Two-slab all-optical spring

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It is demonstrated that a waveguide consisting of two dielectric slabs may become an all-optical spring when guiding a superposition of two transverse evanescent modes. Both slabs are transversally trapped in stable equilibrium due to the optical forces developed. A condition for stable equilibrium on the wavenumbers of the two modes is expressed analytically. The spring constant characterizing the system is shown to have a maximal value as a function of the equilibrium distance between the slabs and their width. © 2007 Optical Society of America

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Electromagnetic forces on neutral bodies may prove to be the foundation of a variety of future optomechanical systems. In addition to the vast research devoted to the manipulation of small particles by laser light,[1] one of the subjects that have been investigated is the trapping of a mirror in a stable equilibrium state by using radiation pressure. A typical configuration is that of a Fabry–Perot cavity consisting of two mirrors, where the radiation pressure on one of the mirrors may be balanced by an external mechanical force.[2,3] This scheme has been recently experimentally characterized as an optical spring.[4]

In contrast to the Fabry–Perot system where radiation is incident perpendicularly upon each mirror, waveguide eigenmodes propagate in the longitudinal direction while exerting pressure on the guiding structure in the transverse direction.[5–8] It was recently shown that two mirrors guiding light between them may experience both attractive and repulsive forces according to the transverse behavior of the mode they guide.[7] In fact, transverse propagating modes, namely, eigenmodes with real transverse wavenumbers, were found to always be repulsive. On the other hand, transverse evanescent modes may be repulsive or attractive, depending on whether their transverse fields are odd or even functions of the transverse coordinate, respectively.

When *both* an attractive and a repulsive mode are propagating in a two-mirror waveguide, the total transverse force may trap each mirror in a stable equilibrium state. To some extent, this effect resembles the optical binding of dielectric particles by scattered laser light.[9] Since only optical forces are responsible for the equilibrium, we may consider the system an all-optical spring. Such a stable equilibrium has been demonstrated for a waveguide consisting of two Bragg mirrors.[7] Although the diverse properties of Bragg mirrors may be exploited for controlling the radiation pressure in a two-mirror waveguide,[5,7] for the realization of an all-optical spring, Bragg reflection may not be necessary. Instead, the two transverse evanescent modes that are required may be guided by total internal reflection.

In this Letter the optical forces in a two-slab system consisting of two infinite lossless dielectric slabs, as illustrated in Fig. 1, are investigated. By using a total internal reflection mechanism, the transverse oscillations of the field in the Bragg reflector are avoided, and the radiation pressure effects for a given power are enhanced. It is demonstrated for the first time to our knowledge that this configuration may become an all-optical spring, and a general analytic expression for the condition for stable equilibrium to occur is developed, as well as an expression for the spring constant. In addition to the obvious advantage in implementation, the simplicity of the system allows us to gain insight into the stable equilibrium phenomenon.

Derivation of the dispersion relations in the two-slab system illustrated in Fig. 1 may be found in Ref. 10, and recently the propagation of light in this type of nano-waveguide was demonstrated.[11] In what follows, we focus on the lowest even TM mode (E_x is even) and the lowest odd TE mode (H_x is odd); both are assumed to be at wavelength λ_0 with corresponding angular frequency $\omega_0 = 2\pi c/\lambda_0$, propagating in the z direction. Obviously, in a practical device that has a finite size in the y direction, the modes are hybrid rather than being pure TE or TM. However, the larger this dimension is, the more accurate the present analysis becomes.

For each mode, the Maxwell stress tensor[12] is used to compute the force per unit area per unit power ($\mathrm{N m^{-2} W^{-1}}$) $\mathcal{F}_\mathrm{A}$ and $\mathcal{F}_\mathrm{R}$ for the attractive TM and for the repulsive TE, respectively; negative values of the force represent attraction, whereas positive values represent repulsion. Figure 2 shows contours of the two normalized forces as a function of the distance between the slabs D and the slabs' width Δ, for slabs of permittivity $\varepsilon_r = 3.45^2 \simeq 11.9$. The normalization is by $(c\lambda_0\Delta_y)^{-1}$, where Δ_y is the width in the y direction through which the power flows. It is seen that the

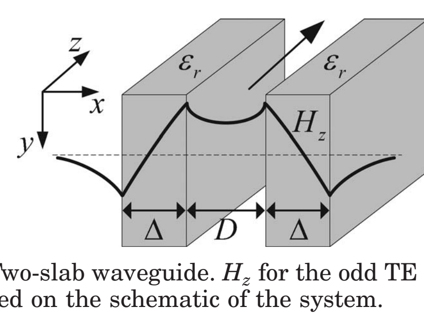

Fig. 1. Two-slab waveguide. H_z for the odd TE mode is superimposed on the schematic of the system.

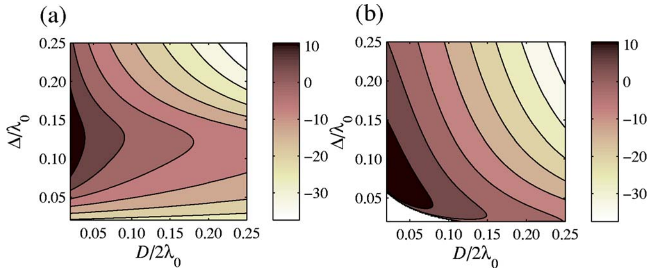

Fig. 2. (Color online) Contours of the transverse force per unit area per unit power in the two-slab waveguide normalized by $(c\lambda_0\Delta_y)^{-1}$ as a function of the slabs' width and the distance between the slabs. (a) Even TM attractive force $(-\mathcal{F}_A)$ (dB). (b) Odd TE repulsive force $\mathcal{F}_B$ (dB).

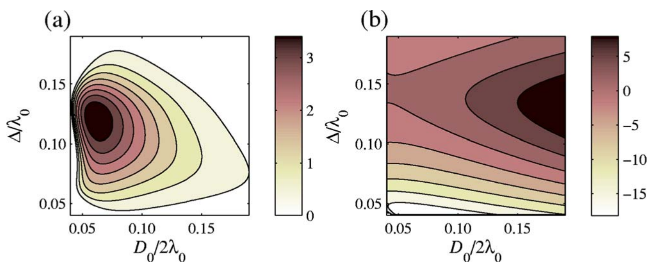

Fig. 3. (Color online) (a) Spring constant K of the two-slab waveguide normalized by $K_0 \triangleq (c\lambda_0^2\Delta_y)^{-1}$ as a function of the slabs' width and the equilibrium distance between the slabs. Negative values of K were replaced by zeros. (b) Power ratio α (dB) of the repulsive and attractive modes.

force generally tends to be stronger for shorter distances between the slabs.

Since there is no interaction between the TM and TE modes, if both are excited in the waveguide, the total force is the sum of each of their forces. Denoting the ratio between the power of the repulsive mode to the power of the attractive mode by $\alpha \triangleq P_{TE}/P_{TM}$, the total force per unit area per unit power is given by

$$\mathcal{F}_T(D) = [\mathcal{F}_A(D) + \alpha\mathcal{F}_R(D)]/(1+\alpha). \tag{1}$$

For some arbitrary distance between the mirrors D_0 to become an equilibrium point, i.e., $\mathcal{F}_T(D_0)=0$, the power ratio α must satisfy the equation

$$\alpha = -\mathcal{F}_A(D_0)/\mathcal{F}_R(D_0). \tag{2}$$

At the equilibrium point D_0, the spring constant may be defined by

$$K = -\left.\frac{d\mathcal{F}_T}{dD}\right|_{D=D_0}, \tag{3}$$

so that $K>0$ corresponds to stable equilibrium, whereas $K<0$ represents unstable states.

Using Eqs. (1)–(3), we may compute the values of K as a function of the slabs' width Δ and the equilibrium point D_0, assuming that the dependence of α on D is negligible. In Fig. 3, we focus on the range of parameters for which a stable equilibrium is obtained. The contours of the spring constant K normalized by $K_0 \triangleq (c\lambda_0^2\Delta_y)^{-1}$ are shown in Fig. 3(a), and the corresponding values of the power ratio α are presented in Fig. 3(b). A maximal value of the spring constant is obtained for $\Delta \simeq 0.12\lambda_0$ and $D_0 \simeq 0.12\lambda_0$, and the optimal spring constant is $K_{max}/K_0 \simeq 3.8$ with a power ratio $\alpha \simeq 1.25$. As a specific example, consider $\lambda_0 = 1.55\ \mu m$ and $\Delta_y = 10\lambda_0$, for which $K_{max} \simeq 3.4 \times 10^8\ Nm^{-3}\ W^{-1}$. Taking the density of Si, which is $\rho \simeq 2.3\ g/cm^3$, and total flowing power $P=1$ W, the corresponding mechanical resonance is $\sqrt{K_{max}P/(\rho\Delta)} \simeq 0.9$ MHz. Oscillation in such a relatively low frequency ensures that the light frequency is unchanged, thus justifying the above analysis under static conditions. In addition, transient phenomena leave the waveguide at the group velocity of the modes and consequently cannot build up in time.

As the permittivity of the slabs ϵ_r is increased, the forces of each mode become stronger, and with them the maximal spring constant that may be obtained. In Fig. 4(a), the maximal spring constant K_{max} is plotted as a function of ϵ_r, beginning with $\epsilon_r=2.1$, 11.9, and then 20–50 in increments of 10. In this range of ϵ_r, the spring constant increases from $K_{max}/K_0 \simeq 0.08$ for $\epsilon_r=2.1$ to $K_{max}/K_0 \simeq 36.3$ for $\epsilon_r = 50$. For each maximal value of K, the corresponding values of the slabs' width and the equilibrium distance are given in Fig. 4(b). As expected, these two geometric quantities decrease with ϵ_r.

Further insight into the effect of the spring may be gained by examining the expression for the force as a derivative of the energy. For one mode, the resulting force per unit area per unit power is given by[5,6,13]

$$\mathcal{F} = -\frac{1}{\omega_0\Delta_y}\frac{\partial\omega}{\partial D}\frac{W}{P}, \tag{4}$$

where W is the energy per unit length stored in the waveguide, P is the total flowing power, $\omega(\beta,D)$ is the angular frequency of the mode, and β is its longitudinal wavenumber. At a specific frequency ω_0, we have $\omega(\beta,D)=\omega_0$, and by differentiating this implicit function, the force may be expressed in the form

$$\mathcal{F} = -\frac{1}{\omega_0\Delta_y}\left(-\frac{\partial\beta}{\partial D}\frac{\partial\omega}{\partial\beta}\right)\frac{W}{P} = \frac{1}{\omega_0\Delta_0}\frac{\partial\beta}{\partial D}, \tag{5}$$

where the equality between the energy and the group velocities, i.e., $\partial\omega/\partial\beta = P/W$, was used. Hence the dispersion function $\beta(D)$ at ω_0 contains all the informa-

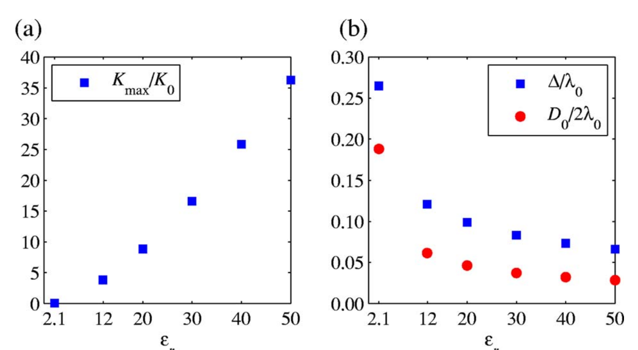

Fig. 4. (Color online) (a) Maximal spring constant of the two-slab waveguide as a function of the slabs' permittivity ϵ_r. (b) Slabs' width and equilibrium distance corresponding to the optimal spring, as a function of ϵ_r.

tion on the transverse pressure per unit power, and it is proportional to the negative of a potential function from which the force can be derived. Accordingly, for an attractive (even) mode $\beta'_{\mathrm{A}}<0$, and for a repulsive (odd) mode $\beta'_{\mathrm{R}}>0$, where the derivative with respect to D is denoted by a prime.

When both an attractive and a repulsive mode are present, as discussed above, the total force may be derived by $\mathcal{F}=\beta'_{\mathrm{T}}/\omega_0\Delta_y$, where $\beta_{\mathrm{T}}\triangleq(\beta_{\mathrm{A}}+\alpha\beta_{\mathrm{R}})/(1+\alpha)$. In Fig. 5(a), the functions β_{A}, β_{R}, and β_{T} for the optimal spring of $\epsilon_r=3.45^2$ are presented. The stable equilibrium region of β_{T} around $D_0\simeq 0.12\lambda_0$ is magnified in Fig. 5(b), showing a maximum which is, in fact, the minimum of the potential well in which the slab is trapped; the minimum of β_{T} represents an unstable equilibrium point. The higher the permittivity is, the deeper and narrower the potential well of the optimal spring becomes. For $\epsilon_r=50$, the potential well is about twice as narrow and twice as deep as the potential well of Fig. 5(b) ($\epsilon_r\simeq 11.9$).

It is now possible to obtain an explicit condition for stable equilibrium on behavior of the two modes' dispersion functions. For equilibrium at some separation D_0, the force must vanish, which requires that

$$\beta'_{\mathrm{A}}+\alpha\beta'_{\mathrm{R}}=0. \tag{6}$$

The equilibrium is stable if the slope is negative, so that

$$\beta''_{\mathrm{A}}+\alpha\beta''_{\mathrm{R}}<0, \tag{7}$$

or alternatively, the spring constant reads, using Eqs. (3), (5), and (6),

$$K=\frac{1}{\omega_0\Delta_y}\frac{\beta'_{\mathrm{A}}\beta''_{\mathrm{R}}-\beta'_{\mathrm{R}}\beta''_{\mathrm{A}}}{\beta'_{\mathrm{R}}-\beta'_{\mathrm{A}}}, \tag{8}$$

and $K>0$ for stable equilibrium points. The expression of Eq. (8) is among the important results of this

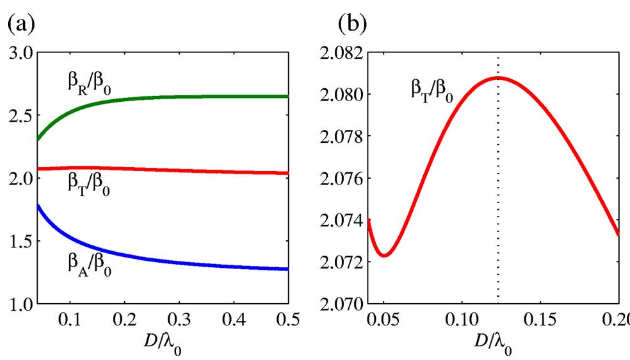

Fig. 5. (Color online) (a) Longitudinal wavenumber functions of the attractive mode β_{A}, the repulsive mode β_{R}, and their weighted sum β_{T}, normalized by $\beta_0\triangleq\omega_0/c$, for the optimal spring with $\epsilon_r=3.45^2$. (b) Magnification of the stable equilibrium region of β_{A}. The equilibrium distance is indicated by a vertical dotted line.

study, and it is valid for any type of mirror used. Finally, the explicit condition for stable equilibrium on the dispersion of the modes, bearing in mind that $\beta'_{\mathrm{R}}>0$ and $\beta'_{\mathrm{A}}<0$, is given by $\beta'_{\mathrm{A}}\beta''_{\mathrm{R}}>\beta'_{\mathrm{R}}\beta''_{\mathrm{A}}$, or

$$(\beta'_{\mathrm{R}}/\beta'_{\mathrm{A}})'>0. \tag{9}$$

The above inequality states that the ratio between the absolute values of the repulsive and attractive forces should decrease with D for a stable equilibrium to be formed.

In conclusion, a two-slab optical spring formed by a superposition of two propagating eigenmodes was presented and investigated, and an analytic expression for the spring constant was developed. The spring constant exhibits a maximal value as a function of the equilibrium distance and the slabs' width. The volume outside the waveguide is almost free of radiation, suggesting that objects may be placed there and apply mechanical forces against the optical spring, which could be used, for instance, for force measurement.

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