Nonlinear filtering by use of intensity-dependent polarization rotation in birefringent fibers

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We analyze the transmissivity of a nonlinear filter that is based on intensity-dependent polarization rotation in a birefringent fiber. It is shown that the transmissivity of the element depends not only on the intensity of the incident light but also on the time behavior of its amplitude. Such an element can be used as a derivator, an element that transmits only variations in the input pulse. The filter can also be used for obtaining lasers that generate a train of intense noiselike pulses with a broadband spectrum and a short coherence length.

Strong optical fields induce intensity-dependent changes in their polarization when passing through isotropic or anisotropic nonlinear media.[1,2] Nonlinear polarization effects in optical fibers were used to obtain optical shutters[3] and artificial saturable absorbers, which are useful for passive mode locking of fiber lasers.[4,5] The analysis of these nonlinear elements was performed under the assumption that the nonlinear medium is isotropic[3,5] or has weak linear birefringence.[6] Therefore the effect of the polarization-mode delay (PMD) was neglected[5,6] and the transmission of the device was assumed to depend only on the instantaneous intensity of the incident wave. However, experimental measurements indicate that a significant birefringence may exist in fiber lasers, probably owing to nonuniformities in the fiber amplifier.[7,8] The measured PMD in those experiments was of the order of 80–100 fs per 1 m of the erbium-doped fiber amplifier. Such large delays can have a significant effect on nonlinear transmission characteristics and can be used for obtaining novel nonlinear filters.

In this Letter we propose and analyze a particular nonlinear filter that uses polarization rotation in a birefringent fiber to obtain a nonlinear high-pass filter, or an optical derivator. The transmissivity of the element is determined not only by the intensity of the incident wave but also by the temporal structure of its amplitude and phase. The transmission of the element is maximum when the time constant of the amplitude variations is of the order of the PMD of the fiber. An analytical solution is given for cases in which dispersion can be neglected. A numerical analysis indicates that, for anomalous dispersion, solitonic effects such as soliton trapping[2,9] affect the transmission. When this new nonlinear element is used inside a laser cavity, it can produce new and interesting pulsed laser characteristics. We demonstrated an erbium-doped fiber laser with such an element, which generates noiselike pulses with a broad and smooth spectrum that has a 3-dB bandwidth of 44 nm.[7] We discuss below how the nonlinear element promotes such a mode of operation.

Figure 1 shows a schematic of the nonlinear element, which is composed of a birefringent fiber and a polarizer. We assume that the polarization of the incident wave is linear and aligned at an angle θ with respect to the slow (x) axis of the birefringent fiber. A polarizer whose axis is rotated by angle $\theta + 90°$ from the x axis is placed at the output of the fiber for conversion of polarization changes into intensity changes. The device is analyzed by separation of the incident wave into two components, E_j, $j = 1, 2$ (1 and 2 denote the x and the y axes, respectively), that are polarized along the principal axes of the birefringent fiber. The coupled-wave equations for the two components are[2,9,10]

$$\frac{\partial A_j}{\partial z} + \delta_j \frac{\partial A_j}{\partial t} + \frac{i}{2} d \frac{\partial^2 A_j}{\partial t^2} = i\gamma\left(|A_j| + \frac{2}{3}|A_{3-j}|^2\right)A_j\,, \tag{1}$$

where γ is the nonlinear coefficient; d is the dispersion parameter; $\delta = 1/2(1/v_{gx} - 1/v_{gy})$ is the difference between the inverse group velocities of the polarization modes; $\delta_1 = \delta$; $\delta_2 = -\delta$; and A_j is the field envelope, defined as $E_j(z,t) = A_j(z,t)\exp(-i\omega_0 t)\exp(i\beta_j z)$, where ω_0 is the carrier frequency and β_j is the carrier wave number for the j-polarization component. We assume in Eqs. (1) that $2(\beta_2 - \beta_1)l \gg 1$, where l is the fiber length, and therefore we neglect the coherent terms, which are not phase matched.[2] The output field obtained after the light emerging from the fiber passes the polarizer is

$$A_{\text{out}}(t) = -A_1(l,t)\exp(i\beta_1 l)\sin\theta + A_2(l,t)\exp(i\beta_2 l)\cos\theta\,. \tag{2}$$

Before solving Eqs. (1) and (2), we emphasize that in Refs. 3, 5, and 6 the effect of the group-velocity mismatch (δ) was neglected. Therefore, when the power

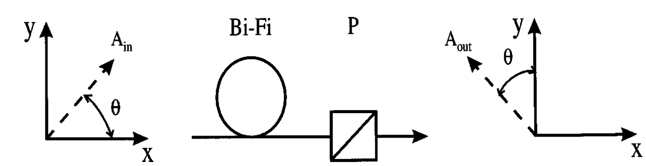

Fig. 1. Schematic description of the nonlinear device: Bi-Fi, birefringent fiber with principal axes x and y; P, polarizer. The input is assumed to be linearly polarized at an angle of θ with the x axis and is perpendicular to the polarizer axis.

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of the input wave was equally split between the two principal axes ($\theta = 45°$), the nonlinear phases accumulated for both polarization components were equal, and the transmissivity of the element did not depend on the power. Mathematically, the nonlinear phase difference between the two polarization components then equals $\phi_{\mathrm{NL}} = \gamma l/3(|A_1|^2 - |A_2|^2)$,[3] and therefore, for $|A_1| = |A_2|$, $\phi_{\mathrm{NL}} = 0$. We show that if the effect of group-velocity mismatch is taken into account the transmissivity depends on input intensity, even in the case of $\theta = 45°$, and the result is a derivator. Below we concentrate on this particular case. We note that, when the birefringence is small and the nonlinear refractive-index change is of the order of the linear birefringence, polarization rotation can occur even in the case of $\theta = 45°$.[11] This effect is caused by the coherent terms in the wave equations, which were neglected here because strong birefringence was assumed.

To obtain an analytical solution for Eqs. (1) we neglect the dispersion term ($d = 0$), and thus the pulse envelope is preserved. By transformation of the time coordinates for the two polarization components, $\tau_j = t - \delta_j z$, an analytical solution can be obtained:

$$A_j(l,t) = A_j(0, t - \delta_j l)\exp[i\gamma|A_j(0, t - \delta_j l)|^2 l] \times \exp\left[i\frac{2}{3}\gamma\int_{-l}^{+l}\frac{1}{2}|A_{3-j}(0, t - \delta z)|^2 \mathrm{d}z\right]. \tag{3}$$

Equations (2) and (3) indicate that when $\theta = 45°$, i.e., for identical input pulses, only self-phase modulation affects the transmittivity. Cross-phase modulation, described by the phase term that includes the constant 2/3, does not affect the output intensity because it adds equal phases for both polarization components.

The output of the filter depends on variation of the signal over a period of the PMD. Assuming an input wave $A(t) = A_m(t)\exp[i\phi(t)]$, we let ΔA_m and $\Delta\phi$ be the amplitude and the phase changes over that period, respectively, so that $A(t + 2\delta l) = [A_m(t) + \Delta A_m(t)]\exp[\phi(t) + \Delta\phi(t)]$. Then, for $(\beta_2 - \beta_1)l = 2m\pi$ (where m is an integer), we obtain

$$I_{\mathrm{out}}(t + 2\delta l) = \frac{\Delta^2 A_m(t)}{4} + [A_m{}^2(t) + A_m(t)\Delta A_m(t)] \times \sin^2\left[\frac{\phi_{\mathrm{NL}}(t) + \Delta\phi(t)}{2}\right], \tag{4}$$

where $\phi_{\mathrm{NL}} = \gamma l(2A_m\Delta A_m + \Delta^2 A_m)$. Equation (4) indicates that the nonlinear phase plays a role similar to that of the linear phase difference. Note that the nonlinear phase vanishes when $\Delta A_m = 0$, regardless of the actual intensity of the input light.

If the input pulse changes slowly during a time period of the PMD, and if the nonlinear phase is small, one can obtain

$$I_{\mathrm{out}} = (\delta l)^2[I^{-1}I'^2 + 4I\phi'^2 + 4\gamma l I' I(2\phi' + \gamma l I')], \tag{5}$$

where $I'(t)$ and $\phi'(t)$ are the time derivatives of the intensity and the phase of the input wave, respectively. Equations (4) and (5) indicate that the transmission has linear and nonlinear components that depend on the derivatives of the input wave. No output is obtained if the intensity and the phase are constant in time, i.e., I', $\phi' = 0$. Therefore the device behaves as a derivator, or high-pass filter, which emphasizes amplitude and phase changes. Equation (5) also indicates that the transmission depends differently on the amplitude and the phase changes of the input field.

Figures 2 and 3 show the transmitted intensity as a function of time, normalized to the maximum input intensity, for a Gaussian pulse, calculated for various input intensities (Fig. 2) and for various PMD values (Fig. 3). In the calculations we assumed that $\theta = 45°$, $l = 4/\gamma$, $d = 0$, and $(\beta_2 - \beta_1)l = 2m\pi$. The figures show that the transmitted pulse undergoes significant reshaping. This nonlinear reshaping is also accompanied by spectral broadening. As expected from Eq. (5) the output vanishes at $t = 0$ because the derivative of the input Gaussian pulse is zero at that point. Figure 2 shows that, although the filter transmits even in the linear regime, the transmitted pulse can grow significantly at high powers owing to the nonlinear effect. Figure 3 shows the effect of increasing PMD for a particular pulse power and indicates that the nonlinear effect is most influential when the pulse width and the PMD are comparable.

The results shown in Figs. 2 and 3 could be understood intuitively by consideration of the effect of PMD. At $\theta = 45°$ the input wave is separated into two equal components, which are polarized along the principal axes of the birefringent fiber. When the pulse is short compared with the difference in the propagation times, two separate orthogonally polarized pulses emerge [dashed curve in Fig. 3(b)]. The intensity of each pulse behind the polarizer equals 1/4 of the incident intensity, independently of power. When the pulse width is much longer than the PMD, the two com-

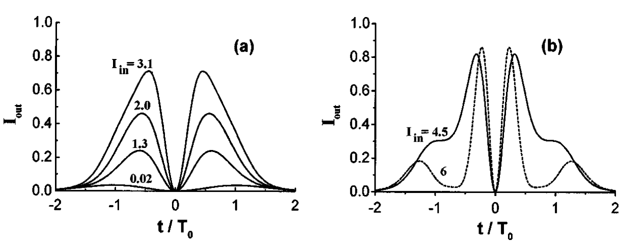

Fig. 2. Normalized output intensity $I_{\mathrm{out}}(t)/I_{\mathrm{in}}(0)$, where $I_{\mathrm{out}}(t)$ is the output intensity for an input Gaussian pulse calculated for various input intensities $I_{\mathrm{in}}(0)$: (a) 0.02, 1.3, 2.0, 3.1; (b) 4.5, 6. $2\delta l/T_0 = 0.64$, where T_0 is the width of the Gaussian pulse, $\theta = 45°$, $\gamma = 1$, $l = 4/\gamma$, and $d = 0$.

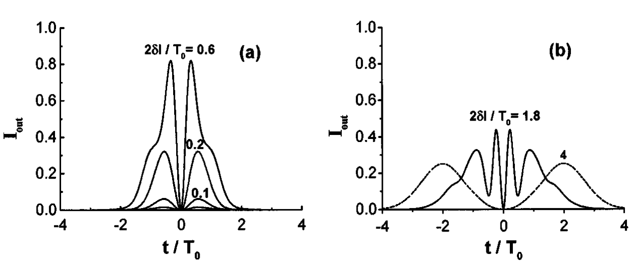

Fig. 3. Normalized output intensity $I_{\mathrm{out}}(t)/I_{\mathrm{in}}(0)$ for a Gaussian pulse calculated for various normalized PMD's, $2\delta l/T_0$: (a) 0.04, 0.1, 0.2, 0.6; (b) 1.8, 4. $I_{\mathrm{in}} = 3$, $\theta = 45°$, $\gamma = 1$, $l = 4/\gamma$, and $d = 0$.

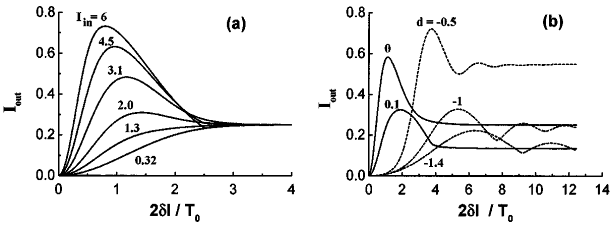

Fig. 4. Maximum normalized output intensity versus the normalized PMD $2\delta/T_0$. (a) $d = 0$, input intensities $I_{\text{in}} = 0.1$, 0.32, 1.3, 2.4, 4.5, 6. (b) $I_{\text{in}} = 2.0$; dispersion parameter $d = -1.4, -1, -0.5, 0, 0.1$; $l = 4/\gamma$.

ponents accumulate similar nonlinear phases. Again, the output polarization does not depend on the input power, and the polarizers can be set to block the output wave [bottom curve in Fig. 3(a)]. However, when the pulse width is of the order of the PMD, the two components slide along each other. The nonlinear phases that the two components accumulate are different because they depend on the instantaneous intensities. The output wave behind the polarizer is a superposition of the two components, and thus it interferes destructively or constructively, depending on the nonlinear phase, leading to waveforms such as those shown in Figs. 2 and 3.

Figure 4(a) shows the maximum transient intensity for the Gaussian pulse versus the PMD for various input intensities. Again it is observed that the maximum transmissivity is obtained when the pulse width is of the order of the PMD. The term $\Delta A_m(t)$ in Eq. (4) becomes significant then, and the output is maximized. To determine the effect of dispersion, we solved Eq. (1) numerically. The input pulse was assumed to have a sech profile. Figure 4(b) shows that dispersion affects the results described above. Unlike in the dispersionless case, cross-phase modulation now also plays a role and modifies the output waveform. Positive dispersion broadens the pulse by spreading its frequency components in the time domain. The spreading decreases the maximum transmissivity because it decreases the instantaneous intensity. When the dispersion is negative, solitons can form and propagate. In this case solitonic effects such as pulse compression[12] can increase the peak intensity of the propagated waves [as shown in the top curve of Fig. 4(b)]. The interaction between the two polarization components can cause soliton trapping.[2,10] Soliton trapping binds the two polarization components and increases the values of the PMD at which the maximum transmissivity is obtained; however, soliton trapping also decreases the maximum transmissivity because the frequency of the two polarization components becomes different and the two waves cannot constructively interfere. Soliton effects are also responsible for the oscillatory curves in Fig. 4(b).

We have seen that the transmission of the nonlinear element depends on the intensity of the input light as well as on its temporal profile. Such a nonlinear element could have interesting effects when placed inside a laser. We have shown[7] that this nonlinear behavior can lead to mode-locked pulsed operation, where each pulse is a noise burst. The nonlinear element in the laser favors pulsed operation. However, the laser cannot support ultrashort pulses, in part because of the PMD, which splits each pulse into two pulses as explained above. On the other hand, the laser does not support long, narrow-band pulses that are longer than the PMD, because the transmissivity of such pulses is small when it is aligned as a derivator. Hence the only stable mode of operation available is the formation of noiselike pulses, which are transmitted well by such a derivator. We also expect that in certain configurations PMD in a laser can produce a burst of short pulses whose repetition rate is determined by the PMD. For example, the transmission of a train of pulses with alternate phases and a repetition rate of 1/PMD is 1 (except for the first and the last pulses) when $\theta = 45°$.

We have analyzed a novel nonlinear filter that is based on nonlinear polarization rotation in birefringent fibers. The device functions as a nonlinearly enhanced derivator. The transmissivity of the device vanishes when the intensity and the phase of the input pulse do not change in time and is maximized when significant amplitude changes occur within a time period of PMD. Strong enhancement of transmissivity is obtained at high powers. When the filter is placed inside a laser cavity it can cause the laser to generate noiselike pulses with broad spectra and high intensities. We believe that the new nonlinear filter may be important for systems that require a derivator, i.e., nonlinear elements that enhance rapid amplitude changes.

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