## Filtering behavior of a self-induced three-mirror cavity formed by intracavity wave mixing in a saturable absorber

Moshe Horowitz, Ron Daisy, and Baruch Fischer

Department of Electrical Engineering, Advanced Opto Electronics Center, Technion–Israel Institute of Technology, Haifa 32000, Israel

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Wave mixing with a self-induced grating in a saturable absorber, which is inserted into a resonator, gives special filtering behavior. In a linear Fabry-Perot cavity with gain, the grating forms a self-matched, three-mirror laser cavity that can discriminate between modes whose frequency difference is much lower than the grating bandwidth. The effect was used to obtain single-mode lasers with narrow linewidths. The self-induced grating can dynamically adjust itself to small disturbances and changes in the cavity parameters. © 1996 Optical Society of America

Single-mode lasers with very narrow linewidths can be attractive for many applications. We recently suggested and demonstrated<sup>1,2</sup> a simple method for obtaining this goal. It was experimentally shown that wave mixing in an erbium-doped fiber laser that contains a saturable absorber (an umpumped section of an erbium-doped fiber) can be used to drive the laser to single-mode operation with narrow linewidth whose upper limit was measured to be 5 kHz. With this method it is possible to obtain linewidths of the order of a few hundred hertz, as was recently demonstrated in Ref. 3. In our previous study<sup>1,2</sup> we presented a general explanation, but not a quantitative analysis, for the filtering behavior.

In this Letter we show that an intracavity saturable absorber, in which the grating is induced by wave mixing of the counterpropagating beams, forms a special self-matched three-mirror cavity, with unique frequency-narrowing properties. We present an analysis of such a system and show that its filtering power is much stronger than that of a simple grating, thus permitting its use for obtaining single-mode lasers with narrow linewidths, even when the bandwidth of the grating is significantly larger than the free spectral range of the laser cavity or the frequency difference between two adjacent cavity modes. Therefore it can be also applicable for long laser cavities. In our previous experiment<sup>1,2</sup> the length of the saturable absorber, and hence of the grating, was  $\approx 20$  cm, and the cavity length was  $\approx 10$  m; therefore the estimated bandwidth of the grating, 312 MHz, was much larger than the free spectral range of the cavity, 9.26 MHz. The nonlinear grating can adjust its central frequency and the phase of its periodic spatial distribution to drifts of the cavity parameters, thus providing narrow bandwidths without any matching or controlling needs.

We analyze the effect of the absorption grating in a laser with a linear cavity, in which the locations of the minima and maxima of the intensity pattern formed by each of the laser modes are well defined along the laser, because of the boundary conditions dictated by the mirrors. When the laser is pumped above threshold, the effect of the absorption grating can be understood by considering the losses of different cavity modes in

the intracavity absorber. The interference pattern of the counterpropagating waves induces a nonuniform periodic absorption pattern, or absorption grating, as a result of the saturation effect. The average loss of a mode is determined by the spatial overlap between the absorption grating and the interference pattern of the waves. When only one mode exists the losses are minimized, because where the intensity of the mode is high the absorption is low, whereas in regions where the absorption is high the losses are ineffective, because the intensity there is low. Other modes have larger overlap with the absorption pattern and therefore have higher losses. These losses depend on the wavelength of the modes and on the relative phases between their interference patterns and the grating. In fact, the maximum loss is obtained for a probe of two counterpropagating waves of which the interference pattern is exactly 180° out of phase with the interference pattern of the strong waves that wrote the grating. It should be noted that the loss in this case is maximum despite the fact that each of the counterpropagating waves exactly fulfills the Bragg condition and can be reflected by the grating. Moreover, it can also be shown that the existence of an absorption grating causes an increase in the losses of other modes with different frequencies that do not fulfill the Bragg condition. It is worthwhile to point out that these effects are somewhat different in a ring cavity. There the relative phases between the intensity patterns of different modes can be arbitrary, and the effect on the mode losses is diminished. Therefore the grating causes discrimination only between optical modes whose frequency differences are of the order of the Bragg bandwidth. Hence, the lack of tight boundary conditions, which exist in a linear cavity because of the mirrors, can be a reason for the larger linewidth of a ring laser compared with a linear cavity laser.

The filtering behavior of the induced grating can be further understood in the following two ways: (a) The waves that are reflected from a grating in the saturable absorber are in phase with the corresponding copropagating waves that wrote the grating. Consider an additional weak pair of counterpropagating waves,

an incident wave and its reflection from the mirror. These waves are reflected by the grating induced by the former strong counterpropagating pair of waves, if their frequency is inside the bandwidth of the grating (the Bragg zone). The reflection of each of the weak waves interferes with the corresponding strong wave and can enhance or attenuate it, depending on the relative phase between them. The strong dependence of the grating reflectivity on the phases of the incident waves can be used to obtain parametric amplifiers and lasers with squeezed light. (b) A different point of view of understanding the filtering effects is based on consideration of a complex three-mirror cavity, such as the cleaved-coupled-cavity diode laser.<sup>4</sup> In our case this is a self-induced three-mirror cavity, formed by the two cavity mirrors and between them the distributed mirror provided by the grating. The weakest losses are obtained for a mode of the main cavity that is also a mode of the two internal subcavities, formed by the reflections from the grating and the corresponding cavity mirrors. Since the grating is self-induced, the resonant conditions of all three cavities are automatically fulfilled for the mode that induces the grating. A different mode that does not fulfill the resonance condition of the two internal cavities has higher losses. Therefore, the system is a self-matched three-mirror laser cavity for the main strong mode, and it is mismatched for others that are slightly detuned. Moreover, such a grating can adjust itself, especially its periodicity, for changes in the cavity, such as those originating from thermal drifts, if the time constant of the changes is longer than the nonlinear response time. (This time is of the order of 10 ms for gratings that are induced by light with a wavelength of 1550 nm in erbium-doped fibers.) Therefore, selfinduced gratings can relax the need for control and stabilization of the internal cavities. We also note that similar filtering behavior can also be obtained by refractive-index nonlinearities. However, these are relatively weak in laser amplifiers, or absorbers, when the laser wavelength is close to the resonance transition of the active medium.<sup>5</sup>

In order to analyze the filtering effects of the selfinduced grating, we have solved the coupled-wave equations and calculated the wavelength dependence of the reflectivity from a saturable absorber with an attached mirror, shown in the inset of Fig. 1. The grating is induced in the saturable absorber by the counterpropagating waves, the incident wave and its reflection from the mirror. We calculated the filtering behavior of the system for another weak pair of counterpropagating waves, using methods similar to those in Refs. 2 and 6. We denote the amplitude of the waves  $A_{\pm j}$  (j = 1, 2) (where  $\pm$  denotes the waves that move along the  $\pm z$  axis, j = 1 corresponds to the strong waves that write the grating, and j = 2 corresponds to the probe waves) and obtain the following coupled-wave equations<sup>2</sup>:

$$\frac{\mathrm{d}A_{\pm 1}}{\mathrm{d}z} = \mp \frac{\gamma_0 (1+i\Omega)}{2(a^2-b^2)^{1/2}} \left[ 1 - \frac{a - (a^2 - b^2)^{1/2}}{2|A_{\pm 1}|^2} \right] A_{\pm 1},$$
(1)

$$\begin{aligned} \frac{\mathrm{d}A_{\pm 2}}{\mathrm{d}z} &= \ \mp \left. \frac{\gamma_0 (1+i\Omega)}{2(a^2-b^2)^{1/2}} \right| A_{\pm 2} - \frac{a-(a^2-b^2)^{1/2}}{2} \\ &\times \frac{A_{\mp 2}}{A_{\pm 1}^* A_{\mp 1}} \exp(\mp 2i\Delta kz \pm \phi_{21}) \right], \end{aligned} \tag{2}$$

where  $a = 1 + \Omega^2 + |A_{+1}|^2 + |A_{-1}|^2$ ,  $b = 2|A_{+1}A_{-1}|$ ,  $\Delta k$  is the difference between the wave numbers of the two pairs of waves,  $\phi_{21}$  is the phase difference between their intensity patterns,  $\gamma_0$  is the absorption coefficient, and  $\Omega$  is the frequency detuning from resonance. We can conclude from these equations that the reflections from the grating [the second term in Eq. (2)] can constructively or destructively interfere with the incident waves that propagate along the absorber [the first term in Eq. (2)], depending on the relative phase ( $\phi_{21}$ ) between the interference patterns of the two pairs of beams. Therefore the weak wave can be attenuated by the wave mixing even when its frequency is inside the Bragg bandwidth of the grating.

Figure 1 shows the reflectivity of the probe wave intensity versus the accumulated phase  $\Delta k l_1$ , as explained below. We have assumed that the relative phase,  $\phi_{21}$ , at the input side of the absorber is zero. The width of the envelope of the reflection curve is determined by the Bragg condition. The first zero of the envelope is obtained for  $\Delta k l_g = \pi$ , where  $l_g$  is the length of the grating. The fast periodic changes of the reflection result from the cavity formed between the grating and the mirror, whose length is  $l_1$ . The first zero is obtained when  $2\Delta k l_1 \approx \pi$ . Thus Fig. 1 demonstrates that the reflectivity can be increased or decreased, compared with the Bragg condition filtering, depending on the relative phase between the interference patterns. In our cavity this phase is formed for waves



Fig. 1. Reflectivity of a probe wave from a grating that was induced in a saturable absorber  $(\gamma_0 l_g = -1.5)$  with a mirror (r = 0.95) by strong coupled waves as a function of the phase difference  $\Delta k l_1$ , where  $l_1$  is the distance between the mirror and the absorber. The length of the absorber (grating) is  $l_g = l_1/5$ , and the input intensity of the strong wave is four times stronger than the saturation intensity. The inset shows the saturable absorber–mirror system (SA is the saturable absorber, M is the mirror).



Fig. 2. Transmissivity of a weak wave (through the cavity with the saturable absorber in which the self-induced grating was formed by the strong self-matched mode) as a function of phase  $\Delta kl$  (*l* is the total length of the cavity). The distances between the absorber and the mirrors are 1 and 9 m, the length of the saturable absorber is 0.2 m, and the intensity of the strong wave at the input of the absorber is four times stronger than the saturation intensity. The inset shows the two mirrors (M<sub>1</sub>, M<sub>2</sub>) and the saturable absorber cavity (SA).

with different wavelengths mainly as a result of the propagation between the absorber and the mirror.

Now we turn to the analysis of the linear Fabry– Perot cavity, shown in the inset of Fig. 2, which contains the saturable absorber. It is a three-mirror laser cavity, with two end mirrors and, between them, the self-induced grating in the saturable absorber. Figure 2 shows the transmissivity through such a cavity, with the mode structure. The reflectivity of one of the end mirrors with the saturable absorber was calculated by Eqs. (1) and (2). The result was used as an effective reflectivity for one of the mirrors of the Fabry-Perot cavity, which is formed with the sec-

ond mirror of the cavity. (In our laser the effect of the amplifier without two-wave mixing<sup>2</sup> can be taken into account by multiplying the reflectivity of the second cavity mirror by the amplifier gain.) In the calculation, the distances between the absorber's two ends and the cavity mirrors,  $l_1$  and  $l_2$ , were taken to be 1 and 10 m, respectively, and the absorber (grating) length was  $l_g = 0.2$  m. The envelope of the transmissivity curve (the mode strength) in Fig. 2 is determined by  $l_2$ ,  $l_1$ , and the grating length. When the absorber length is much shorter than the cavity length the periodicity of this envelope is increased by approximately  $(l_1 + l_2)/l_1$ (where  $l_1 < l_2$ ) compared with the case for which no grating is induced. Thus, as the ratio  $l_2/l_1$  increases, the grating can reject more modes. However, the difference between the transmissivity of adjacent modes is decreased. Figure 2 also indicates that the grating can prevent mode hopping even in cases for which the laser is strongly pumped above the threshold and the intracavity light intensity is high. (The intensity of the wave at the input of the absorber was taken to be four times stronger than the saturation intensity.) In these cases the losses that are due to the existence of the absorber can be low.

In conclusion, we have presented a self-matched three-mirror cavity obtained by a saturable absorber inside a laser cavity and analyzed its filtering behavior. Because the grating is self-induced it can provide a narrow linewidth, with relaxed needs for control and stabilization of cavity parameters.

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