Quality factor and absorption bandwidth of electrically small lossy structures

Ofer Markish1, Yehuda Leviatan1

1Department of Electrical Engineering, Technion, Israel Institute of Technology, Haifa 32000, Israel
E-mail: leviatan@ee.technion.ac.il

Abstract: A new quality factor ($Q_{abs}$) characterising power absorption by electrically small structures ranging from antennas to lossy objects is defined. In analogy to the radiation quality factor ($Q_{rad}$), which is evaluated using the antenna radiated fields in transmitting mode, $Q_{abs}$ is evaluated using the fields scattered by the absorbing structure. Similar to the known relationship between the antenna matching bandwidth ($B_{match}$) and $Q_{rad}$, it is rigorously shown that for an electrically small lossless receiving antenna $Q_{abs}$ is inversely proportional to the absorption bandwidth ($B_{abs}$) of the antenna. Based on a circuit model, it is then conjectured that the same $B_{abs}$–$Q_{abs}$ relation is also valid in the cases of electrically small lossy antennas and objects that do not have terminals. Numerical examples are shown to demonstrate the validity of the presented $B_{abs}$–$Q_{abs}$ relation.

1 Introduction

The theory dealing with the radiation quality factor ($Q_{rad}$) and its minimum value for an electrically small transmitting antenna is well established (see e.g. [1–4]). In saying that the antenna is ‘electrically small’ we mean that it can fit inside a sphere (Chu sphere) whose radius is smaller than the wavelength divided by 2 (or in some definitions by 4π) and thus the antenna radiation can be well described by the first-order spherical modes [5, 6]. It is shown in [7] that assuming the antenna has a well-defined single resonance, there is an inverse relationship between the matching bandwidth ($B_{match}$) of the antenna and $Q_{rad}$. The ability to predict $B_{match}$ based on the knowledge of $Q_{rad}$, a quantity that is evaluated merely at one frequency, is clearly attractive. The complimentary theory for receiving antennas and absorbers has received somewhat less attention. In [8], the characteristics allowing for such structures to attain optimal absorption at a certain frequency were studied. In [9–11], the subject of bounds on cumulative absorption over a frequency interval by antennas and lossy structures was tackled by the use of polarisability dyadics. In this paper, we continue with the subject of absorption by lossy structures but not in the context of the use of polarisability dyadics. In this paper, we continue with the ‘electrically small’ we mean that it can fit inside a sphere (Chu sphere) whose radius is smaller than the wavelength divided by 2.

2 $B_{match}$ – $Q_{rad}$ relation

With the view towards defining a new quality factor for absorption and establishing a relation between $B_{abs}$ and $Q_{abs}$, we first review the known relationship between $B_{match}$ and $Q_{rad}$ (see e.g. [7]). The tools used in this section will serve us in the following sections.

An arbitrarily-shaped electrically small lossless transmitting antenna is depicted schematically in Fig. 1a. A time-harmonic voltage source $v_{tx}$ with internal impedance $R_{tx}$ is feeding the antenna at an angular frequency $\omega$ through a transmission line with characteristic impedance $Z_{c}$. A tuning inductor or capacitor with reactance $X_{tune}(\omega)$ is connected to the antenna terminals and the volume around the tuning element is denoted by $V_{tune}$. The antenna is enclosed within a Chu sphere of radius $a$, and we denote this volume by $V_{chu}$. The antenna input impedance is $Z_{ant} = R_{rad} + jX_{ant}$, where $R_{rad}$ is the antenna radiation resistance and $X_{ant}$ is the antenna reactance. The voltage across the tuning element is $v_{tune}^{\prime}$ and the reflection coefficient $\Gamma(\omega)$ is given by

$$\Gamma(\omega) = \frac{Z_{tune}(\omega) + jX_{tune}(\omega)}{Z_{tune}(\omega) + jX_{tune}(\omega) + Z_{ant}(\omega)} .$$

(1)