# New Technique to Accurately Interpolate the Complex Reflection Spectrum of Fiber Bragg Gratings

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***Abstract*—We demonstrate theoretically a new method to accurately interpolate the complex reflection spectrum of fiber Bragg gratings with a finite length at any desired frequency resolution. The required sampling resolution is significantly smaller than can be expected by directly using the sampling theorem for obtaining a low-error characterization of the reflection spectrum. A further decrease in the required sampling resolution by a factor of two is obtained by sampling both the complex reflection and the complex transmission functions. The new reconstruction technique may enable to significantly reduce the time needed to characterize fiber Bragg gratings and to interrogate fiber Bragg sensors.**

***Index Terms*—Gratings, optical fiber devices, periodic structures.**

## I. Introduction

FIBER Bragg gratings are important elements in optical communication systems as well as in systems for optical metrology. Fiber Bragg gratings are usually characterized by their complex reflection spectrum [1]–[3]. The complex reflection spectrum is also used to extract the structure of fiber Bragg gratings and fiber Bragg sensors [4], [5]. In order to accurately characterize the complex reflection spectrum of fiber Bragg gratings, the reflection function should be sampled with a high enough resolution. The Fourier transform of the complex reflection spectrum is equal to the impulse response function of the grating. According to sampling theorem, an error-free characterization of an arbitrary reflection spectrum can be obtained when the corresponding impulse response has a finite duration [6]. In this case the required sampling rate in the frequency domain for an error-free characterization of the reflection spectrum should be higher than the duration of the impulse response divided by $2\pi$. However, since for most interesting cases the impulse response function of a fiber Bragg grating is not equal to zero over an infinite interval length, errors due to aliasing effects cannot be avoided. In practice, the errors in characterizing fiber Bragg gratings can be minimized by using a high enough spectral resolution; however, when the resolution is increased the duration of the reflection spectrum measurement also significantly increases. Moreover, when the reflectivity of the grating increases, the effective duration of the impulse response also increases. Therefore, one may think that the resolution needed to characterize fiber Bragg gratings depends on the grating reflectivity.

In this paper, we demonstrate theoretically a new method to accurately interpolate the complex reflection spectrum of fiber Bragg gratings with a finite length at any desired frequency resolution. The sampling resolution is significantly smaller than can be expected by using directly the sampling theorem for obtaining a low-error characterization of the reflection spectrum. Our method is based on the mathematical properties of the reflection spectrum of fiber Bragg gratings. We show that the reflection spectrum can be presented by two frequency dependent functions whose Fourier transforms have a finite duration. Similar mathematical properties were previously used for simplifying the synthesis of discrete scattering structures [7], [8] and of thin film optics [9]. The properties of the reflection spectrum of a fiber Bragg grating with a known length were used to obtain the best fit between the desired reflection spectrum and the reflection spectrum of the synthesized grating.

The reflection spectrum can be reconstructed from samples of the reflection spectrum or from samples of both the reflection and the transmission spectra of the grating. When both the reflection and the transmission spectra are sampled, the required spectral resolution is half than required when only the reflection spectrum is sampled. According to sampling theorem, the sampling of a general spectral function with an infinite impulse response duration will result in an error in the reconstruction of the spectrum. However, since the reflection spectrum is not a general spectral function and has unique properties, it can be sampled with a finite spectral resolution without adding sampling errors. Moreover, the sampling resolution required by our technique is significantly smaller than can be expected by using directly the sampling theorem for obtaining a low-error characterization of the reflection spectrum.

When only the reflection spectrum is sampled, our technique may not be suitable for reconstructing gratings with a very high reflectivity. In the case where only the reflection spectrum is sampled, the transmission spectrum is calculated from the sampled reflection spectrum. The calculation of the transmission spectrum involves highly nonlinear operations, when the grating reflectivity is high. Therefore, noise or inaccuracies added to the reflection spectrum may prevent an accurate calculation of

Manuscript received December 2, 2003; revised April 2, 2004. This work was supported in part by the Division for Research Funds of the Israeli Ministry of Science and in part by the Ollendorff Center of the Department of Electrical Engineering at the Technion.

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Digital Object Identifier 10.1109/JQE.2004.831650

the transmission spectrum and cause high errors in the reconstructed spectrum of such gratings. When both the complex reflection and transmission spectra are measured, our method is more robust to noise or inaccuracies in the sampled data.

This paper is organized as follows. In Section II we describe the mathematical properties of the reflection and transmission spectra of fiber Bragg gratings. In Section III we show how to accurately interpolate the reflection spectrum by using its mathematical properties given in Section II. In Section IV we give numerical examples to our interpolation method.

## II. Theoretical Background

In this section, we give the mathematical model for our spectrum interpolation technique. Using this model, we show that the reflection spectrum of fiber Bragg gratings can be expressed as a ratio of two functions, whose Fourier transform have finite support. Our analysis is given for the general case where the grating is analyzed using a continuous model.

We assume that the propagation of the waves in the grating is described by the coupled-mode equations [10], [11]

$$\begin{aligned}\frac{du_1(k,z)}{dz}+iku_1(k,z)&=q(z)u_2(k,z)\\ \frac{du_2(k,z)}{dz}-iku_2(k,z)&=q^*(z)u_1(k,z)\end{aligned}\tag{1}$$

where $k=2\pi n_{\mathrm{avg}}(1/\lambda-1/\lambda_B)$ is the wavenumber detuning, $\lambda_B$ is the central (Bragg) wavelength, $\lambda$ is the wavelength of the incident wave, $n_{\mathrm{avg}}$ is the average refractive index of the grating, $u_1(k,z)$ and $u_2(k,z)$ are the complex amplitudes of the backward and the forward propagating waves, respectively, and $q(z)$ is the complex coupling coefficient of the grating [12].

By using the vectorial notation, $U(k,z)=(u_1(k,z), u_2(k,z))^t$, we introduce four solutions to the coupled-mode equations, (1): $\Phi(k,z)$, $\overline{\Phi}(k,z)$, $\Psi(k,z)$, and $\overline{\Psi}(k,z)$, with the following boundary conditions [13]:

$$\begin{aligned}\Phi(k,z=0)&=\begin{pmatrix}1\\0\end{pmatrix} & \Psi(k,z=L)&=\begin{pmatrix}0\\1\end{pmatrix}\\ \overline{\Phi}(k,z=0)&=\begin{pmatrix}0\\1\end{pmatrix} & \overline{\Psi}(k,z=L)&=\begin{pmatrix}1\\0\end{pmatrix}.\end{aligned}\tag{2}$$

Since the two solutions $\Phi(k,z)$ and $\overline{\Phi}(k,z)$ are independent [13], the solution $\Psi(k,z)$ can be expressed as a linear combination of the solutions $\Phi(k,z)$ and $\overline{\Phi}(k,z)$

$$\Psi(k,z)=a(k)\overline{\Phi}(k,z)+b(k)\Phi(k,z).\tag{3}$$

In order to define the complex transmission and reflection functions of the grating, we refer to the solution of (1), $U(k,z)=(u_1(k,z),u_2(k,z))^t=\Psi(k,z)/a(k)$. The solution $U(k,z)$ satisfies the boundary conditions $u_2(k,z=0)=1$ and $u_1(k,z=L)=0$, and, therefore, it describes the scattering of a forward-propagating wave that enters from the left side of the grating, $z=0$. Therefore, the complex reflection coefficient $r(k)$ and the complex transmission coefficient $t(k)$ can be defined using the complex functions $a(k)$ and $b(k)$ [13], [14]

$$\begin{aligned}r(k)&=\frac{b(k)}{a(k)}\\ t(k)&=\frac{1}{a(k)}.\end{aligned}\tag{4}$$

We assume that the grating exists only in the region $[0,L]$ and present the solution $\Psi(k,z)$ in following integral form:

$$\Psi(k,z)=\begin{pmatrix}0\\1\end{pmatrix}\exp[ik(z-L)]+\int_{z-L}^{L-z}K(\tau,z)\exp(ik\tau)d\tau,\quad 0\le z\le L.\tag{5}$$

Next, we substitute (5) into the coupled mode equations, (1). Using the theory of characteristics [15], as performed in [13] for an infinite grating, it can be shown that the function $K(\tau,z)$ exists and is unique. Substituting $z=0$ in (3), we obtain that $\Psi(k,z=0)=(b(k),a(k))^t$. Therefore, the functions $a(k)$ and $b(k)$ can also be represented in an integral form

$$\begin{aligned}a(k)&=\exp(-ikL)+\int_{-\infty}^{\infty}\alpha(\tau)\exp(ik\tau)d\tau\\ b(k)&=\int_{-\infty}^{\infty}\beta(\tau)\exp(ik\tau)d\tau\end{aligned}\tag{6}$$

where the functions $\alpha(\tau)$ and $\beta(\tau)$ are equal to zero for $|\tau|>L$. Equation (6) shows that the functions $\alpha(\tau)$ and $\beta(\tau)$ are equal to the Fourier transform of the functions $a(k)-\exp(-ikL)$ and $b(k)$, respectively. Similar results, obtained for a discrete scattering model and for thin film optical structures, were used in [7]–[9] for synthesis applications. In Section III we show that the results obtained in this section can be also used for interpolation of the reflection spectrum from the sampled data.

According to sampling theorem [6], the maximum sampling period that is required for a full characterization of the functions $a(k)$ and $b(k)$ is equal to $\Delta k=\pi/L$. When the functions $a(k)$ and $b(k)$ are sampled with sufficient resolution (i.e., small enough sampling period), they can be calculated with any desired resolution by padding the time domain functions $\alpha(\tau)$ and $b(\tau)$ with zeros and calculating the Fourier transform of the result [6]. Then, the complex reflection spectrum of the grating, $r(k)$, can be found at the desired resolution using (4). In Section III we show how to calculate the functions $a(k)$ and $b(k)$ when both the reflection spectrum and transmission spectrum are measured, and when only the reflection spectrum is measured.

## III. Reconstruction of the Complex Reflection Spectrum

In the previous section we have shown that the complex reflection spectrum can be obtained from the functions $a(k)$ and $b(k)$. The functions $a(k)$ and $b(k)$ can be sampled without any loss of information since their corresponding Fourier transforms, $\alpha(\tau)$ and $\beta(\tau)$, are nonzero only in a limited time interval. Therefore, when the amplitude and the phase of both the complex reflection and the complex transmission functions are known with a spectral sampling period smaller than $\Delta k = \pi/L$, the functions $a(k)$ and $b(k)$ can be directly calculated from (4) at any desired resolution. However, in most experiments only the complex reflection function is used to characterize the grating. In this case, we will show that the complex transmission function needed to extract the functions $a(k)$ and $b(k)$, can be calculated from the complex reflection function when the sampling period of the reflection spectrum is equal to or smaller than $\Delta k = \pi/(2L)$. We note that the complex transmission function needed to calculate the functions $a(k)$ and $b(k)$ may be different from the actual transmission function of the grating. The actual transmission function may be affected by coupling to cladding modes [16] that are not included in the conventional analysis of gratings, based on two-mode coupled wave equations (1). However, as demonstrated in previous work, the two-mode coupled equations give an excellent agreement between the theoretical and the experimental complex reflection spectra (see for example [11]).

The amplitude of the transmission function can be found from the reflection spectrum using the conservation of energy, obtained from the coupled-mode equations

$$|r(k)|^2 + |t(k)|^2 = 1. \quad (7)$$

The transmission function is a causal function, and, therefore, the Hilbert transform can be used to connect the real and the imaginary parts of the function. Since we need a connection between the amplitude and the phase of the transmission function, we perform the Hilbert transform on the logarithm of the transmission function

$$\log[t(k)\exp(-ikL)] = \log|t(k)| + i\arg[t(k)] - ikL. \quad (8)$$

Since the function $t(k)\exp(-ikL)$ is a minimum phase-shift function [17], the real and imaginary parts of its logarithm are connected through the Hilbert transform [18]. We use the discrete Hilbert transform [19] to obtain the phase of the transmission from the function, $\log|t(k)|$. The numerical calculation of the discrete Hilbert transform of the function $\log|t(k)|$ is performed by multiplying the Fourier transform of the function $\log|t(k)|$ by a discrete step function in the time domain [19]. However, since the function $t(k)$ and, hence, the function $\log|t(k)|$ has an infinite duration in the time domain, aliasing effects in calculating the Hilbert transform cannot be avoided. The error due to aliasing effects can be reduced as needed since the function $\log|t(k)|$ can be accurately calculated at any desired resolution.

The spectral resolution of the function $\log|t(k)|$ determines the maximum time interval where the Fourier transform of the function $\log|t(k)|$ is calculated. The function $\log|t(k)|$ can be directly calculated from the amplitude of the reflection spectrum using the connection $\log|t(k)| = 1/2\log(1/|a(k)|^2)$ and (4) and (7). Since the Fourier transform of the function $|a(k)|^2$ is nonzero only in the time interval $-2L \le \tau \le 2L$, the function $|a(k)|^2$ can be accurately calculated at any desired resolution, when the reflection spectrum is sampled with a period smaller than or equal to $\Delta k = \pi/(2L)$. We note that the resolution required to sample the function $|a(k)|^2$ is twice than the resolution needed to sample the function $a(k)$ since the Fourier transform of the function $|a(k)|^2$ is nonzero in the time interval $-2L \le \tau \le 2L$. Therefore, when only the reflection spectrum is measured, the overall sampling period, required by our method, is equal to $\Delta k = \pi/(2L)$. By padding the Fourier transform of the function $|a(k)|^2$ with zeros, we can find the function $\log|t(k)|$ with a sufficient resolution for calculating the Hilbert transform with an error that can be reduced as needed. Therefore, the resolution of the function $\log|t(k)|$ should be chosen to be high enough to minimize errors caused by aliasing effects.

Once the complex transmission function is calculated, we can obtain the functions $a(k)$ and $b(k)$, and find the reflection coefficient, $r(k)$, with any desired spectral resolution. We note that when only the complex reflection spectrum of the grating is measured, the required sampling period for our technique is equal to $\Delta k = \pi/(2L)$. Alternatively, the sampling period needed to characterize the grating equals $\Delta k = \pi/L$ when both the complex reflection and the complex transmission functions of the grating are simultaneously sampled. In both methods the minimum number of points required to fully characterize the grating is the same.

In order to calculate the maximum wavelength sampling period, $\Delta\lambda$, from the maximum wavenumber sampling period, $\Delta k$, we use the following connection: $\Delta\lambda \approx \Delta k\lambda_B^2/2\pi n_{\text{avg}}$. When only the complex reflection function is sampled, the maximum wavelength sampling period needed to fully characterize the grating using our method is equal to

$$\Delta\lambda \approx \frac{\lambda_B^2}{4n_{\text{avg}}L}. \quad (9)$$

The sampling period given in (9) depends only on the grating length and not on the grating reflectivity. However, as shown in the next section, the extraction of the complex transmission function from the complex reflection function may become inaccurate when analyzing gratings with a very high reflectivity. When the function $b(k)$ is directly obtained from both the complex transmission and the complex reflection functions of the grating, a full characterization of the grating can be attained with twice the sampling period given in (9) (i.e., half the resolution). When a weakly reflecting grating is analyzed, most of the impulse response energy is obtained in a time duration of $2L$. Therefore, the maximum wavelength sampling period, required in this case, is also twice the period given in (9). However, since the impulse response function of a grating is not equal to zero over an infinite interval, a small error is added to the reconstructed spectrum even when the grating reflectivity is low.

## IV. Numerical Examples

In order to demonstrate our reconstruction method, we analyzed the reflection spectrum of a uniform grating with a length of $L = 4$ mm, a maximum reflectivity of 99%, a Bragg wavelength, $\lambda_B = 1.55\ \mu$m, and an average refractive index, $n_{\text{avg}} = 1.45$. The complex reflection function of the

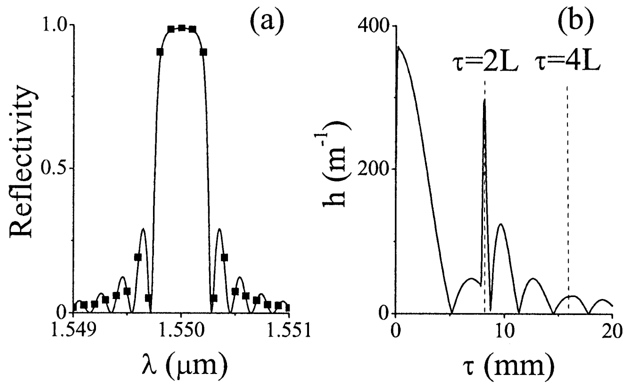


Fig. 1. (a) The reflectivity and (b) the impulse response function of a uniform fiber Bragg grating with a maximum reflectivity of 99% and a length of $L = 4$ mm. The sampled points used to characterize the reflection function with a sampling period of $\Delta\lambda = 0.1$ nm are marked in the figure.

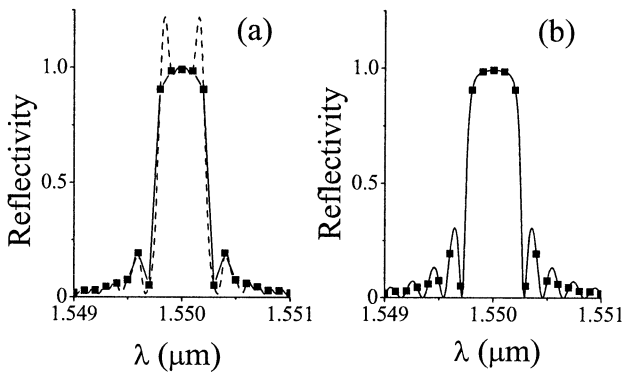


Fig. 2. (a) Reconstructed spectrum of the grating analyzed in Fig. 1, obtained by a linear interpolation (solid line) and by sinc interpolation as used in the sampling theorem (dashed line), and (b) the reconstructed spectrum obtained using the technique described in this paper. The reflection spectrum was reconstructed from the sampled points shown in Fig. 1(a).

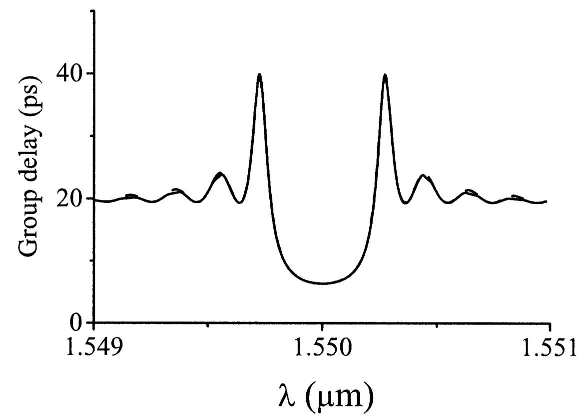


Fig. 3. Group delay of the grating analyzed in Fig. 1, calculated using an explicit expression for a uniform grating (dashed line) and calculated from the reflection spectrum obtained by using our reconstruction technique when the sampling period was equal to that given in (9), $\Delta\lambda = 0.1$ nm (solid line).

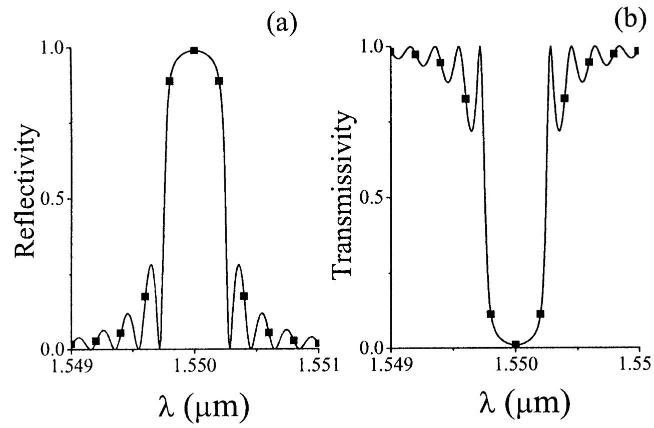


Fig. 4. (a) The reflectivity and (b) the transmissivity of the grating analyzed in Fig. 1 calculated from the samples of both the complex reflection and transmission spectra. The sampling period was equal to $\Delta\lambda = 0.2$ nm. The sampled points of the reflection and transmission spectra are marked in the figure.

grating was obtained using an explicit expression [11]. Fig. 1 shows the reflectivity and the impulse response function of the grating. The figure clearly shows that the duration of the impulse response is significantly longer than the minimum propagation time of the reflected light along the grating, $2L$. The complex reflection spectrum was sampled with the period, given in (9), $\Delta\lambda = 0.1$ nm that corresponds to a time duration $\tau = 4L$. The sampled points are shown in Fig. 1(a).

Fig. 2(a) shows a linear interpolation of the grating reflectivity performed using the sampled points shown in Fig. 1(a). The figure also shows an interpolation of the reflectivity function using sinc functions as used in the sampling theorem [6]. The two interpolation methods shown in Fig. 2(a) give a significant error in the reconstructed grating reflectivity since the sampling resolution is lower than needed according to sampling theorem. Fig. 2(b) shows the interpolation of the grating reflectivity using the technique described in this paper. The figure shows that an excellent reconstruction was obtained. The group delay of the grating was also reconstructed from the complex reflection spectrum. Fig. 3 shows the reconstructed group delay of the reflection spectrum $t_{\rm GD} = (n_{\rm avg}/c)d\phi(k)/dk$, where $\phi(k)$ is the phase of the reflection spectrum, $n_{\rm avg}$ is the average refractive index, and $c$ is the speed of light in vacuum. The figure compares the reconstructed group delay (solid curve) to the accurate group delay (dashed curve) calculated by using an explicit expression [11]. The phase of the reflection is not defined at frequencies in which the reflectivity is equal to zero. Therefore, we omitted in Fig. 3 the group delay that was calculated for frequencies which correspond to a reflectivity lower than $-20$ dB. Fig. 3 shows an excellent agreement between the accurate and the reconstructed group delays. The calculation of the group delay requires a derivative operation which magnifies the interpolation errors. Therefore, the group delay could not have been calculated when a linear or a sinc interpolation were used.

In the next example, we demonstrate our interpolation method when both the complex reflection spectrum and the complex transmission spectrum are known. We analyzed the reflection spectrum and transmission spectrum of the uniform grating given in the previous example. The reflection and the transmission spectra were sampled with twice the sampling period given in (9), $\Delta\lambda = 0.2$ nm, which corresponds to a time duration of $\tau = 2L$. Fig. 4 shows the interpolated reflectivity and transmissivity. The sampled points of the reflection and transmission spectra are also shown in the figure. The figure

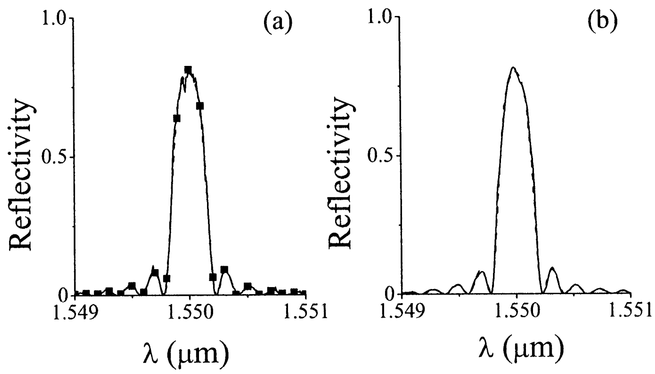


Fig. 5. (a) Noisy reflectivity and (b) the interpolated reflectivity of a uniform fiber Bragg grating with a maximum reflectivity of 80% and length of $L = 4$ mm. The noise was represented by identical independent random variables that were added to the calculated complex reflection spectrum of the grating. The random variables had a Gaussian distributions with a zero mean, a standard deviation of 0.02 and a uniformly distributed phase in the region $[-\pi, \pi]$. The complex reflection function was sampled with a sampling period of $\Delta\lambda = 10^{-1}$ nm, and the sampled points are marked in Fig. 5(a).

shows that an excellent reconstruction is obtained when both the complex transmission and the complex reflection functions are sampled with twice the sampling period given in (9).

When a grating is characterized experimentally, noise is added to the sampled data. The technique for reconstructing the reflectivity of fiber Bragg gratings described in this paper is not sensitive to noise as long as the grating reflectivity is not too high. When only the complex reflection function is sampled, the calculation of the complex transmission function from the complex reflection function may become sensitive to noise when the grating reflectivity is high. When the reflectivity of the grating is high, the amplitude of the function $t(k)$ is close to zero at the Bragg zone and the calculation of the logarithm function in (8) becomes sensitive to noise. In this case, the Hilbert transform cannot be calculated accurately and the technique gives a large error. When both the complex transmission and the complex reflection spectra are sampled, the sensitivity of our method to noise is significantly reduced. In this case, the only operation which may be sensitive to noise is the calculation of the function $a(k) = 1/t(k)$.

In the next example, we analyze a uniform grating with a length of $L = 4$ mm and a maximum reflectivity of 80%. We calculated the reflection spectrum of the grating using an explicit expression given in [11]. Then, we added to each point of the spectrum a complex random variable that represents a noise added in the experiment. The noise variables had a Gaussian distribution with a zero mean, a standard deviation of 0.02, and a uniformly distributed phase in the region $[-\pi, \pi]$. After adding the noise, the reflection spectrum was sampled with a sampling period $\Delta\lambda = 0.1$ nm as required by (9). Fig. 5(a) shows the grating reflectivity and the sampled points. Fig. 5(b) shows the reconstructed grating reflectivity. The figures also show the exact reflectivity of the grating calculated according to [11]. The figure shows a very good quantitative agreement between the interpolated and the accurate grating reflectivities. The average standard deviation of the interpolated complex reflection function was approximately 0.0205. The average standard deviation was calculated by performing a thousand simulations and averaging the error obtained in the thousand simulations across all frequencies where the complex reflection spectrum was interpolated.

When the reflectivity is increased, inaccuracies in the reconstructed spectrum are magnified due to the high nonlinearities in our method. When the grating reflectivity was increased to about 90% we were not able to reconstruct the reflection spectrum from the sampled reflection data. An accurate reconstruction in this case was obtained when the standard deviation of the noise variables was lower than $5 \times 10^{-3}$.

## V. Conclusions

We have demonstrated theoretically a new method to accurately interpolate the complex reflection spectrum of fiber Bragg gratings with a finite length at any desired frequency resolution. The required sampling resolution is significantly smaller than can be expected by using directly the sampling theorem for obtaining a low-error characterization of the reflection spectrum. A further decrease in the required sampling resolution, by a factor of two, is obtained by sampling both the complex reflection and the complex transmission functions. Our new reconstruction technique may become sensitive to noise when only the complex reflection spectrum is measured in the case of highly reflecting fiber Bragg gratings. The new reconstruction technique may enable to significantly reduce the time needed to characterize fiber Bragg gratings and to interrogate fiber Bragg sensors.

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