ICECA

International Conference
Enumerative Combinatorics and Applications
University of Haifa – Virtual – August 25-27, 2025

SCHRIJVER'S ϑ-FUNCTION NEED NOT UPPER BOUND THE SHANNON CAPACITY OF A GRAPH

IGAL SASON

Andrew and Erna Viterbi Faculty of Electrical and Computer Engineering, Department of Mathematics, Technion, Haifa 3200003, Israel

The Shannon capacity of a graph, as introduced in Shannon's seminal paper (1956) on zero-error communication, plays a key role in understanding the synergy and interaction between zero-error information theory and graph theory. The significance of the Shannon capacity of graphs, and the hardness of its computability in general, are highlighted in various survey papers.

This talk addresses an open question concerning a variant of the Lovász ϑ function of a graph, which was introduced by Schrijver and independently by McEliece *et al.* (1978). The question of whether this variant provides an upper bound on the Shannon capacity of a graph was explicitly stated by Bi and Tang (2019). This talk presents an explicit example of a Tanner graph on 32 vertices, which shows that, in contrast to the Lovász ϑ function, this variant does not necessarily upper bound the Shannon capacity of a graph. The example resolves this question, and it clarifies a subtle but significant distinction between these two closely related graph invariants. The detailed example is provided at https://arxiv.org/abs/2505.07778.