106929 - Selected Topics in Analysis 2

Instructor: Igal Sason (office: Meyer building 652, e-mail: sason@ee.technion.ac.il).

Time and place: Ulman 601 - Monday 10:30-12:30; Ulman 607 - Tuesday 14:30-15:30.

Course outline (tentative):

1. Elementary inequalities with applications:

   - Cauchy-Schwarz, Jensen's and arithmetic-geometric-mean inequalities;

   - Probability theory: inequalities related to the characteristic function;

   - Discrete geometry: finite sets in the 3-dimensional Euclidean space & projections;

   - Extremal graph problems:

     - Definitions, extremal graph problems with forbidden subgraphs: C_4, C_3 (K_3);
     - Turan's theorem (forbidden subgraph is K_p, with an integer $p \geq 2$);
     - Construction of a graph on n vertices without C_4 subgraphs;
     - The friendship theorem (on friends and politicians);
     - Probabilistic approach for graphs with large girth and chromatic number.

   - Discrete geometry: incidences, the Szemerédi-Trotter theorem, and a derivation of a weaker version via the Cauchy-Schwarz inequality.

2. The arithmetic mean - geometric mean (AM-GM) of Gauss, and applications.

3. Permanent of matrices: Definitions, applications, and two theorems; an asymptotically tight result for the number of Latin squares of order n.

4. Three theorems in extremal set theory: Sperner's theorem; Erdos-Ko-Rado theorem, and Kneser Graphs; sunflower lemma and applications.

5. On polynomials with only real roots.

6. Elements of majorization theory and Schur convexity with applications.

7. Topics in analytical number theory: the infinitude of primes; Bertrand's postulate; binomial coefficients are almost never powers; harmonic and prime-harmonic sums.

REFERENCES

[1] M. Aigner and G. M. Ziegler, *Proofs from the Book*, Springer, 6th edition, 2018.

[2] M. Steele, *The Cauchy-Schwarz Master Class*, Cambridge University Press, 2004.