Experimental reconstruction of a long-period grating from its core-to-core transmission spectrum

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We demonstrate, for the first time to our knowledge, reconstruction of the structure of a long-period grating from its measured core-to-core transmission spectrum intensity. The reconstruction is obtained by writing an auxiliary grating in cascade to the interrogated grating. Our reconstruction technique is based on using the Hilbert transform and a phase-retrieval algorithm. Using our method, we have reconstructed the structure of a uniform long-period grating with a 47% coupling efficiency. © 2005 Optical Society of America

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The problem of reconstructing the structure of fiber Bragg gratings (FBGs) has been extensively studied in the past decade.[1,2] Reconstruction techniques of FBGs have been used for improving the writing process of the gratings[3] as well as for developing distributed fiber Bragg sensors.[4] In contrast, we are aware of no methods that have been demonstrated for experimental reconstruction of long-period gratings (LPGs) from their transmission spectrum. A reconstruction technique for LPGs is important for developing novel distributed fiber sensors and for improving the writing process of such gratings.

The reconstruction of LPGs is unique when both the complex core-to-core and the complex core-to-cladding transmission spectra are known.[5] While the complex core-to-core transmission spectrum can be measured by applying the same technique used in FBGs,[2] the measurement of the complex core-to-cladding transmission spectrum can be performed only outside the fiber and would require complicated bulk optics. Recently, we have demonstrated a new theoretical method for extracting the core-to-cladding transmission spectrum of LPGs from their complex core-to-core transmission spectrum[6] by using a phase-retrieval algorithm. The method requires writing an auxiliary grating in cascade to the interrogated grating. To obtain a good reconstruction in practical cases where the measured spectrum contains noise, the length of the auxiliary grating should be considerably shorter than the length of the interrogated grating, and the complex core-to-core transmission spectrum of the auxiliary grating should be measured.[6]

In this Letter we demonstrate, for the first time to our knowledge, the experimental reconstruction of a LPG from its core-to-core transmission spectrum. To simplify the experimental system, we use the Hilbert transform and limit our measurement to interrogated gratings that do not have a high coupling efficiency. In such gratings, only the intensity of the core-to-core transmission spectrum of the cascaded structure, rather than the complex core-to-core transmission spectrum, is required for the reconstruction. We also use a weak auxiliary grating, which enables us to reconstruct the interrogated grating without measuring the core-to-core transmission spectrum of the auxiliary grating. The reconstruction technique described in this Letter can also be directly applied to interrogate strong gratings. However, in this case, the complex core-to-core transmission spectrum of the cascaded structure should be measured.

Our experimental system is shown in Fig. 1. The interrogated grating had a uniform profile with a length of $L_2 = 15$ cm, a period of 528 μm, and a maximum coupling of $\sim$47%. The auxiliary grating had a uniform profile with a length of $L_1 = 1.5$ cm, a period of 512 μm, and a maximum coupling of $\sim$15%. The distance between the gratings was approximately $L_f = 25$ cm. Both gratings in our cascaded structure were designed to couple between the core mode and the LP_{04} cladding mode around the wavelength of 1560 nm. The gratings were written in a hydrogen-loaded SMF-28 by using the method described in Ref. 7 and were annealed afterward. Because of the annealing process, the core-to-core transmission spec-

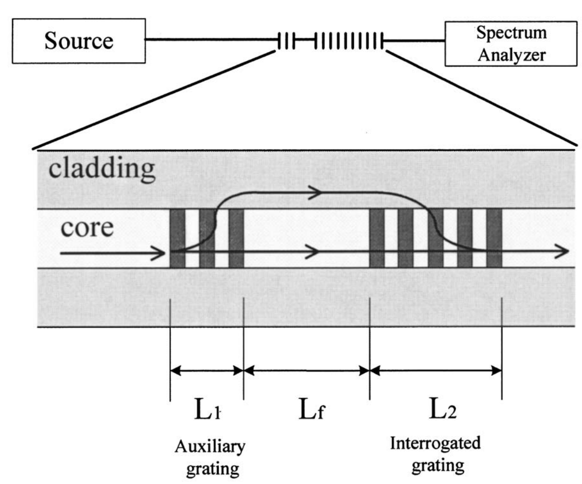

Fig. 1. Experimental system used to reconstruct the interrogated grating structure. An auxiliary grating is written to obtain a unique reconstruction.

trum of the auxiliary grating could not be accurately measured.

We denote the core-to-core and core-to-cladding transmission spectra of the nth grating $(n=1,2)$ by $a_n(k)$ and $b_n(k)$, respectively. The core-to-core transmission function of the cascaded structure is given by

$$a_{\text{tot}}(k) = a_1(k)a_2(k)\exp(-ikL_f) - b_1(k)b_2^*(k)\exp(ikL_f). \tag{1}$$

In our experiment, we measure the intensity spectrum $|a_{\text{tot}}(k)|^2$ and use it to reconstruct the interrogated grating. The core-to-core and the core-to-cladding transmission spectra of the nth grating $(n=1,2)$ can be represented in the following form[6]:

$$a_n(k) = \exp(-ikL_n) + \int_{-L_n}^{L_n} \alpha_n(\tau)\exp(ik\tau)\mathrm{d}\tau,$$

$$b_n(k) = \int_{-L_n}^{L_n} \beta_n(\tau)\exp(ik\tau)\mathrm{d}\tau, \tag{2}$$

where L_n is the length of the nth grating. Equations (2) show that the functions $a_n(k)$ and $b_n(k)$ $(n=1,2)$ have finite support in the time domain. Therefore, the Fourier transform of the two elements in Eq. (1), $a_1(k)a_2(k)\exp(-ikL_f)$ and $b_1(k)b_2^*(k)\exp(ikL_f)$ have finite support that do not overlap when the following condition is fulfilled:

$$L_f > (L_1 + L_2). \tag{3}$$

Since L_f in our experiment was chosen to fulfill the condition in inequality (3), the functions $a_1(k)a_2(k)$ and $b_1(k)b_2(k)^*$ could be extracted from the function $a_{\text{tot}}(k)$ by filtering in the time domain.

In our analysis we use the preservation of power relation:

$$|a_n(k)|^2 + |b_n(k)|^2 = 1, \tag{4}$$

which can be justified from the measured data. Using the Cauchy–Schwartz inequality and Eq. (4), for $n=1,2$ we obtain

$$|a_1(k)a_2(k)| + |b_1(k)b_2(k)| \leq |a_n(k)|^2 + |b_n(k)|^2 = 1. \tag{5}$$

For our measured data, the function $|a_1(k)a_2(k)| + |b_1(k)b_2(k)|$ had an oscillatory behavior. The value of the local maxima of the function varied between 0.957 and 0.991 for the frequency regime where the coupling of the interrogated grating was significant and approached 1 outside this regime. This result indicates that the upper bound for the losses changes between $1-0.991=0.9\%$ and $1-0.957=4.3\%$.

Using Eq. (4), we obtain the result that the function $|b_1(k)|^2+|b_2(k)|^2$ can be calculated from the elements in the function $a_{\text{tot}}(k)$:

$$|b_1(k)|^2 + |b_2(k)|^2 = 1 - |a_1(k)a_2(k)|^2 + |b_1(k)b_2(k)|^2. \tag{6}$$

The extracted functions $|b_1(k)|^2+|b_2(k)|^2$ and $b_1(k)b_2^*(k)$ uniquely define the functions $b_1(k)$ and $b_2(k)$ as proved in Ref. 8. To extract the functions $b_1(k)$ and $b_2(k)$, we first calculate the amplitude of the function $b_1(k)+b_2(k)\exp(ikL_f)$. Then we use the separated hybrid input–output (SHIO) phase-retrieval algorithm[9] to uniquely extract the phase of the function $b_1(k)+b_2(k)\exp(ikL_f)$ from its amplitude, as described in Ref. 6. The SHIO algorithm allows extraction of the phase of a spectral function from its amplitude by using the support constraints of the Fourier transform of the function. The idea behind the SHIO algorithm is to find the optimal phase of the spectral function, which minimizes the energy outside the region where the Fourier transform of the function exists.[9] After using the SHIO algorithm, we calculate the functions $b_1(k)$ and $b_2(k)$ by filtering the function $b_1(k)+b_2(k)\exp(ikL_f)$ in the time domain. Then, the function $|a_1(k)|$ is calculated from the functions $b_1(k)$ by using Eq. (4).

The function $a(k)$ of a LPG is a minimum phase-shift function if it is not equal to zero in the upper half of the complex plane of k.[10] For a minimum phase-shift function $a(k)$, the phase of the function can be reconstructed from its amplitude by using the Hilbert transform. We define the function $a(k,z)$ as the amplitude of the core mode, where $a(k)=a(k,z=L)$. When z is small enough, the function $a(k,z)$ is a minimum phase-shift function.[11] Since the function $a(k,z)$ is a continuous function of z, a sufficient condition that ensures that the function $a(k)$ is a minimum phase-shift function is

$$\min_{k,z}|a(k,z)| > 0. \tag{7}$$

In our case, the auxiliary grating was designed to have a maximum coupling of only 15%, and, therefore, its core-to-core transmission function $a_1(k)$ is a minimum phase-shift function. In addition, since our interrogated grating had a moderate coupling efficiency (~47%), the maximum coupling across the cascaded structure did not exceed 80%, and thus the cascaded structure also fulfilled the minimum phase-shift condition given in inequality (7). Therefore, we could calculate the phase of the core-to-core transmission function $a_{\text{tot}}(k)$ from its amplitude without measuring it directly. However, when the cascaded structure does not fulfill the minimum phase-shift condition, the phase of the function $a_{\text{tot}}(k)$ should be measured. After extracting the phase of the function $a_1(k)$, we use the function $a_1(k)$ to calculate the phase of the function $a_2(k)$ from the known product $a_1(k)a_2(k)$. The functions $a_2(k)$ and $b_2(k)$ are then used to reconstruct the grating structure by using a layer-peeling algorithm.[5]

The intensity of the core-to-core transmission spectrum of the cascaded structure was measured in the wavelength region 1510–1610 nm with a resolution of 5 pm and is shown in Fig. 2. For each point of the

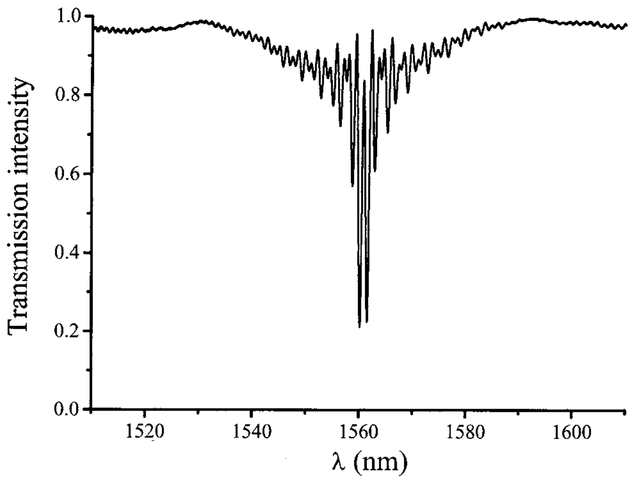

Fig. 2. Intensity of the core-to-core transmission spectrum of the cascaded structure $|a_{\mathrm{tot}}(\lambda)|^2$, measured with a bandwidth of 100 nm and a spectral resolution of 5 pm.

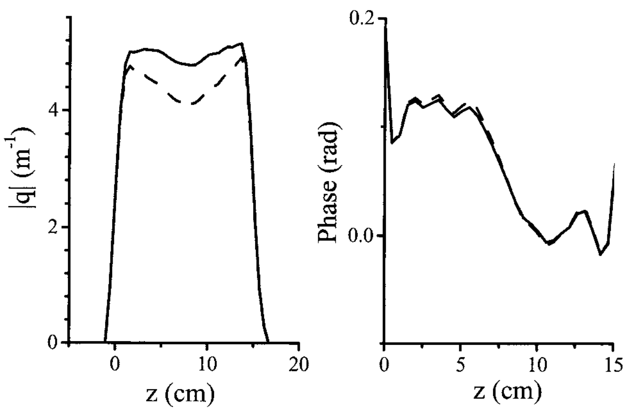

Fig. 3. (a) Amplitude and (b) phase of the coupling coefficient of the interrogated grating, reconstructed by using a layer-peeling algorithm (solid curve) and Born approximation (dashed curve).

spectrum, the wavenumber detuning $k=\pi[\Delta n_{\mathrm{eff}}(\lambda)/\lambda-\Delta n_{\mathrm{eff}}(\lambda_c)/\lambda_c]$ should be calculated, where $\Delta n_{\mathrm{eff}}(\lambda)$ is the difference in the effective refractive indices of the core mode and the LP_{04} cladding mode at the wavelength λ, and $\lambda_c \simeq 1558$ nm is the central wavelength of the interrogated grating. Numerical simulations show that the dependence of $\Delta n_{\mathrm{eff}}(\lambda)$ on the wavelength λ is approximately linear over the 100 nm bandwidth: $\Delta n_{\mathrm{eff}}(\lambda)=\Delta n_{\mathrm{eff}}(\lambda_c)+C(\lambda-\lambda_c)$. Thus, the wavenumber detuning can be approximated by

$$k \simeq \frac{\pi}{\lambda_c}\left[\frac{\Delta n_{\mathrm{eff}}(\lambda_c)}{\lambda_c}-C\right](\lambda_c-\lambda). \quad (8)$$

The effective refractive index difference at the central wavelength of the grating was calculated from the resonance condition: $\Delta n_{\mathrm{eff}}(\lambda_c)=\lambda_c/\Lambda=2.95\times10^{-3}$, where Λ is the period of the interrogated grating. The constant C was then extrapolated from Eq. (8) by requiring that the reconstructed grating would have a length of 15 cm. We obtained from the measured data that $C=-1.2\times10^{-6}$ (1/nm). For comparison, we calculated the value of C numerically for a SMF-28 by using the IFO-Gratings software by Optiwave and obtained $C=-1.19\times10^{-6}$ (1/nm). If the constant C is neglected, a 35% error is obtained in estimating the length of the reconstructed grating.

Figure 3(a) shows the amplitude of the reconstructed coupling coefficient obtained by using the Born approximation[6] (dashed curve) and by the layer-peeling algorithm[5] (solid curve). The figure shows that the Born approximation did not yield a uniform profile, while the layer-peeling algorithm gave a uniform profile with only 5% divergence from uniformity. Figure 3(b) shows the phase obtained by the two reconstruction methods. The figure shows that the phase of the coupling coefficient is relatively constant with less than 0.3 rad change along the whole grating.

The resolution of the reconstruction Δz is determined by the wavenumber bandwidth of the spectrum, denoted BW: $\Delta z=\pi/\mathrm{BW}$. In our experiment the wavenumber bandwidth was equal to 620 m^{-1} (100 nm), and the spatial resolution was 5 mm—~10 grating periods. The maximal length of the cascaded structure that could be theoretically interrogated by our system is given by $\pi/(2\Delta k)$, where Δk is the wavenumber resolution. For our wavelength resolution, $\Delta k=0.031\ \mathrm{m}^{-1}$ ($\Delta\lambda=5$ pm), the maximum interrogated length is ~50 m.

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