{\max} > 2$, for all $b_{\max}$.

Similarly, in $I_\perp$,

$$\alpha_\perp^{\max} = \frac{2}{1 + a(b_{\max})p} \quad . \tag{22}$$

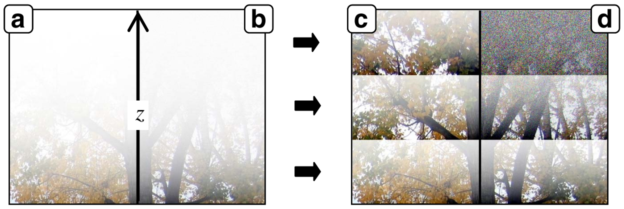

Figure 3. (a) A noise-free hazy image, simulated by linearly changing $b \in [0.2, 6]$. Airlight acts as local bias. (b) A slightly noisy version, simulated using the noise parameters of Nikon D100 at ISO 200. (c) Regional contrast stretch of (a) reveals the objects and details. (d) Regional contrast stretch of (b) does not recover small details at large b over noise, as explained in [21].

Recall that $p \geq 0$. Thus, Eqs. (21,22) imply that $1 \leq \alpha_\perp^{\max} \leq 2$, $\alpha_\perp^{\max} \leq \alpha_\parallel^{\max}$. We use Eqs. (21,22) in Secs. 5 and 6, to calculate the SNR when polarization-filtered frames are well exposed.

4. SNR in Haze

Without noise, even a small intensity change over a background could be stretched to reveal objects and details. Moreover, haze can be removed also by using a physical model [12, 15], sometimes using prior scene knowledge [7]. Figs. 3a,c demonstrate piecewise contrast-stretch on a synthetic utopian noise-free hazy image [21]. Even in parts that appear blank in Fig. 3a, visibility is retrieved in Fig. 3c. These parts correspond to more distant scene regions, where the accumulated airlight is higher. Yet, actions such as contrast stretching affect both the signal and the noise. This is demonstrated in Fig. 3b. There, noise following the model in Eq. (15) is introduced. Now, objects are lost in the parts corresponding to distant regions [21], despite regional contrast stretch in Fig. 3d. Thus, the main degradation in haze is increase of noise, and not the deterministic airlight.

Typically, there is interest to distinguish objects, e.g., cars, over a nearby background, or distinguish finer details, e.g., digits over a license plate. The ability to distinguish an object/detail depends on the radiance difference between the object and its background. It also depends on the amount of noise. Let L^{back} be the object's background radiance, in ideal, undisturbed conditions. Due to attenuation in Eq. (2), the difference in their image values corresponds to $D - D^{\text{back}}$.

We use the goal of object-vs-background distinction in order to define the *signal* [21] of interest[5]

$$S(\mathbf{x}) = D(\mathbf{x}) - D^{\text{back}}(\text{near } \mathbf{x}) \ , \tag{23}$$

[5] Other definitions of the signal may be suggested. As long as the signal following these definitions remains linearly dependent on the acquisition time (e.g., the maximum value in the image or the image STD), the results presented in the next sections do not change: the defined signal cancels out in the SNR comparison.

in the problem of differentiating an object over a background. Thus, we define the SNR as the ratio of Eq. (23) with the noise standard deviation (STD). Based on Eq. (15), this SNR is

$$R_{\text{unpolarized}}(t) = \frac{S(\mathbf{x})}{\sqrt{[D(\mathbf{x}) + A(\mathbf{x})]/g}} \ . \tag{24}$$

Constraining the Acquisition Time

Let us distinguish between exposure time and acquisition time:

Definitions: *Exposure time* is the time span at which the camera detectors are exposed when acquiring a *single* frame. *Acquisition time* spans the whole imaging process, which generally consists of multiframe acquisition.

For a fair SNR comparison, different methods should be examined under an *equal acquisition time*. Suppose a polarization-based method requires a total acquisition time of $\alpha_{\text{tot}} t$, where $\alpha_{\text{tot}} > 1$. The same time has to be allocated for unpolarized frame acquisition. Recall that an unpolarized frame is well-exposed at exposure time of t. Thus, this time cannot be simply increased to utilize the available extended time $\alpha_{\text{tot}} t$. Such an increase would lead to saturation. The solution is to use the available time to acquire multiple frames, with exposure times that sum up to $\alpha_{\text{tot}} t$. These frames are then scaled by the exposure time and averaged to yield a single image.

This procedure may increase the resulting SNR, if time is properly allocated to the frames. What is the best way for dividing the available time? It is easy show that when photon noise dominates (Eq. 14), the best way for noise reduction is to divide the available time $\alpha_{\text{tot}} t$ to equal units. Then, each of the averaged frames has the same exposure time. The resulting SNR in the averaged image increases by $\sqrt{\alpha_{\text{tot}}}$:

$$R_{\text{unpolarized}}(\alpha_{\text{tot}} t) = \sqrt{\alpha_{\text{tot}}} \frac{S(\mathbf{x})}{\sqrt{(A(\mathbf{x}) + D(\mathbf{x}))/g}} \ . \tag{25}$$

5. Is There a Benefit in Using $I_\parallel$?

As discussed in Sec. 2, the airlight in the polarization filtered image $I_\parallel$ is weaker. Therefore, polarization may improve the SNR in the acquired image. Following Eqs. (6,23)

$$S_\parallel(\mathbf{x}) = D_\parallel(\mathbf{x}) - D_\parallel^{\text{back}}(\text{near } \mathbf{x}) \ , \tag{26}$$

since the object is assumed to be unpolarized. Using Eqs. (6,7,15,24,26), the SNR in $I_\parallel$ is

$$R_\parallel(\alpha_\parallel t) = \frac{S_\parallel}{\sqrt{(D_\parallel + A_\parallel)/g}} = \frac{S\sqrt{g\alpha_\parallel/2}}{\sqrt{[D + A(1-p)]}} . \tag{27}$$

Now we compare $R_\parallel$ to the SNR in the unpolarized image.

5.1. Using $I_{\parallel}$ under Equal Acquisition Time

In this section we compare the SNR in the polarization-filtered frame $I_{\parallel}$ to the SNR in an unpolarized image (or frame average), taken in the same acquisition time $\alpha_{\text{tot}} = \alpha_{\parallel}$. The SNR change induced by a polarizer is

$$C_{\parallel} \equiv \frac{R_{\parallel}(\alpha_{\parallel} t)}{R_{\text{unpolarized}}(\alpha_{\parallel} t)} \quad . \qquad (28)$$

From Eqs. (6,25-28),

$$C_{\parallel} = 1/\sqrt{2[1 - a(b)p]} \quad . \qquad (29)$$

Eq. (29) does not depend[6] on the specific camera noise parameters κ and g, i.e., it is valid for *every* sensor and settings that adhere to the assumptions in Eqs. (13,14).

Fig. 4[left] plots $C_{\parallel}$ as a function of p for two values of b. Recall from Eq. (20) that a increases with b. Thus, $C_{\parallel}$ in Eq. (29) monotonically increases with p and b. Therefore, the potential for improvement in the distant scene parts is the most significant. As the SNR change depends on the object distance, it *varies* across the image.

For the purpose of distinguishing a signal out of noise, when is it beneficial to use a polarizer? If $C_{\parallel} > 1$, the polarizer yields SNR improvement, i.e., better visibility [21]. If $C_{\parallel} < 1$, it means that mounting a polarizer only worsens the situation and the visibility is harmed. Based on Eq. (29), $C_{\parallel} > 1$ if the airlight DOP satisfies

$$p > 1/[2a(b)] \quad . \qquad (30)$$

Specifically, in distant regions, $a \approx 1$. Then, Eq. (30) leads to:

Conclusion 1: *Under the constraint of equal acquisition time, using a polarizer is beneficial for enhancing the SNR only when* $p > 0.5$.

Following [9], $p \leq 0.75$. Thus, a polarizer is beneficial only for $a > 2/3$, which corresponds to an optical depth of $b = 0.36$. Therefore,

Conclusion 2: *For any DOP, in the range* $b < 0.36$*, it is never worth using a polarizer, for enhancing the SNR under the constraint of equal acquisition time.*

Fig. 5 shows an example from an experiment. Both image I^{total} and $I_{\parallel}$ were taken using the same acquisition time $\alpha_{\text{tot}} t = 1/250$sec. Both are contrast stretched. The DOP is $p \approx 0.2$. The noise in $I_{\parallel}$ is higher, thus deteriorating the ability to discriminate objects.

5.2. Single Frame Constraint

The analysis leading to Eq. (29) sets equal acquisition times, i.e., it assumes that it is possible to take multiple unpolarized frames, optimally allocate the exposure time to

[6]Apparently Eq. (29) is singular at $p = 1$ if $a = 1$. However, $p = 1$ is outside the natural domain, as detailed in Sec. 5.3.

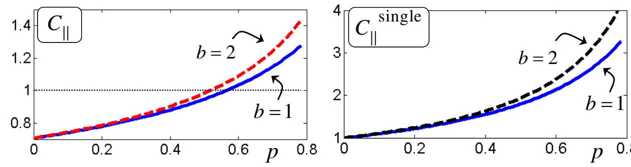

Figure 4. $C_{\parallel}$ and $C_{\parallel}^{\text{single}}$ as a function of p, based on Eqs. (29) and (34). Under a single-frame constraint, if the distances are uniform, it is always beneficial to use a polarizer, since $C_{\parallel}^{\text{single}} > 1$.

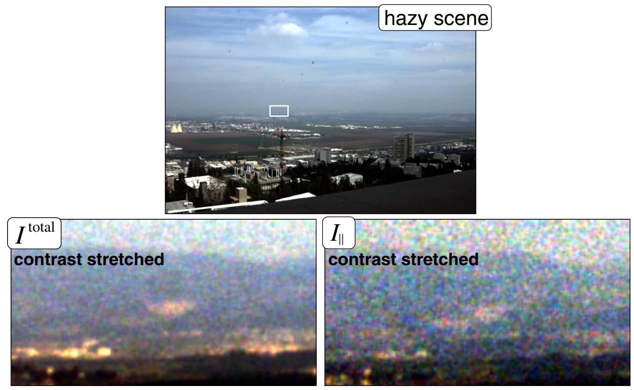

Figure 5. [Top] Hazy scene, $p \approx 20\%$. [Bottom] A distant region, cropped from the FOV. Both cropped frames were photographed in the same acquisition time $\alpha_{\text{total}} t = 1/250$sec. [Left] An average of two unpolarized images, each exposed at $1/500$sec. [Right] A single polarized image, $p \approx 20\%$, exposed at $1/250$sec. The polarized image is noisier, and as a result, fine details are lost.

each frame and finally average the frames, as detailed in Sec. 4. However, this acquisition mode is not always applicable. For example, when shooting video, the acquisition interval is constrained to the video frame rate, e.g., $30[Hz]$. Only a single frame can be taken at each acquisition interval. However, the exposure time of each individual frame varies within the bound of the video period.

Thus, now we examine the SNR gain when we use just a single unpolarized frame

$$C_{\parallel}^{\text{single}} \equiv \frac{R_{\parallel}(\alpha_{\parallel} t)}{R_{\text{unpolarized}}(t)} \quad . \qquad (31)$$

As shown in App. A, mounting a polarizer enables an increase of the exposure time without saturation. Then, the benefit of mounting a polarizer is higher than in Sec. 5.1, as we now show. From Eqs. (6,24,31)

$$C_{\parallel}^{\text{single}} = \sqrt{\frac{\alpha_{\parallel}}{2[1 - a(b)p]}} \quad . \qquad (32)$$

The maximum possible SNR results from the longest exposure time, $\alpha_{\parallel}^{\max}$, which is given in Eq. (21). Plugging Eq. (21) in Eq. (32) yields

$$C_{\parallel}^{\text{single}} = \{[1 - a(b_{\max})p][1 - a(b)p]\}^{-\frac{1}{2}} \quad , \qquad (33)$$

where $b_{\max}$ is the maximal optical depth in the field of view. As expected, for $p = 0$, no gain is achieved.

If the object distances in the field of view are approximately uniform, the haze effects are approximately uniform. Then, contrast stretch counters these effects. At a uniform range, $a(b_{\max}) \approx a(b)$. Then, Eq. (33) becomes

$$C_{\|}^{\text{single}} = 1/[1 - a(b_{\max})p] \ . \tag{34}$$

Fig. 4[right] plots $C_{\|}^{\text{single}}$ given by Eq. (34), as a function of p. Here it is always beneficial to use a polarizer.

Conclusion 3: *Using just a single frame, polarization is beneficial in a FOV having a uniform distance, with a flexible exposure time.*

Polarization can be beneficial also in a different way. Instead of increasing the SNR, polarization can be used to decrease the exposure time, i.e., achieve

$$\alpha_{\|} < 1 \ . \tag{35}$$

This is advantageous, for example, when imaging objects in motion. To avoid worsening the SNR ($C_{\|}^{\text{single}} < 1$), Eq. (32) requires

$$\alpha_{\|} > 2[1 - a(b)p] \ . \tag{36}$$

At long distances (where polarization has the best potential), $a \approx 1$ (Fig. 2[Left]). There, to satisfy both Eqs. (35) and (36), we need $p \geq 0.5$.

Conclusion 4: *Using just a single frame, when $p \geq 0.5$, polarization is beneficial for shortening the exposure time.*

5.3. Valid Range

Note that plugging $p = 1$, $a = 1$ in Eqs. (29,32) apparently results in $C_{\|} \to \infty$. This happens because following Sec. 3, the assumption of airlight dominance in the image $I_{\|}$ is valid for $p < 0.75$. The value of $p > 0.75$, and specifically $p = 1$, does not appear in natural haze and the analysis in Sec. 3 does not hold there. Moreover, acquiring a polarized image with $p = 1$ means that only D exists in $I_{\|}$. At $a = 1$, the limit is $D \to 0$, which means that the signal is dark. In these conditions, Eq. (14) is invalid. Therefore, we stress that Eqs. (29,32) and also the results in the subsequent sections are under three assumptions: $p \leq 0.75$; domination by airlight; and Eq. (14).

6. Is There a Benefit in Dehazing?

Consider polarization-based dehazing within a fixed acquisition time. In this section we show that this rarely results in SNR gain.

6.1. Pointwise Recovery

In this section we examine the benefit of using pointwise dehazing (Eq. 10), i.e., no spatial denoising methods are applied to the airlight. To find the SNR in the dehazed image, we need the noise variance in $\hat{D}(\mathbf{x})$. Here the noise in $I_{\|}$ and $I_{\perp}$ is uncorrelated. Based on Eq. (10), a first order approximation [14, 20] yields,

$$\sigma^2_{\hat{D}(\mathbf{x})} = \left(\frac{\sigma_{\|}}{\alpha_{\|}}\right)^2 \left(\frac{1+p}{p}\right)^2 + \left(\frac{\sigma_{\perp}}{\alpha_{\perp}}\right)^2 \left(\frac{1-p}{p}\right)^2 \ , \tag{37}$$

where $\sigma_{\|}$ and $\sigma_{\perp}$ are the noise variances in $I_{\|}$ and $I_{\perp}$, respectively. Let C_{dehaze} be the SNR change due to polarization-based dehazing. Using Eq. (37) and a derivation in App. B,

$$C_{\text{dehaze}} = \sqrt{2/\alpha_{\text{tot}}}\, p \left\{ \left[\frac{(1+p)^2}{\alpha_{\|}} + \frac{(1-p)^2}{\alpha_{\perp}}\right] - pa(b)\left[\frac{(1+p)^2}{\alpha_{\|}} - \frac{(1-p)^2}{\alpha_{\perp}}\right]\right\}^{-\frac{1}{2}} \ , \tag{38}$$

for any relative exposure times $\alpha_{\|}$ and $\alpha_{\perp}$. Now, we explore two choices for $\alpha_{\|}$ and $\alpha_{\perp}$.

6.1.1 Equal Exposure Times

A convenient choice for the acquisition process is keeping the exposure time constant $\alpha \equiv \alpha_{\perp} = \alpha_{\|}$. Here, the total acquisition time is $\alpha_{\text{tot}} t = 2\alpha t$. Then, Eq. (38) yields

$$C_{\text{dehaze}}^{\text{same}} = \frac{p}{\sqrt{2\{1 + p^2[1 - 2a(b)]\}}} \ . \tag{39}$$

Note that $p = 0$ yields $C^{\text{dehaze}} = 0$. This happens because for $p = 0$, Eq. (10) yields $\hat{D}(\mathbf{x}) = 0$, i.e., the signal is completely deleted by the algorithm.

Now, is it beneficial to dehaze based on two polarization-filtered frames and Eq. (10)? In other words, can we obtain SNR improvement, i.e., $C_{\text{dehaze}}^{\text{same}} > 1$? Examining Eq. (39), this occurs only if

$$p > \sqrt{2/[4a(b) - 1]} \ . \tag{40}$$

Since $a \leq 1$, Eq. (40) means that even in the most distant ranges, improvement is achieved only for $p > 0.8$, but this value is outside the natural domain.

Conclusion 5: *For SNR improvement, it is* never *beneficial to use pointwise polarization-based dehazing (Eq. 10) with equal exposure times.*

This conclusion is demonstrated by an example in an experiment. Fig. 6 shows I^{total} and $\hat{D}$ of the same distant scene. Both are contrast stretched. The DOP is $p \approx 0.24$. The exposure times are equal, $\alpha_{\|} t = \alpha_{\perp} t = 1/200$sec, such that $\alpha_{\text{tot}} t = 1/100$sec. The noise in $\hat{D}$ is higher, thus deteriorating the ability to discriminate objects. In I^{total}, more objects are seen, such as the mountain ridge in the background.

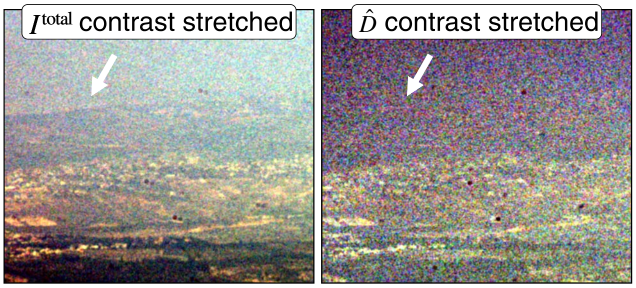

Figure 6. A real world experiment, $p \approx 0.24$. I^{total} and D from Eq. (10), both are contrast-stretched. Images were taken with Nikon D100, ISO 200, $t_{\|} = t_{\perp} = 1/200$sec.

6.1.2 Automatic Exposure

We now examine the choice of using the maximal possible exposure times (Eqs. 21,22) in each of the frames. Here

$$\alpha_{\text{tot}} = \alpha_{\|}^{\max} + \alpha_{\perp}^{\max} = 4/\{1 - [pa(b_{\max})]^2\} \quad . \tag{41}$$

Recall that for a scene with approximately uniform distances, $a(b_{\max}) \approx a(b)$. Then, Eqs. (38,41) yield

$$C_{\text{dehaze}}^{\text{auto}} = \frac{p}{\sqrt{1 - [pa(b_{\max})]^2}} \quad . \tag{42}$$

The value of $C_{\text{dehaze}}^{\text{auto}}$ given in Eq. (42) monotonically increases with a and p. Thus, the SNR gain is maximal when $p = 0.75$, $a = 1$. However, there $C_{\text{dehaze}}^{\text{auto}} = 1.11$, which is a rather small gain. Moreover, practically, $C_{\text{dehaze}}^{\text{auto}} > 1$ only in rare scenarios. Since $a \leq 1$, Eq. (42) yields $C_{\text{dehaze}}^{\text{auto}} > 1$ only for $p > 0.7$. This is a very high threshold for p, which is rarely passed. To sum up, dehazing with automatic exposure times is beneficial only for very high levels of p, and also then the gain is very small, and thus questionable.

Conclusion 6: *With optimal exposure times, pointwise dehazing results in SNR gain only in rare cases.*

6.2. Denoised Airlight

Sec. 6.1 did not take into account noise reduction operations. In this section we consider the benefit of using a denoised airlight for the estimation of D, as in Eq. (11) and in single-image methods such as [7], where A is estimated independently. Let the noise in $\hat{A}^{\text{denoised}}$ be negligible due to denoising as in [6, 14]. Then, the noise in $\hat{D}^{\text{smooth}A}$ equals the one in I^{total} (Eqs. 1,11). Polarization-based dehazing by Eq. (11) thus does not improve the SNR, relative to acquisition of unpolarized frames in the same period.

Let us examine if we can achieve an even better SNR. Let $\hat{A}^{\text{denoised}}$ be subtracted from $I_{\|}$, as opposed to Eq. (11), in which $\hat{A}^{\text{denoised}}$ is subtracted from I^{total}:

$$\hat{D}_{\|}^{\text{smooth}A} = I_{\|} - \hat{A}^{\text{denoised}}(1-p)\alpha_{\|}/2 \quad , \tag{43}$$

based on Eq. (6). In this case, $R_{\|}$ is the upper bound for the SNR of $\hat{D}_{\|}^{\text{smooth}A}$. Then,

Conclusion 7: *The SNR of $\hat{D}_{\|}^{smoothA}$ matches the best of either $R_{\|}$ or $R_{\text{unpolarized}}$. Determination which is the better one is set by conclusions 1,2 and 3.*

7. Discussion

In this theoretical analysis we made a fundamental comparison of methods for imaging in haze. The comparison is done under constraints of acquisition time. In most cases, the analysis showed that using polarization is not beneficial in the context of distinguishing an object out of noise. Our conclusions *do not* contradict previous dehazing studies [11, 15]. Our analysis assumed that haze can be treated by regional contrast stretch (see Sec. 4 and Fig. 3). This is applicable only in regions having rather uniform distances. Moreover, regional contrast stretch using sliding windows often creates halos and does not restore colors. On the other hand, polarization-based dehazing does not need regional contrast stretch. Moreover, polarization-based dehazing yields an estimate of the attenuation map. This leads to attenuation compensation, recovering scene colors and a distance map.

This analysis applies to natural illumination in haze. Underwater, the DOP may be larger, which may change some results of the SNR analysis. An extension to artificial illumination needs to follow. The case of artificial illumination differs from natural illumination by the significant DOPs of the objects, higher DOP of the backscatter, and a different dependency of the backscatter on the range [20]. Therefore, future analysis of artificial illumination may yield different conclusions regarding the benefit of polarization.

A. Automatic Exposure

A mounted polarizer attenuates part of the light and therefore enables an increase in exposure time. In this appendix we calculate what is the possible increase in exposure time, while avoiding saturation. The exact expression for the possible change in exposure time depends on the specific scene. Therefore, this analysis yields a rough assessment, rather than an exact number for each case. The saturation graylevel is $V = 2^{\mathtt{B}} - 1$, where $\mathtt{B}$ is the number of bits per pixel. Denote the pixel coordinate vector where I reaches its maximum as $\mathbf{m}$, i.e., $\mathbf{m} = \operatorname{argmax}_{\mathbf{x}} I(\mathbf{x})$. If I is well exposed, it is almost saturated. Then,

$$V = I(\mathbf{m}) = A(\mathbf{m}) + D(\mathbf{m}) \quad . \tag{44}$$

Denote $b_{\max}$ as the maximal optical depth imaged in a scene. Recall that we look at a scene where airlight dominates. Airlight increases with distance. Thus the brightest pixel is assumed to be at an optical depth $b(\mathbf{m}) = b_{\max}$. Therefore, $A(\mathbf{m}) = a(b_{\max}, \mu_{\max})V$ in the unpolarized image, where $\mu_{\max}$ is the value of μ at the brightest pixel. Recall from Sec. 3 that μ has only little influence on a.

Now, let us use a polarizer. Also here, we seek an optimal exposure, i.e. the maximal exposure below image saturation. The brightest pixels in I and in $I_\parallel$ are located at the same coordinate, as long as they are inside the range where the polarized airlight dominates the direct component as well (Sec. 3). Thus, based on Eq. (7)

$$\max_{\mathbf{x}} I_\parallel = I_\parallel(\mathbf{m}) = \alpha_\parallel[V - pA(\mathbf{m})]/2 \tag{45}$$

$$\max_{\mathbf{x}} I_\perp = I_\perp(\mathbf{m}) = \alpha_\perp[V + pA(\mathbf{m})]/2 \ . \tag{46}$$

To avoid saturation, we require $\max_{\mathbf{x}} I_\parallel \leq V$ and $\max_{\mathbf{x}} I_\perp \leq V$. Then, combining Eqs. (44-46) yields Eqs. (21,22).

B. SNR Improvement in Dehazing

In this appendix we calculate the SNR gain obtained by pointwise dehazing. Using Eqs. (7,15), the noise variances $\sigma_\parallel^2$ and $\sigma_\perp^2$ are

$$\sigma_\parallel^2 = \frac{I_\parallel}{g} \approx \frac{\alpha_\parallel\left[D(\mathbf{x}) + (1-p)A(\mathbf{x})\right]}{2g} \ ,$$
$$\sigma_\perp^2 = \frac{I_\perp}{g} \approx \frac{\alpha_\perp\left[D(\mathbf{x}) + (1+p)A(\mathbf{x})\right]}{2g} \ . \tag{47}$$

Then, plugging Eq. (47) in Eq. (37) yields

$$\sigma_{\hat{D}}^2 = \frac{1}{2gp^2}\left\{(A+D)\left[\frac{(1+p)^2}{\alpha_\parallel} + \frac{(1-p)^2}{\alpha_\perp}\right] - pA\left[\frac{(1+p)^2}{\alpha_\parallel} - \frac{(1-p)^2}{\alpha_\perp}\right]\right\}. \tag{48}$$

Define the SNR in the dehazed image as

$$R_{\mathrm{dehaze}} \equiv S/\sigma_{\hat{D}} \ , \tag{49}$$

where S is the signal in the dehazed image (Eq. 23). Define the change in SNR caused by dehazing as

$$C_{\mathrm{dehaze}} \equiv R_{\mathrm{dehaze}}(\alpha_{\mathrm{tot}}t)/R_{\mathrm{unpolarized}}(\alpha_{\mathrm{tot}}t) \ . \tag{50}$$

Combining Eqs. (25,48-50) yields Eq. (38).

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