Pulse propagation in a fiber Bragg grating written in a slow saturable fiber amplifier

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We have developed a model to study nonlinear pulse propagation in a fiber Bragg grating written in an erbium-doped fiber amplifier. The saturation effect in such amplifiers depends on the accumulated energy along the pulse rather than on the pulse instantaneous power. We have shown that the gain saturation effect cannot be neglected when Bragg solitons are amplified by erbium-doped fiber amplifiers. The slow saturation of the amplifier limits the output pulse power, and it tends to split the amplified pulse into several pulses. We have shown that when the propagation velocity of the amplified pulses decreases, the amplifier gain per unit length increases. © 2009 Optical Society of America

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A Bragg soliton (BS) is a solitary wave that can propagate in a fiber Bragg grating (FBG) in the presence of Kerr nonlinearity. Such solitons were predicted theoretically [1–4] and then observed in experiments [5,6]. Due to high dispersion in FBGs [5], the peak power of BSs can be high, of the order of tens of kilowatts. Such pulses are advantageous for nonlinear applications such as optical frequency conversion. One of the main effects that limit the propagation of BSs and especially the propagation of BSs with a low group velocity is the high loss in FBGs [6]. Another effect that limits the generation of intense BSs is the loss in the soliton launching due to reflections. FBGs that are written in fiber amplifiers can be used to overcome the grating loss and to obtain intense pulses. The propagation of a BS along an unsaturated amplifier was analyzed in [7]. The propagation of BSs in amplifying media with fast saturable absorption were studied in [8].

Erbium-doped fiber amplifiers (EDFAs) have become essential components in optical communication systems because of their efficient pumping, low insertion loss, polarization-independent gain, high output saturation power, and near-quantum-limited noise [9,10]. Therefore, EDFAs are attractive to amplify BSs. However, the high energy of BSs can saturate the EDFA gain. The saturation energy of EDFAs may be increased by increasing the effective mode area. However, the increase of the mode area also decreases the nonlinear effect, and hence the pulse energy should also be increased to obtain BSs. Therefore, the saturation effect of BSs written in EDFA can significantly affect the propagation of the solitons.

In this Letter we describe a model to study the propagation of pulses and BSs in Bragg gratings written in EDFA (BG-EDFA). We show that since the energy of a typical BS is high, the saturation effect in BG-EDFAs cannot be neglected. Because of the slow saturation of EDFAs, the gain depends on the accumulated energy of the amplified pulses rather than on their instantaneous power, as occurs in fast saturable amplifiers. We study the dynamics of pulse amplification in BG-EDFAs and show that the gain saturation limits the pulse amplification by splitting the amplified pulse into several pulses. The slow saturation also causes a deformation in the amplified pulse. We discuss the effect of the slow velocity of BSs on soliton amplification in BG-EDFAs and show that slow velocity allows the amplifier gain to be increased for a given amplifier length.

The propagation of strong optical pulses in BG-EDFA can be analyzed by adding to the nonlinear coupled-mode equations [3,4] the gain and the change in the refractive index that are caused by the EDFA [9,10]:

$$i\partial_z u + i{V_g}^{-1}\partial_t u + [\sigma(z) + \sigma_g(z,t)]u + \kappa(z)v + \Gamma(|u|^2 + 2|v|^2)u - j\frac{1}{2}g(z,t)u = 0, \tag{1a}$$

$$-i\partial_z v + i{V_g}^{-1}\partial_t v + [\sigma(z) + \sigma_g(z,t)]v + \kappa(z)u + \Gamma(|v|^2 + 2|u|^2)v - j\frac{1}{2}g(z,t)v = 0, \tag{1b}$$

where $u(z,t)$ and $v(z,t)$ are the slowly varying amplitudes of the coupled waves with the positive and the negative phase velocity, respectively; $\kappa(z)$ is the coupling coefficient; $\sigma(z)$ is proportional to the average refractive index change along the grating [7]; $\sigma_g(z,t) = 2\pi/\lambda_B \Delta n_g(z,t)$, where $\Delta n_g(z,t)$ is the refractive index change caused by the EDFA and λ_B is the Bragg wavelength; $g(z,t)$ is the amplifier gain coefficient; Γ is the nonlinear Kerr coefficient; and $V_g = c/n_{\mathrm{eff}}$ is the group velocity in the absence of the grating, where c is the velocity of light in vacuum and n_{eff} is the effective refractive index of the fiber. In the general case the coefficients κ and σ depend on the gating location as required to model apodization and chirp in gratings [6].

EDFA can be modeled as a three-level energy system [9,10]. We assume that the EDFA is pumped by a 980 nm light source and that the bandwidth of the amplified signal is significantly narrower than the

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bandwidth of the amplifier. Hence the EDFA can be analyzed by using the rate equations [9,10]:

$$\partial_t N_2(z,t) = -\partial_t N_1(z,t) = \frac{P_p(z,t)}{P_{p,sat}}\frac{N_1(z,t)}{\tau} + \frac{P_s(z,t)}{P_{s,sat}}\frac{N_1(z,t)-\eta_s N_2(z,t)}{(1+\eta_s)\tau} - \frac{N_2(z,t)}{\tau}, \tag{2a}$$

$$\partial_z P_p(z,t) = -\alpha_p N_1(z,t) P_p(z,t), \tag{2b}$$

where $N_i(z,t)$ $(i=1,2)$ is the atomic population in the ith energy level, P_p (P_s) is the pump (signal) power, $P_{p,sat}$ ($P_{s,sat}$) is the pump (signal) saturation power, η_s is the ratio of emission to absorption cross sections at the wavelength of the amplified signal, α_p is the pump absorption coefficient, and τ is the spontaneous decay time [9]. Assuming that the amplified spontaneous emission can be neglected, the steady-state solution for the small-signal gain, $g_0(z)$, is obtained from Eqs. (2) by setting $P_s=0$.

The signal amplitude gain coefficient is given by $g(z,t)=\alpha_s[\eta_s N_2(z,t)-N_1(z,t)]$, where α_s is the signal absorption coefficient. The average refractive index along the BG-EDFA changes slightly as a function of the population density by the amount $\Delta n_g(z,t)$. This effect shifts the local bandgap of the grating and can cause interesting effects when high-energy pulses are amplified. We will not study this effect in this Letter, and we will assume that the central wavelength of the optical pulse is around $\lambda = 1533$ nm, and thus that the magnitude of Δn_g is negligible [9].

Equation (2a) can be simplified if we consider the amplification of a single BS that propagates along the BG-EDFA. The spontaneous decay time in the EDFA, τ, is of the order of 10 msec [9]. In a standard EDFA $P_{p,sat}\simeq 1$ mW [9] and $P_p \lesssim 1$ W. Therefore, the pumping characteristic time $\tau P_{p,sat}/P_p$ is of the order of 10 μs. The duration of optical BSs used in experiments is typically less than 10 ns, and their propagation velocity is greater than about 0.2 of the group velocity in the absence of the grating. Therefore, both the spontaneous emission and the pumping can be neglected during the propagation of a single BS in any location along the BG-EDFA. In this case the solution of Eq. (2a) becomes

$$g(z,t) = g_0(z)\exp\left[\frac{-\int_{-\infty}^{t} P_s(z,s)\mathrm{d}s}{E_{sat}}\right], \tag{3}$$

where $E_{sat}=P_{s,sat}\tau$ is the saturation energy and $P_s = |u(z,t)|^2+|v(z,t)|^2$. In the derivation of Eqs. (1) and (3), the transverse change in the gain profile due to saturation is neglected. A similar model was successfully validated by comparing the theory to experiments [10].

To study the nonlinear pulse propagation in BG-EDFA we solved Eqs. (1) by using a numerical simulation that extends the split-step method described in [11] by adding the saturation effect given in Eq. (3). The parameters of the BG-EDFA are $n_{eff}=1.45$, $\kappa=9000$ m^{-1}, $\Gamma=5$ m^{-1} kW^{-1}, $\sigma=0$, $\lambda_B=1533$ nm, $P_{s,sat}=104$ μW [9], $\tau=10$ ms [10], and hence $E_{sat}=1.04$ μJ. Our initial pulse at time $t=0$ is a BS solution in a uniform FBG ($g=0$). We have simulated the pulse propagation for four different detunings of the soliton frequency with respect to the Bragg frequency. The temporal FWHM was 0.5 ns for all the simulations. Since the temporal FWHM of the initial soliton was kept constant, the decrease in the detuning resulted in slower and more powerful solitons. Therefore, the input energy of the pulses was significantly increased as the pulse detuning was decreased. Figure 1 shows the temporal profiles of the input pulses after a propagation of $L=15$ cm along the fiber. The initial parameters of the pulses as well as the output energies are given in the figure caption.

Figure 1(a) shows the output pulse intensity as a function of time when the output energy E_L is smaller than the saturation energy—$E_L=0.4E_{sat}$. The output pulse becomes asymmetric in time owing to the amplifier saturation, since the leading edge of the pulse experiences a higher gain than its trailing edge. When the pulse velocity decreases, the initial pulse energy increases, and the output energy becomes greater than the saturation energy. The temporal intensity of the output wave in these cases is shown in Figs. 1(b)–1(d). The figures show that the output wave contains several pulses. Figure 2 shows the pulse power at different locations along the BG-EDFA as a function of the reduced time $\tau=t-z/\nu_0$, where ν_0 is the group velocity of the input pulse. The initial pulse parameters are the same as used in Fig. 1(c). Figures 1 and 2 indicate that the new pulses are formed from the front edge of the initial pulse. The pulse amplification and the saturation effect cause a deformation in the propagating pulse. The pulse deformation causes the generation of a new pulse from

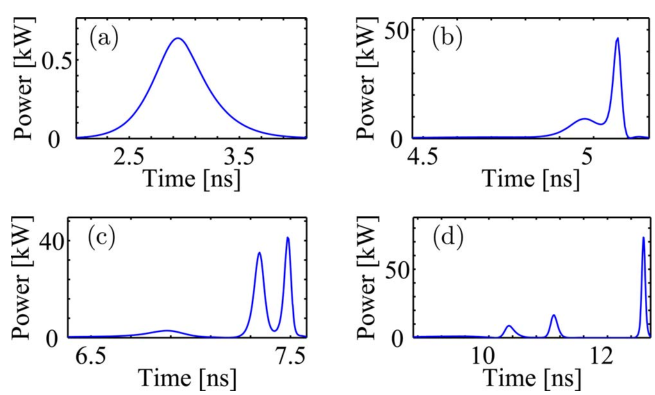

Fig. 1. (Color online) Pulse power as a function of time after a propagation of $L=15$ cm in BG-EDFA for input pulses with the same FWHM duration of 0.5 ns and with different carrier frequencies. The peak power P_0, energy E_0, velocity ν_0, and detuning from the Bragg frequency f_0 of the input pulses are, as $[P_0, E_0, \nu_0, f_0]$, (a) [43 W, $0.02E_{sat}$, $0.3V_g$, 1952 GHz], (b) [186 W, $0.1E_{sat}$, $0.15V_g$, 1883 GHz], (c) [423 W, $0.23E_{sat}$, $0.1V_g$, 1871 GHz], (d) [1.2 kW, $0.65E_{sat}$, $0.06V_g$, 1865 GHz]. The energy of the output pulses E_L equals (a) $0.4E_{sat}$, (b) $2.8E_{sat}$, (c) $5.1E_{sat}$, and (d) $11E_{sat}$.

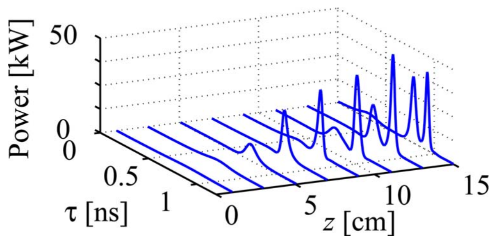

Fig. 2. (Color online) Pulse power at different locations along the amplifier as a function of the reduced time $\tau = t - z/v_0$ where v_0 is the group velocity of the input pulse. The parameters of the input pulse are the same as used in Fig. 1(c).

the front edge of the pulse, since this part of the pulse experiences the maximum gain. The generation of several pulses significantly decreases the maximum gain that can be obtained from the amplifier. To verify that the splitting of the pulses is caused by the saturation effect, we have simulated the pulse propagation when the saturation is not taken into account. Figure 3(a) shows the output pulse when the saturation effect is neglected. The input soliton had the same parameters as used in Fig. 1(c), but the amplifier length was only $L=6$ cm [Fig. 3(a)] and $L=7$ cm [Fig. 3(b)]. Figure 3(a) shows that after propagating $L=6$ cm the output pulse peak power was 100 kW, and the pulse energy was about the same as obtained in Fig. 1(c). Figure 3(b) shows that when the length of the amplifier is further increased to $L=7$ cm the maximum pulse power becomes very high, of the order of 200 kW, and pulse breakup is observed, similar to the pulse breakup observed in [7].

The splitting of solitons that propagate in EDFA with anomalous dispersion was reported in [12]. The soliton propagation in that work was governed by the modified Ginzburg–Landau equation. The splitting of the pulses was obtained by gain dispersion that was due to the limited bandwidth of the amplifier. In our system, we did not take this effect into account, because the bandwidth of the pulses is of the order of few tens of gigahertz. This pulse bandwidth is significantly lower than the bandwidth of the EDFA and the bandwidth of the pulses used in [12]. In our system, because of the high energy of BSs compared with the energy of nonlinear Schrödinger solitons with the same duration, the saturation effect becomes dominant, and it causes the splitting of the propagating pulses.

The total energy that passes through the z location of the amplifier equals $E_s(z)=\int_{-\infty}^{+\infty} P_s(z,\tau)\mathrm{d}\tau$. For a single pulse this energy is equal to the pulse energy

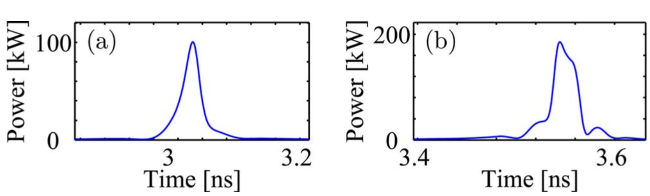

Fig. 3. (Color online) Pulse power P as a function of time when saturation effect is neglected after a propagation of (a) $L=6$ cm and (b) $L=7$ cm. The initial pulse parameters are identical to that used in Fig. 1(c).

at location z. The change in the total energy along the EDFA can be analytically calculated by using Eqs. (1a), (1b), and (3):

$$\frac{\mathrm{d}}{\mathrm{d}z}[\langle v_e\rangle(z)E_s(z)] = g_0(z)E_{\mathrm{sat}}\left\{1-\exp\left[-\frac{E_s(z)}{E_{\mathrm{sat}}}\right]\right\}, \tag{4}$$

where

$$\langle v_e\rangle(z) = \frac{\int_{-\infty}^{+\infty}\{v_e(z,\tau)[|u(z,\tau)|^2+|v(z,\tau)|^2]\}\mathrm{d}\tau}{\int_{-\infty}^{+\infty}[|u(z,\tau)|^2+|v(z,\tau)|^2]\mathrm{d}\tau}, \tag{5}$$

and $v_e(z,t)=(|u|^2-|v|^2)/(|u|^2+|v|^2)$ is the normalized energy velocity at time t and location z [13]. Hence, $\langle v_e\rangle(z)$ can be interpreted as the energy velocity at location z that is time averaged according to the instantaneous power. For a single BS that propagates in a gainless uniform grating the velocity $\langle v_e\rangle$ is equal to the group velocity of the BS. In the results shown in Figs. 1 and 2 the pulse approximately maintained its velocity prior to its splitting. In regions where the average velocity $\langle v_e\rangle$ is approximately constant, Eq. (4) shows that the amplification of the pulse energy $\mathrm{d}E(z)/\mathrm{d}z$ depends on the effective gain coefficient $g_{\mathrm{eff}}(z)\triangleq g_0(z)/\langle v_e\rangle$ rather than on the gain coefficient $g_0(z)$. Therefore, the energy amplification increases as the velocity $\langle v_e\rangle$ decreases.

In conclusion, we have developed a model to study nonlinear pulse propagation in FBG written in a slow saturable amplifier, such as EDFA. The model results show that gain saturation cannot be neglected when a BS is amplified by EDFA. The slow saturation limits the output pulse power, and it also tends to split the amplified pulse into several pulses.

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