Generation of Complex Microwave and Millimeter-Wave Pulses Using Dispersion and Kerr Effect in Optical Fiber Systems

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***Abstract*—We study a new method to synthesize high-frequency complex microwave and millimeter-wave pulses using dispersion, Kerr effect, and group velocity delay in optical fiber systems. The profile of the generated pulses can be controlled by changing the parameters of the optical system. Nonlinear propagation effect in fibers can be used to generate electrical pulses with an extremely broad spread spectrum. Soliton trapping can be used to generate electrical pulses with a controllable frequency. Implicit results are given when dispersion or nonlinear effect can be neglected. Generation of electrical pulses with a controllable microwave frequency is demonstrated experimentally using a Mach–Zehnder interferometer and a chirped fiber Bragg grating.**

***Index Terms*—Interference, millimeter-wave devices, nonlinear optics, optical fiber devices, optical propagation.**

I. Introduction

OPTOELECTRONIC devices have been used to generate continuous-wave radio-frequency signals using a laser with an external electronic feedback [1], an electronic circuit controlled by the photoconductivity effect [2], an optical wave mixing in hetrojunction bipolar transistor [3], an optical frequency comb generator with a uni-travelling-carrier photodiode [4], and a low-frequency square wave modulation of a laser diode [5]. The electrical wave generated in those works was a low-noise continuous-wave signal with a frequency that could be controlled over a broad frequency regime. Complex radio frequency pulses were generated using spectral shaping of a supercontinuum source followed by a dispersive element.

In this paper, we study a novel optical method to synthesize complex microwave and millimeter-wave (MW/MMW) pulses. The method is based on using linear and nonlinear propagation effects in optical fiber systems in order to generate complex MW/MMW pulses that can be controlled by adjusting the parameters of the optical system. The effect is theoretically analyzed in an optical system based on pulse propagation in a birefringent fiber. Implicit results are given when the dispersion or the nonlinear effect can be neglected. Dispersion can be used to generate MW/MMW pulses with a frequency that can be controlled by changing the parameters of the optical system. When the nonlinear effect in the fiber becomes significant, MW/MMW pulses with a complex frequency modulation can be generated. Since the response time of nonlinear Kerr effect is on the order of a few femtoseconds, broadband MW/MMW pulses can be obtained without the need for expensive electronics. Soliton trapping effect can be used to generate electrical pulses with a frequency that can be controlled by changing the input power of the soliton. We experimentally demonstrated the generation of electrical pulses modulated at a controllable microwave frequency using a Mach–Zehnder interferometer and a chirped fiber grating.

The optical technique described in this paper enables generation of pulses modulated with a complex frequency profile that can be optically controlled. The electrical pulses can have an extremely broad spectrum that cannot be obtained using conventional electronic systems. Broad spectrum pulses, generated by the technique described in this paper, may be important in radars based on pulse compression [6] as well as in communication systems based on spread-spectrum technique [7]. Pulse compression in radars enables one to obtain a large radiated energy simultaneously with a high range resolution. The technique also enables reduction in the effect of clutter, multipath interference, and electronic countermeasures [6]. Spread-spectrum signals used in communication systems are used in order to suppress interference effect caused by multipath propagation, jammers, and other users of the channel. The use of Spread-spectrum pulses also enable one to hide signals and to achieve message privacy in the presence of other listeners. The extremely broad spectrum of the pulses, generated by the technique described in this paper, can significantly improve the performance of broad spectrum radars and communication systems. Pulses that are generated using nonlinear Kerr effect, as described in Sections III and IV, are modulated with a complex frequency profile. Such pulses enable one to efficiently hide signals, to suppress interference effects caused by jammers, and to reduce the sidelobe intensity of the electrical pulses obtained at the output of the matched filter in the receiver. The compression of the complex pulses in the receiver can be obtained by using a surface acoustic wave dispersive delay line and digital methods [6] as well as by using novel optical methods, described elsewhere.

II. General Description of the System

In this paper, we show that the interference of optical pulses that propagate in a fiber system with dispersion, group delay,

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and Kerr effect can be used to generate electrical pulses modulated with a complex frequency profile. We choose to analyze the generation of the electrical pulses in an optical system based on a birefringence fiber. Similar results can be obtained in other optical systems such as in a Mach–Zehnder interferometer with a chirped fiber grating, as described in Section VI.

Fig. 1 shows a schematic description of the system. The system is based on the propagation of an optical pulse in a birefringent fiber. The propagation of the input pulse in the birefringent fiber can be analyzed by separating the input wave into two polarization components, each aligned along the principal axes of the birefringent fiber. Each polarization component changes while propagating inside the fiber due to group velocity delay, group velocity dispersion, and nonlinear Kerr effect. The combination of nonlinear and linear propagation effects in the birefringence fiber changes the polarization state of the optical pulses at the output of the fiber. The polarization changes are converted into electrical pulses using a polarizer and an optical detector. Pulse propagation in a high birefringent fiber was previously used for nonlinear shaping of optical pulses in order to generate noiselike pulses in lasers [8]. The nonlinear filter, obtained by the birefringent fiber and the polarizer, was used to enhance intensity and phase changes in the noiselike pulses. Nonlinear propagation of pulses in a weakly birefringent fiber, where the effect of differential group delay can be neglected, was used to obtain an intensity-dependent optical filter [9]–[10].

In our system, we assume that the polarization of the input wave is linear and aligned at an angle θ with respect to the slow axis of the birefringent fiber x. A polarizer aligned at an angle ϕ with respect to the y-axis of the birefringent fiber is placed at the output of the fiber in order to convert polarization changes into intensity changes.

The system can be analyzed by separating the incident wave into two polarization components $E_{x,y}$, aligned along the two principal axes of the birefringent fiber x and y, respectively. The coupled wave equations for the two polarization components are [11]

$$\begin{aligned}\frac{\partial A_x}{\partial z} + \delta\frac{\partial A_x}{\partial t} + \frac{i}{2}\beta_2\frac{\partial^2 A_x}{\partial t^2} &= i\gamma\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_x\\ \frac{\partial A_y}{\partial z} - \delta\frac{\partial A_y}{\partial t} + \frac{i}{2}\beta_2\frac{\partial^2 A_y}{\partial t^2} &= i\gamma\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_y\end{aligned} \quad (1)$$

where γ is the nonlinear coefficient, β_2 is the dispersion coefficient, $2\delta = (1/v_{gx} - 1/v_{gy})$ is the difference between the inverse group velocities of the two polarization components, and A_x, A_y are the envelopes of the field components polarized along the x- and the y-axis, respectively, defined as $E_{x,y}(z,t) = A_{x,y}(z,t)\exp(-i\omega_0 t)\exp(i\beta_{x,y}z)$, where ω_0 is the carrier frequency and $\beta_{x,y}$ is the carrier wave number for the x and y polarization components, respectively. Since the dispersion effect does not strongly depend on the fiber birefringence, we assume that the dispersion coefficient is the same for the two polarization components. We also assume in (1) that the phase mismatch between the two polarization components is high, $2(\beta_x - \beta_y)l \gg 1$, where l is the fiber length, and therefore we can neglect the coherent terms $i\gamma A_x^* A_y^2\exp\{-2i(\beta_x - \beta_y)z\}/3$ and $i\gamma A_y^* A_x^2\exp\{2i(\beta_x - \beta_y)z\}/3$. We neglect attenuation effect along the fiber since a fiber with a relatively short length can be used to implement the optical system.

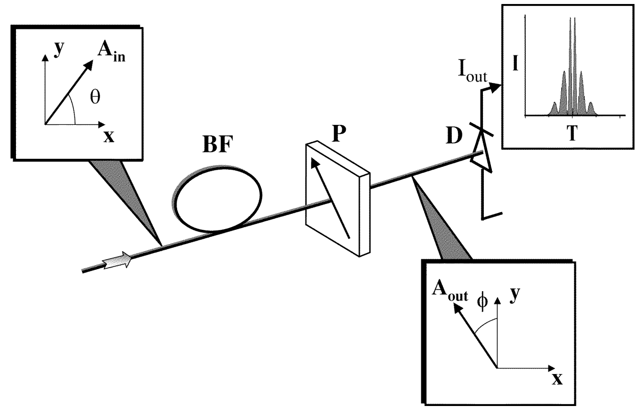

Fig. 1. Schematic description of an optical system used to generate complex microwave pulses. BF is a birefringent fiber with principal axes x and y; P is a polarizer aligned at an angle ϕ with respect to the y-axis of the birefringent fiber. The input wave has a linear polarization, aligned at an angle θ with respect to the x axis of the fiber.

We transform the time frame of each polarization component into a time frame that moves with the pulse at its group velocity $\tau_x = t - \delta z$, $\tau_y = t + \delta z$, and obtain

$$\begin{aligned}\frac{\partial A_x}{\partial z} + \frac{i}{2}\beta_2\frac{\partial^2 A_x}{\partial \tau_x^2} &= i\gamma\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_x\\ \frac{\partial A_y}{\partial z} + \frac{i}{2}\beta_2\frac{\partial^2 A_y}{\partial \tau_y^2} &= i\gamma\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_y.\end{aligned} \quad (2)$$

The output field after passing the polarizer equals

$$\begin{aligned}A_{\text{out}}(t) = &-A_x(l,t)\exp(i\beta_x l)\sin(\phi)\\ &+A_y(l,t)\exp(i\beta_y l)\cos(\phi).\end{aligned} \quad (3)$$

Equation (3) shows that the output field after passing the polarizer is a linear combination or an interference between the fields of the two polarization components. Since the interference between the two polarization components is affected by the parameters of the input pulse as well as by linear and nonlinear propagation effects in the fiber, it is possible to control and to tailor the properties of the electrical pulses as shown below. The intensity at the output of the polarizer can change on a slow time scale as required to generate microwave or millimeter-wave pulses. We analyzed numerically the system using the split step Fourier method [11]. In order to understand the dependence of the electrical pulses on the parameters of the optical system, we also give explicit solutions to (3) when nonlinear effect or dispersion can be neglected.

The optical pulses were converted to electrical signals using an optical detector. The detector adds thermal and shot noise to the electrical signal. When the input optical power is high enough, the noise added by the detector does not significantly affect the system performance. The detector also limits the maximum bandwidth of the generated pulses due to its finite response time. Assuming that the noise added by the detector is negligible, the output current of the detector $i(t)$ equals $i(t) =$

$I(t) \star h(t)$, where $h(t)$ is the impulse response of the detector, $I(t)$ is the optical intensity of the pulse, and $\star$ denotes a convolution operator. Due to the large carrier frequency of optical waves, the optical system described in this paper can generate electrical pulses with an extremely broad bandwidth. Therefore, the parameters of the optical system should be chosen in order to ensure that the bandwidth of the optical pulses at the output of the polarizer will be narrower than the bandwidth of the detector. In this case, the shape of the optical and the electrical pulses is approximately the same. We choose the parameters of the optical system described in this paper in order to obtain electrical pulses with a bandwidth narrower than about 50 GHz. Such a bandwidth can now be obtained using commercial detectors. Therefore, we assumed in our calculations that the limited bandwidth of the detector does not significantly affect the output pulse profile. An optical InP/InGaAs uni-travelling-carrier photodiode with a full-width at half-maximum (FWHM) bandwidth of 220 GHz that operates at a wavelength regime of 1.55 μm has been demonstrated [12]. Using such a broadband optical detector in our system will enable us to generate electrical pulses with a broader spectrum than chosen in the current paper. Since the intensity of the detected light is always greater than zero, the amplitude of the electrical pulses has a single polarity. It is possible to obtain electrical pulses with a voltage that changes its polarity on time using a simple electronic circuit such as a mixer or an electrical filter.

When the two polarization components of the optical pulses that propagate in the birefringent fiber do not interact due to cross-phase-modulation effect, the optical system can be implemented using a Mach–Zehnder interferometer instead of a birefringent fiber. In such a device, the interference between pulses that propagate in different arms of the interferometer can be used to generate microwave and millimeter-wave pulses. In Section VI, we experimentally demonstrate the generation of electrical pulses with a controllable microwave frequency using such an interferometer.

III. Microwave Pulses Generated Using Dispersion Effect

When nonlinear effect can be neglected, the coupled wave equations become [11]

$$\begin{aligned} i\frac{\partial A_x(\tau_x,z)}{\partial z} &= \frac{1}{2}\beta_2 \frac{\partial^2 A_x(\tau_x,z)}{\partial \tau_x^2} \\ i\frac{\partial A_y(\tau_y,z)}{\partial z} &= \frac{1}{2}\beta_2 \frac{\partial^2 A_y(\tau_y,z)}{\partial \tau_y^2}. \end{aligned} \tag{4}$$

In order to obtain explicit results, we assume that the input pulse has a Gaussian profile $A(z=0,t) = A_0 \exp(-t^2/2T_0^2)$. We also assume that the input polarization is linear and aligned at an angle $\theta = 45^\circ$ with respect to the x-axis of the birefringent fiber. The amplitude of the two polarization components $A_{x,y}$ after propagating through a fiber with a length l is given by

$$A_{x,y}(l,\tau_{x,y}) = \frac{A_0}{\sqrt{2}} \frac{T_0}{(T_0^2 - i\beta_2 l)^{1/2}} \exp\left(-\frac{\tau_{x,y}^2}{2(T_0^2 - i\beta_2 l)}\right). \tag{5}$$

We assume that the two polarization components constructively interfere at the carrier frequency $(\beta_x - \beta_y)l = 2m\pi$, where m is an integer number. In the case when the axis of the polarizer is perpendicular to the polarization state of the input wave, the output intensity becomes

$$\begin{aligned} I_{\text{out}}(t) = &\frac{|A_0|^2}{4}\frac{T_0}{T(l)}\left\{\exp\left[-\left(\frac{t-\delta l}{T(l)}\right)^2\right]\right. \\ &+ \exp\left[-\left(\frac{t+\delta l}{T(l)}\right)^2\right] \\ &- 2\exp\left[\frac{-\left(t^2 + (\delta l)^2\right)}{T^2(l)}\right] \\ &\left.\cdot \cos\left[\frac{2(\delta l)(\beta_2 l)t}{T_0^2 T^2(l)}\right]\right\} \end{aligned} \tag{6}$$

where $l_D = T_0^2/\beta_2$ is the dispersion length and $T(z=l) = T_0(1 + (l/l_D)^2)^{1/2}$ is the width of the Gaussian pulse at the output of the fiber. The first and second terms in (6) are proportional to the intensities of the two polarization components at the output of the fiber. The third term is caused by the interference of the two polarization components. This term causes the generation of the microwave signal. Fig. 2 shows the output intensity calculated using (6). The figure indicates that the output pulse is modulated with a uniform modulation frequency. The modulation frequency equals 14.2 GHz in Fig. 2(a) and 9.5 GHz in Fig. 2(b). The instantaneous modulation frequency of the electrical signal ω_{RF} can be calculated directly from the third term in (6)

$$\omega_{\text{RF}} = \frac{2\delta l \beta_2 l}{T_0^4 + (\beta_2 l)^2}. \tag{7}$$

Equation (7) shows that the modulation frequency ω_{RF} does not change along the pulse as obtained in Fig. 2. The frequency ω_{RF} can be controlled by changing the group velocity delay of the fiber δl, the dispersion coefficient β_2, and the initial pulse width T_0. The duration of the microwave pulses can be controlled by changing the duration of the input optical pulse, the fiber length, and the dispersion. When the system is implemented using a fiber Bragg grating and a Mach–Zehnder interferometer, as discussed in Section VI, the electrical pulses can be simply controlled by changing the parameters of the grating or the interferometer. The minimum modulation frequency is limited by the pulse duration. The maximum modulation frequency is limited by the response time of the detector.

The results, shown in Fig. 2, can be intuitively understood by considering the effect of dispersion on the optical pulses. Dispersion induces a linear frequency chirp along the pulses since different frequency components that form the pulse propagate with a different phase velocity [11]. The dispersion equally affects the fields of the two polarization components. However, the propagation time of the two polarization components is different due to the birefringence of the fiber. Therefore, at a given time, the optical frequency of the two polarization components

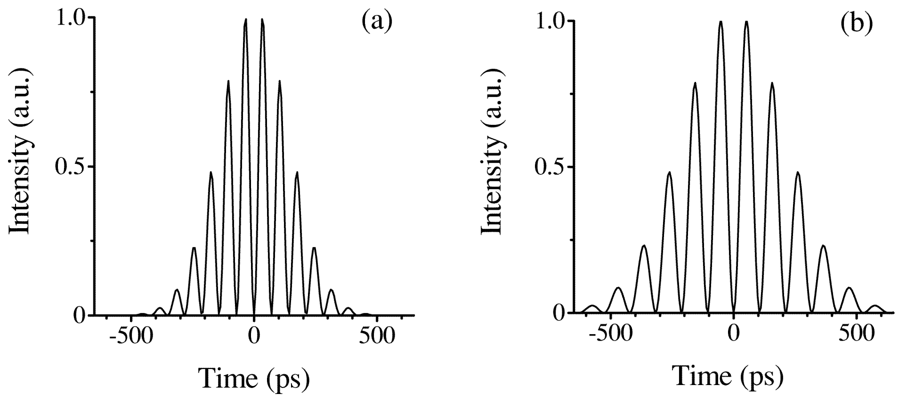

Fig. 2. Output intensity $I_{\text{out}}(t)$ calculated for an input Gaussian pulse with an FWHM $T = 2\ln(2)^{1/2}T_0 = 2.5$ ps that propagates in a birefringent fiber with a length $l = 15$ km, a group delay $\delta l = 13.5$ ps, and a dispersion coefficient (a) $\beta_2 = -20$ ps^{2}km^{-1} and (b) $\beta_2 = -30$ ps^{2}km^{-1}. The modulation frequency of the output pulse equals (a) $f = 14.2$ GHz and (b) $f = 9.5$ GHz.

becomes different and the output polarization state changes in time. The change in the output polarization is converted into an intensity change by the polarizer. Since chirp induced by second-order dispersion is linear, the modulation frequency of the microwave pulse does not change along the pulse.

IV. Microwave Pulses Generated Using Kerr Effect

In the previous section, we have shown that dispersion can be used to generate MW/MMW pulses with a frequency that can be controlled by changing the parameters of the optical system. When a high-intensity optical pulse propagates in a fiber, the optical nonlinear Kerr effect changes the phase and the instantaneous frequency of the pulse and generates new frequency components that do not exist in the input pulse. Therefore, the nonlinear Kerr effect can be used to generate microwave signals modulated with a complex frequency profile. Due to the fast response time of the optical Kerr effect, high-frequency modulation of the electrical pulses can be obtained without the need for a fast modulation of the input optical pulse. In order to understand how nonlinear Kerr effect can be used to generate microwave pulses, we neglect dispersion effect and obtain an implicit solution to (2) and (3).

When dispersion effect can be neglected, the coupled wave equations become

$$\begin{aligned}\frac{\partial A_x}{\partial z} &= i\gamma\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_x\\ \frac{\partial A_y}{\partial z} &= i\gamma\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_y.\end{aligned} \tag{8}$$

The solution of the coupled wave equations is given by [8]

$$\begin{aligned}A_{x,y}(l,t) = {} & A_{x,y}(0,\tau_{x,y})\exp\left[i\gamma\left|A_{x,y}(0,\tau_{x,y})\right|^2 l\right]\\ & \times\exp\left[i\frac{2}{3}\gamma\int_{-l}^{l}\frac{1}{2}\left|A_{y,x}(0,t\pm\delta z)\right|^2 dz\right].\end{aligned} \tag{9}$$

Since we neglect dispersion effect, the pulse intensity profile of each polarization component does not change while propagating in the fiber. In order to simplify the solution, we assume that the input pulse has a linear polarization, aligned at an angle $\theta = 45^\circ$, with respect to the x-axis of the birefringent fiber. In this case, it can be shown that the cross-phase-modulation effect, given by the nonlinear term in (9) that contains the constant 2/3, does not affect the output intensity since it adds equal phases to both polarization components.

After passing a polarizer, aligned at an angle ϕ with respect to the y-axis, the output intensity I_{out} becomes

$$\begin{aligned}I_{\text{out}} = \frac{1}{2}\Big\{ & \sin^2(\phi)\left|A_x(0,\tau_x)\right|^2 + \cos^2(\phi)\left|A_y(0,\tau_y)\right|^2\\ & -\sin(2\phi)\left|A_x(0,\tau_x)A_y(0,\tau_y)\right|\cos(\gamma l\Delta I)\Big\}\end{aligned} \tag{10}$$

where

$$\Delta I = |A_x(0,t-\delta l)|^2 - |A_y(0,t+\delta l)|^2 \tag{11}$$

is the difference between the output intensities of the two polarization components.

Fig. 3 shows the output intensity for an input Gaussian pulse with an FWHM $T = 2\ln(2)^{1/2}T_0 = 200$ ps and a maximum peak power (a) $|A_0|^2 = 4.8083$ W and (b) 6.1833 W that propagates in a birefringent fiber with a length of $l = 2$ km, nonlinear coefficient $\gamma = 5$ km^{-1}W^{-1}, and group delay $\delta l = 20$ ps. The results shown in Fig. 3 indicate that complex microwave pulses can be generated from Gaussian optical pulses. Fig. 3 also shows the instantaneous frequency of the microwave pulse. The modulation of the electrical signal is obtained due to beating between the two polarization components, described mathematically by the third term in (10). Therefore, the instantaneous frequency of the microwave signal ω_{RF} is defined as $\omega_{\text{RF}} = \gamma l d(\Delta I)/dt$. We choose to use the standard expression for the instantaneous frequency [11], defined as the derivative of the relative phase between the two polarization components. The derivative of the relative phase and hence the instantaneous frequency can have a positive or a negative sign. However, we note that since the output intensity in our system depends on the cosine of the phase difference between two output polarization components and since a cosine is an even function, the same output intensity is obtained for a negative or a positive instantaneous frequency. Therefore, in our system, negative or positive instantaneous frequencies have the same effect on the microwave pulse amplitude.

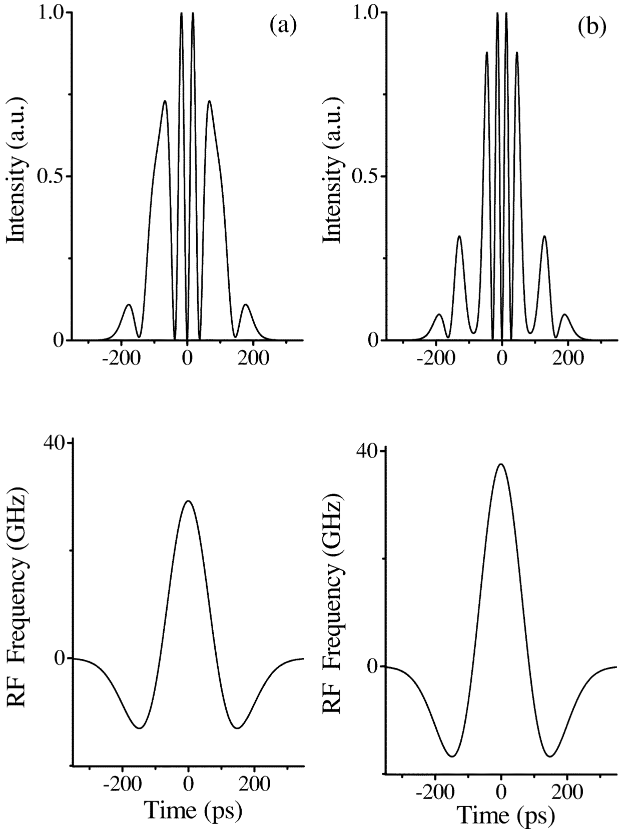

Fig. 3. Output intensity $I_{\rm out}(t)$ and instantaneous modulation frequency $\omega_{\rm RF}$ calculated for a Gaussian pulse with an FWHM $T = 200$ ps and a maximum peak power (a) $|A_0|^2 = 4.8083$ W and (b) 6.1833 W that propagates in a birefringent fiber with a length of $l = 2$ km, a nonlinear coefficient $\gamma = 5$ km^{-1}W^{-1}, and a group delay $\delta l = 20$ ps. The input pulse has a linear polarization rotated at an angle $\theta = 45^\circ$ with respect to the x-axis of the birefringent fiber. The polarizer is aligned at $\phi = 45^\circ$ with respect to the input polarization. The amplitude of the instantaneous frequency profile linearly depends on the input intensity of the pulse.

The results shown in Fig. 3 can be intuitively understood by considering the change in the optical pulse due to the propagation in the nonlinear fiber. The nonlinear effect adds a time-dependent phase to the two polarization components. When the input pulse is linearly polarized at an angle 45° with respect to one of the axes of the birefringence fiber, only self phase modulation effect causes the generation of the microwave pulses. The nonlinear interaction between the two polarization components (cross phase modulation) does not change the polarization of the optical wave that propagates in the fiber since it adds an equal phase to both polarization components. Since the magnitude of the nonlinear effect depends on the instantaneous intensity of the optical pulse, the phase and the instantaneous frequency of the two polarization components change in time. In the case when the input pulse is linearly polarized at an angle 45° with respect to the x-axis of the birefringent fiber, the input and the output fields of the two polarization components are the same. However, since each polarization component propagates with a slightly different group velocity, the output instantaneous frequency of the two polarization components becomes different at a given time. Therefore, changes in the input pulse intensity are converted by the birefringent fiber into polarization changes. By controlling the intensity of the input pulse, it is possible to control the magnitude of the electrical pulse modulation.

When the group velocity delay of the birefringence fiber δl is much shorter than the input pulse duration, the intensity of the output pulse can be expanded using a Taylor series

$$I(t \pm \delta l) = I(t) \pm (\delta l) I^{(1)}(t) + \frac{1}{2}(\delta l)^2 I^{(2)}(t) \pm \frac{1}{6}(\delta l)^3 I^{(3)}(t) + O\left(\delta^4 l^4\right) \quad (12)$$

where $I^{(i)}(t) = dI^{(i)}/dt^i$ represents the ith time derivative of the input pulse intensity $I(t) = |A_x(t, z=0)|^2 = |A_y(t, z=0)|^2$. The instantaneous frequency $\omega_{\rm RF} = \gamma l d(\Delta I)/dt$ can be calculated from (10)–(12)

$$\omega_{\rm RF} = -\gamma l \left[2\delta l I^{(2)}(t) + \frac{1}{3}(\delta l)^3 I^{(4)}(t)\right] + O\left(\delta^5 l^5\right). \quad (13)$$

Equation (13) shows that the instantaneous modulation frequency of the electrical pulse depends on the time derivatives of the optical pulse intensity. Therefore, the instantaneous frequency for a Gaussian pulse, shown in Fig. 3, is similar to the second derivative of a Gaussian function. The magnitude of the instantaneous frequency modulation linearly depends on the intensity of the input Gaussian pulse $|A_0|^2$. Therefore, it is possible to control the magnitude of the instantaneous modulation frequency profile simply by changing the intensity of the input pulse.

The profile of the electrical pulses can also be controlled by changing the angle of the output polarizer axis with respect to the axes of the birefringent fiber. Fig. 4 shows the profile of the microwave pulses for four different orientations of the polarizer. Besides the orientation of the polarizer, the other parameters of the system are identical to that used in Fig. 3(b). The change in the microwave pulse shape when the axis of the output polarizer is rotated can be understood from (10). The equation shows, for example, that when the axis of the polarizer is rotated by 90°, the sign of the beating component that causes the frequency modulation of the electrical pulses (the third term in the equation) is inverted. This result explains the difference between Fig. 4(a) and (c). When the nonlinear effect becomes dominant, the pulse propagation might be significantly affected by modulation instability [13]–[16]. In the case when the dispersion coefficient equals zero, the system is marginally stable. In order to ensure the stability of the system, a normal dispersion may be added, as discussed in the next section. When a small normal dispersion is added in order to ensure the stability of the system, the results do not qualitatively change compared to those described in this section.

V. Microwave and Millimeter-Wave Pulses Generated Using Dispersion and Kerr Effect

In the previous sections, we have shown that dispersion effect can be used to generate microwave pulses with a uniform instantaneous frequency and that Kerr effect can be used to generate microwave pulses modulated with a complex frequency profile that depends on the time derivatives of the input optical pulse intensity. The combination of the two effects enables us

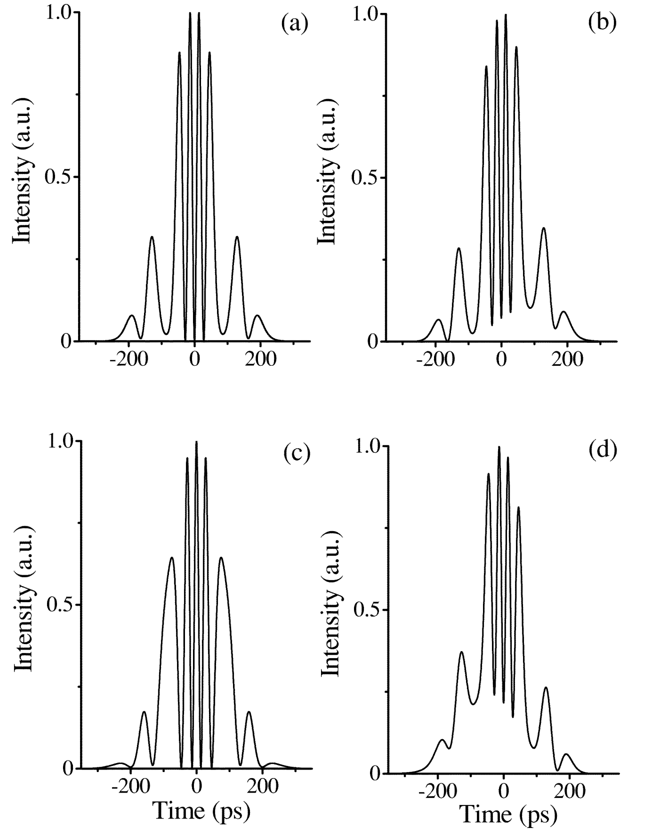

Fig. 4. Output intensity $I_{\rm out}(t)$ calculated for four different orientations of the output polarizer: (a) $\phi = 45^\circ$, (b) $\phi = 30^\circ$, (c) $\phi = 135^\circ$, and (d) $\phi = 70^\circ$. Besides the orientation of the polarizer, the parameters of the system are identical to those used in Fig. 3(b).

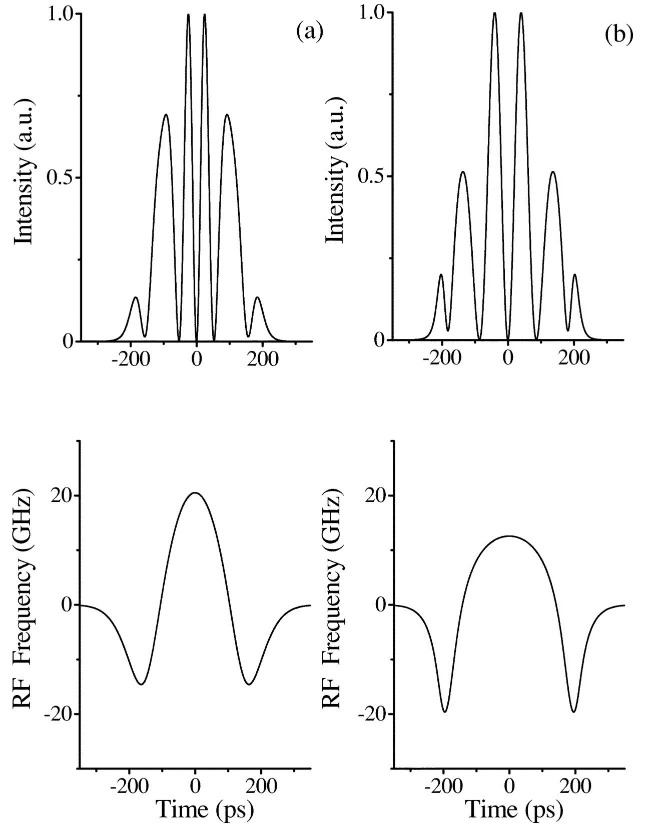

Fig. 5. Output intensity $I_{\rm out}(t)$ and instantaneous modulation frequency $\omega_{\rm RF}$ calculated for a Gaussian pulse with an FWHM $T = 200$ ps that propagates in a fiber with a dispersion coefficient (a) $\beta_2 = 30$ ps^{2}km^{-1} and (b) $\beta_2 = 100$ ps^{2}km^{-1}. Besides the dispersion coefficient of the fiber, the parameters of the system are identical to those used in Fig. 3(a).

to simplify the generation and control of electrical pulses modulated with a complex frequency profile. In order to calculate the output intensity when both dispersion and nonlinear effect become important, (2) was solved numerically using the split step Fourier method [11]. Since the combination of linear and nonlinear effects can form a rich variety of phenomena that can be used to generate microwave pulses, there is a need to develop an algorithm to find the parameters of the optical system and the input optical pulse in order to tailor the profile of the electrical pulses to the requirements. In this paper, we will only focus on the main phenomena that can be used to generate complex microwave pulses. In particular, we show that soliton trapping effect can be used to generate microwave pulses with a frequency that can be controlled by changing the power of the input optical pulses.

We demonstrated in Fig. 3 the generation of electrical pulses due to Kerr effect in the case when dispersion could be neglected. Fig. 5 shows how a normal dispersion changes the results shown in Fig. 3. The figure shows the output intensity and the instantaneous frequency of the microwave pulse $\omega_{\rm RF}$ for a fiber with a normal dispersion coefficient $\beta_2 = 30$ ps^{2}km^{-1} [Fig. 5(a)] and $\beta_2 = 100$ ps^{2}km^{-1} [Fig. 5(b)]. Besides the dispersion coefficient, the parameters of the system are identical to that used for calculating the results shown in Fig. 3(a). We have calculated the instantaneous frequency numerically using the connection $\omega_{\rm RF} = d[\arg\{A_x(l,t)\} - \arg\{A_y(l,t)\}]/dt$. In calculating the instantaneous frequency we assume that the intensity profile of the two polarization components changes smoothly on time and therefore it only affects the envelope of the microwave pulse and not its internal structure.

When the fiber has a normal dispersion coefficient, the optical pulse becomes broader as it propagates in the fiber. Therefore, the pulse intensity and the magnitude of modulation frequency of the microwave pulses decrease as the dispersion coefficient increases. The modulation instability effect in highly birefringent fibers can be obtained in the normal dispersion regime [13]–[14]. However, when the pulse power is greater than the critical power $P_c = 3\delta^2/\gamma\beta_2$, the modulation instability effect disappears. In the example shown in Fig. 5, the critical power equals $P_c = 2$ W [Fig. 5(a)] and $P_c = 0.6$ W [Fig. 5(b)], and therefore the solution shown in Fig. 5 is stable. In the anomalous dispersion regime, the combination of dispersion and nonlinear effect can shorten the duration of the optical pulses [17]–[19]. The decrease in the pulse duration increases the pulse intensity, and therefore the amplitude of modulation frequency of the electrical pulse increases. However, the maximum pulse power may be often limited by modulation instability [15]– [16].

One of the important effects that can be used to generate microwave pulses is soliton trapping [20]–[22]. Two orthogonally polarized solitons can propagate with the same group velocity in a birefringent fiber by shifting their carrier frequencies in opposite directions due to cross phase modulation effect. The frequency shifting of the two polarization com-

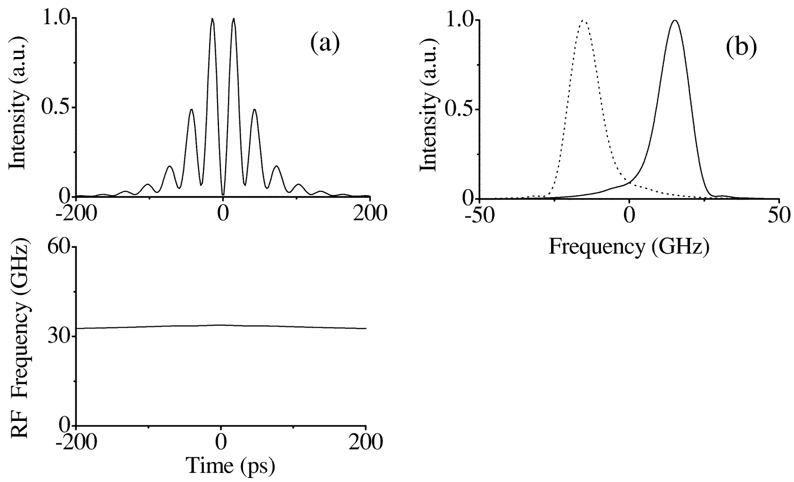

Fig. 6. (a) Output intensity and instantaneous frequency of a microwave pulse and (b) optical spectrum of the two polarization components calculated for an input pulse $A(t) = A_0 \text{sech}(t/T_0)$, with a duration $T_0 = 5$ ps, a maximum peak power $|A_0|^2 = 0.4488$ W that propagates in a fiber with a length of $l = 14$ km, a nonlinear coefficient $\gamma = 5$ km^{-1}W^{-1}, a group delay $\delta l = 78.4$ ps, and a dispersion coefficient $\beta_2 = -56$ ps^{-2}km^{-1}. The average modulation frequency is 33 GHz.

ponents causes a time modulation of the polarization state at the output of the fiber. After passing through the polarizer, the polarization changes are converted into modulation of the electrical pulses. The modulation frequency of the electrical pulse can be controlled by changing the parameters of the fiber such as dispersion or group delay as well as by changing the duration and the intensity of the input soliton.

Fig. 6 shows the output intensity and the instantaneous frequency of the generated pulses and the spectrum of the two polarization components for an input optical pulse with a hyperbolic secant profile $A(t) = A_0 \text{sech}(t/T_0)$, a duration $T_0 = 5$ ps, and a maximum peak power $|A_0|^2 = 0.4488$ W. The input pulse is linearly polarized at an angle $\theta = 45°$ with respect to the x-axis of the birefringent fiber. The birefringent fiber has a length $l = 14$ km, a nonlinear coefficient $\gamma = 5$ km^{-1}W^{-1}, a group delay $\delta l = 78.4$ ps, and a dispersion coefficient $\beta_2 = -56$ ps^{2}km^{-1}. The average modulation frequency of the electrical pulse equals 33 GHz. Small changes in the modulation frequency are obtained along the pulse, as shown in Fig. 6. It is possible to control the modulation frequency simply by changing the power of the input soliton. For example, when the input pulse had a maximum peak power $|A_0|^2 = 0.5935$ W, the average modulation frequency of the millimeter-wave pulse increased to 40 GHz. However, there is not a simple connection between the modulation frequency and the input power, since although the two polarization components are mutually bounded, the central frequency of the two components periodically changes along the fiber. Therefore, when the input intensity increases, the pulse frequency can increase or decrease, depending on the intensity of the input optical pulses and the parameters of the optical system.

VI. Experimental Results

In Section III, we showed theoretically that dispersion can be used to generate MW/MMW pulses with a constant instantaneous frequency that can be controlled by adjusting the fiber birefringence. When the two polarization components of the optical pulses that propagate in the birefringent fiber do not interact due to cross-phase-modulation effect, the optical system can be implemented using a Mach–Zehnder interferometer instead of a birefringent fiber. In a Mach–Zehnder interferometer, pulses that propagate in different arms of the interferometer interfere at the output of the device and can generate microwave or millimeter-wave pulses as obtained in Section III. The group velocity delay added by the Mach–Zehnder interferometer can be controlled by adjusting the length difference between the two arms of the interferometer. Therefore, in a system implemented using a Mach–Zehnder interferometer, the frequency of the electrical pulses can be easily controlled by changing the length of one of the interferometer arms using an optical delay line.

Fig. 7 shows the experimental setup used in our experiment. In order to obtain dispersion effect, we used a chirped fiber Bragg grating. Our source was an erbium-doped ring fiber laser that generated pulses at a repetition rate of 250 MHz, with a duration of 3 ps, at a central wavelength of about 1550 nm. The pulses were transmitted through a circulator and were reflected by a chirped fiber Bragg grating with a length of 10 cm and an average dispersion slope of 0.13 nm/cm around a center wavelength of 1549.8 nm. After passing the grating, the optical pulse duration increases to about 1000 ps. The electrical pulses were generated using a Mach–Zehnder interferometer and a detector. The difference between the lengths of the two arms of the interferometer was adjusted using an optical delay line. In order to increase the modulation depth of the electrical signal, we added in the interferometer a polarization controller and an attenuator. We used an optical detector with a 3-dB electrical bandwidth of 25 GHz. We added at the output of the detector an electrical bandpass filter with a central frequency of 10 GHz and a bandwidth of 5 GHz in order to filter slowly varying components in the electrical pulse caused by grating imperfections and to obtain electrical pulses with a voltage that can change its polarity. The filter increased the electrical pulse duration to about 2000 ps and added a small ripple at the end of the electrical pulse.

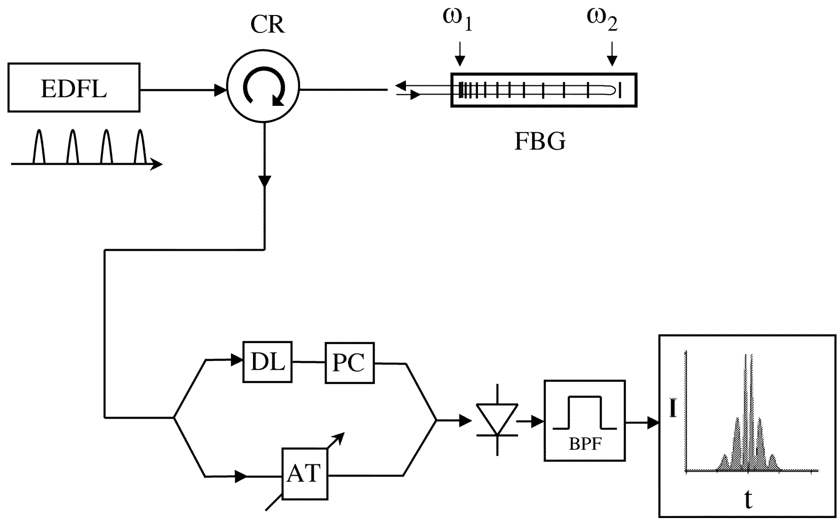

Fig. 7. Schematic description of the optical system used to generate microwave pulses with a uniform instantaneous frequency. CR is a circulator; FBG is a chirped fiber Bragg grating; DL is a delay line; PC is a polarization controller; AT is an attenuator, and BPF is an electrical bandpass filter. The optical source (EDFL) is an actively mode-locked erbium-doped fiber laser. The modulation frequency of the electrical pulses can be controlled by changing the length of the delay line.

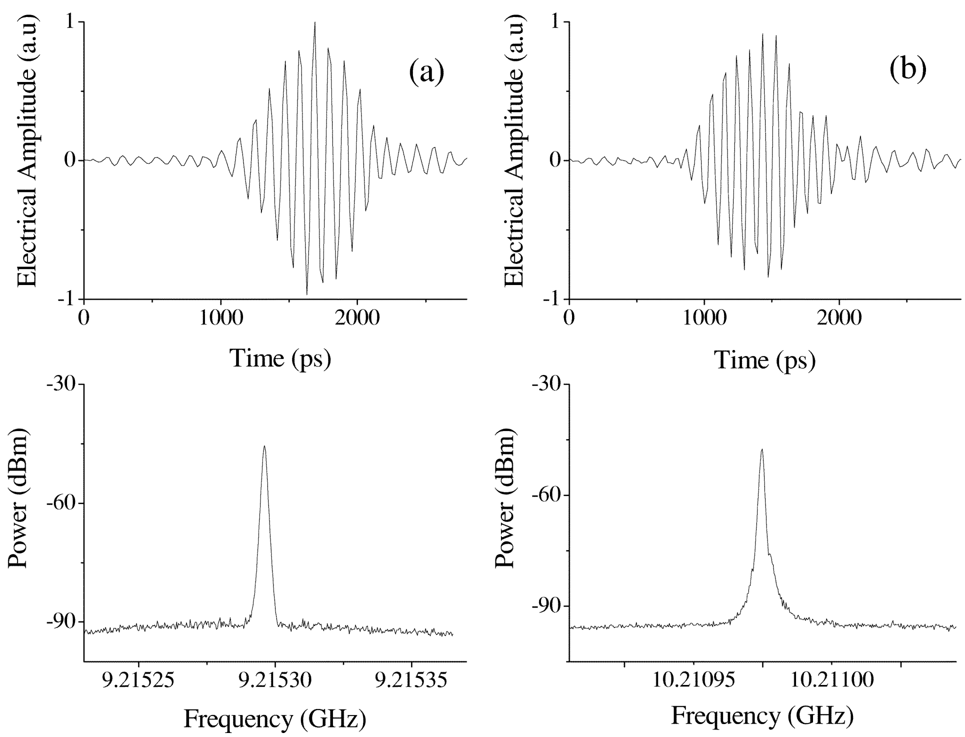

Fig. 8. Output electrical pulses, measured using a sampling scope and an electrical spectrum analyzer, obtained for two different lengths of the optical delay line. The modulation frequency of the pulses equals (a) 9.215 GHz and (b)10.21 GHz.

The mathematical analysis of the device used in the experiment is identical to that given in Section III. The group velocity delay between pulses that propagate in the different arms of the interferometer replaces the group velocity delay δl of the birefringent fiber. Therefore, by changing the difference between the lengths of the two interferometer arms, we could control the frequency of the electrical pulses as described in Section III.

Fig. 8 shows two electrical pulses measured using a sampling scope and an electrical spectrum analyzer. The modulation frequency was changed from 9.215 GHz [Fig. 8(a)] to 10.21 GHz [Fig. 8(b)] by adjusting the length of the optical delay line. The measured linewidth of the microwave signals, shown in Fig. 8(a) and (b), was 400 and 100 Hz, respectively. A small chirp in the frequency of the electrical pulses can be observed. The chirp is mainly caused by imperfections of the fiber Bragg grating. Since the output of an interferometer is sensitive to changes in environmental conditions, the phase of the modulation profile of the electrical pulse slowly changed on a time scale of a few seconds. This effect could be eliminated by isolating the optical system thermally. The small ripple at the end of the pulse was mainly added by the electrical bandpass filter and may be avoided using a filter without tails in its impulse response. The electrical filter also caused the change in the electrical pulse envelope obtained when the modulation frequency was changed.

VII. CONCLUSION

We have studied a new method to synthesize high-frequency complex microwave and millimeter-wave pulses using dispersion, Kerr effect, and group velocity delay in optical fiber systems. The structure of the electrical pulses could be changed by controlling the parameters of the optical systems. Optical Kerr effect could be used to generate electrical pulses with an extremely broad spectrum without the need for a fast electronic circuit. Implicit expressions were given when dispersion or nonlinear effect could be neglected. Soliton trapping could be used to generate electrical pulses with a controllable frequency. Generation of electrical pulses with a controllable microwave frequency was demonstrated experimentally using a Mach–Zehnder interferometer and a chirped fiber Bragg grating.

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Oren Levinson, photograph and biography not available at the time of publication.

Moshe Horowitz, photograph and biography not available at the time of publication.