Coupling effects of signal and pump beams in three-level saturable-gain media

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We theoretically investigate new coupling and indirect wave-mixing effects formed in a three-level atomic gain medium that is optically pumped by two mutually coherent beams that propagate in opposite directions. The interference of the pump beams and the saturation effects caused by the signal waves periodically modulate the gain along the amplifier that is due to spatial hole burning. In cases when the interference patterns of the pump and the signal waves are spatially synchronized, the signal gain becomes dependent on the frequencies and the optical phases of the pump and the signal waves. This dependence can be used for obtaining controllable narrow-band filters and for obtaining single-mode operation in lasers. © 1998 Optical Society of America [S0740-3224(98)00109-X]

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The effect of wave mixing or spatial hole burning in two-level saturable gain or absorber media has been investigated extensively.\textsuperscript{1–6} The interference between waves that propagate in different directions creates periodic modulation or a grating of the gain and the refractive index of the medium as the result of saturation effects. The self-induced gratings can couple and affect the signal waves that induced it. Gain and refractive-index gratings in erbium-doped fiber amplifiers (EDFA’s) have been studied\textsuperscript{7,8} and were used for obtaining optical filters in the 1.5-\mu m-wavelength regime where most optical communication systems operate.

In lasers with linear cavities, spatial hole burning is induced as a result of the interference of two counterpropagating waves that propagate inside the cavity. There are two basic approaches to analyzing the effect of spatial hole burning on laser operation. The first\textsuperscript{9} leads to the result that the average gain of each cavity mode is proportional to the overlap between the gain distribution and the interference pattern of the mode. The second approach\textsuperscript{10} is based on calculating the coupling between the intracavity counterpropagating waves that is due to diffraction from the self-induced grating. Both analyses indicate that in most of the interesting cases the wave mixing tends to decrease the modal gain and the laser power. This result causes multimode operation of lasers to eliminate the effect of the wave mixing by reducing the light coherence length.\textsuperscript{11} The reduction of the modal gain because of saturation effects can also be intuitively explained by consideration of the decrease in the spatial overlap between the gain profile and the interference pattern of the signal as a result of the gain saturation effects. In regions where the intensity is high, the gain is low. The gain is high where the intensity is low; however, those regions make only a small contribution to the overall gain. Alternatively, it can be shown that the reflections from the gain grating destructively interfere with their corresponding copropagating input waves and thus decrease the gain.\textsuperscript{7}

In cases when the pump level is less than the transparency threshold, the medium becomes a saturable absorber. Here the nonlinear wave-mixing effect has a role opposite that in the saturable-gain case,\textsuperscript{11} and the induced grating tends to decrease the losses of the waves that induced it. Therefore an absorber that is put inside a laser cavity can drive the laser to single-mode operation. This effect was theoretically analyzed and experimentally demonstrated in an erbium-doped fiber laser.\textsuperscript{11,12} Single mode operation with a spectral linewidth of less than 5 kHz was obtained as a result of insertion of the absorber into the cavity.\textsuperscript{11} A similar idea was used for obtaining an EDFA laser with a linewidth of less than 950 Hz (Ref. 13) and for obtaining single-frequency operation in ytterbium-doped fiber lasers.\textsuperscript{14}

Here we investigate new wave-mixing effects in a three-level atomic gain medium that is optically pumped by two mutually coherent counterpropagating pump waves that interfere inside the amplifier. In such amplifiers the contribution of the wave-mixing effect to the signal gain depends on the wavelength and on the optical phases of the pump waves. In particular, when the interference patterns of the pump and the signal waves are synchronized (as defined mathematically below), an increase of the signal gain owing to spatial hole burning (or, alternatively, to wave mixing) can be obtained. This result is in contrast to that obtained in an amplifier that is pumped by one beam or by two beams that are mutually incoherent; here the wave-mixing effect does not depend significantly on the pump wavelength, and it tends to decrease the modal gain. The dependence of the signal gain on the wavelength and the phases of the pump and the signal waves can be used for obtaining filters and single-mode operation of lasers. One can control the wavelength of such a laser by changing the wavelength or the phases of the pump waves. Previously,\textsuperscript{15} we analyzed the effect of periodic pumping on the signal gain for the case in which the signal waves do not interfere. In this case the amplifier gain does not depend significantly
on the signal wavelength. However, the interference of the pump waves can cause bistable operation of the laser owing to changes in the saturation behavior of the signal gain.

In our analysis we model an amplifier with a three- level atomic system (Fig. 1) and neglect, for simplicity, the absorption of the pump wave from excited states (i.e., only atoms from the ground state can be excited by the pump). A similar model was used to approximate the behavior of EDFA.

Signal gain coefficient $g$ and pump absorption coefficient $a$ in a three-level energy amplifier can be written as

$$g = N_t \sigma_{as} \frac{\eta I_p - 1}{1 + I_p + I_s},$$

$$a = N_t \sigma_{ap} \frac{1 + I_s \eta_s}{1 + I_p + I_p},$$

where $I_s = I_{signal}/I_{sat}$ is the normalized signal intensity ($I_{sat}$ is the saturation intensity for an unpumped medium), $I_p = I_{pump}/I_{p0}$ is the normalized pump intensity ($I_{p0}$ is the pump intensity that decreases the population in the ground state to half of its initial value without the pumping), $\sigma_{as}$ ($\sigma_{ap}$) is the signal (pump) absorption cross section, $N_t$ is the density of the erbium ions, and $\eta$ is the ratio between the emission and the absorption cross sections. Because of the interference of counterpropagating waves, the intensities of the signal and the pump vary along the amplifier. Assuming that $A_{z+p}$ and $A_{z-s}$ are the amplitudes of the pump and the signal waves, respectively (where $\pm$ corresponds to the wave that propagates along the $\pm z$ axis), the spatial dependence of the gain coefficient can be described as

$$g = N_t \sigma_{as} \frac{\eta_s (I_{pt} + c_p U_p + c_p^* U_p^{-1}) - 1}{1 + I_{pt} + c_p U_p + c_p^* U_p^{-1} + I_{st} + c_s U_s + c_s^* U_s^{-1}},$$

where $U_p = \exp(2i k_0 z)$, $U_s = \exp(2i k_z z)$ ($k_0$ and $k_z$ are the wave numbers of the pump signal and the waves, respectively), $c_p = A_{z+p} A_{z-p}^*$, $c_s = A_{z-s} A_{z+s}^*$, $I_{pt} = I_{z+p} + I_{z-p}$, and $I_{st} = I_{z+s} + I_{z-s}$. The gain coefficient can be expanded into its Fourier components: $g = g_{dc} + g_{1} U_s + g_{2} U_s^* + 0(U_s^2)$ and $a = A_{dc} + A_{1} U_p + A_{2} U_p^* + 0(U_p^2)$, where $g_{dc}$ ($A_{dc}$) is the zero or the average gain (absorption) coefficient for the signal (pump) wave, $g_{\pm 1}$ ($A_{\pm 1}$) are the first Fourier components with spatial frequencies of $\pm 2k_z$ ($\pm 2k_p$), and $0(U_p^2)$ and $0(U_s^2)$ are the higher-order Fourier components. The expressions for the various Fourier components were calculated analytically. However, the complete expressions for the Fourier components are lengthy and are not given in this paper. Only the first-order Fourier component of the grating ($g_{\pm 1}$) fulfills the Bragg condition and can diffract the signal waves. However, in cases when $k_p = m k_z$, where $m$ is an integer ($m > 1$), the grating that is induced by the signal waves contains a Fourier component that causes diffraction of the pump waves. The amplitude of such high-order gratings is usually quite small, and therefore we neglect such diffraction and focus on the effect of the pump interference on the Fourier component $g_{\pm 1}$.

The coupled-wave equations for the pump and the signal waves are

$$\frac{dA_{+s}}{dz} = N_t \sigma_{as} (g_{dc} A_{+s} + g_{-1} A_{-s}),$$

$$\frac{dA_{-s}}{dz} = -N_t \sigma_{as} (g_{dc} A_{-s} + g_{1} A_{+s}),$$

$$\frac{dA_{+p}}{dz} = N_t \sigma_{ap} (a_{dc} A_{+p} + a_{-1} A_{-p}),$$

$$\frac{dA_{-p}}{dz} = -N_t \sigma_{ap} (a_{dc} A_{-p} + a_{1} A_{+p}).$$

Equations (4)–(7) were solved numerically. The effect of the periodic pumping on the signal gain is shown below for the case in which the interference patterns of the pump and the signal waves are synchronized and the ratio between the wavelengths of the signal and the pump wave is $\lambda_s/\lambda_p = 2$. In an erbium-doped fiber this ratio is obtained, for example, when the wavelengths of the pump and the signal are approximately 780–800 and $\sim$1550 nm, respectively, outside the fiber. (The ratio between the wavelengths outside and inside the fiber is slightly different because of the dispersion.) An optical gain in EDFA in the 1550-nm regime can be obtained by pumping the amplifier in the 780–800-nm regime, as assumed in our analysis.

Figure 2 demonstrates the wave-mixing effect for the case when the peaks of the signal interference pattern ($I_s$) coincide with intensity minima of the pump interference pattern ($I_p$). The gain distribution for the depleted case in which the pumps interfere and the intensity of the signal is low (compared with the saturation intensity) is shown in Fig. 2 ($g_{as}$). The change in the sign of the gain coefficient shown in the figure indicates that the periodically pumped medium can be considered a combination of amplifying regions (which are located where the pump intensity is high) and absorbing regions (located where the pump intensity is low). The gain grating in this case does not diffract the signal because its periodicity, which equals that of the pump interference pattern, is lower than that of the signal interference pattern, and hence the Bragg condition is not met.

When the signal intensity is increased, the gain profile is changed because of saturation effects. The saturation
average gain constant (\(g_{dc}\)). The figure indicates that the average gain coefficient (\(g_{dc}\)) is significantly decreased owing to saturation effects. Figure 3 also indicates that the gain obtained for the case of Fig. 2 is higher than that obtained for the case when the signal waves are mutually incoherent and do not interfere. Because of the existence of amplifying and absorbing regions in the bidirectionally pumped amplifier, the grating can have a spatial phase shift compared with the interference pattern of the signal wave. In this case the gain coefficient has an imaginary part, \(\text{Im}(g_{dc})\), which affects the signal phase as expected from the Kramers– Kronig relation for a narrow-band filter.

The effect of wave mixing on modal gain demonstrated in Fig. 2 depends on the signal power and on the relative phase between the signal and the pump interference patterns. For example, one can invert the sign of the first Fourier component (\(g_1\)) (compared with that in Fig. 2) by changing the optical phase of one of the signal beams by \(\pi/2\). In this case, demonstrated in Fig. 4, the peaks of the signal interference pattern coincide with the intensity maxima of the pump interference pattern. Therefore the sign of \(g_1\) is negative (as shown in Fig. 3) and the signal gain is significantly lower than that of Fig. 2 and the incoherent case.

In general, synchronization between the interference patterns of the signal and the pump waves can be obtained when the ratio between the wavelengths of the signal and the pump (inside the medium) is \(\lambda_s/\lambda_p = m/n\), where \(m\) and \(n\) are integer numbers. However, generally speaking, the effect of the pump interference is significantly decreased as \(m\) and \(n\) are increased because the saturation begins to affect both amplifying and absorbing regions in the amplifier and thus the wave-mixing effect

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**Fig. 2.** Wave intensities and spatial gain profiles along the amplifier with \(\lambda_s = 2\lambda_p\), where the maxima of the signal interference pattern coincide with minima of the pumping intensity. \(I_s\) and \(I_p\) show the spatial distribution of the pump and the signal interference pattern; \(g_{sa}\) is the gain for the undepleted case in which the signal intensity is much smaller than the saturation intensity (\(I_s = 0.01\)), and \(g\) is the gain for a signal with a high intensity (\(I_{st} = 10\)). \(I_{pt} = 1.2\), \(N_i\sigma_{sw} = 1\), \(\eta_s = 1.18\).

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**Fig. 3.** Dependence of the average gain coefficient (\(g_{dc}\)), the first Fourier coefficient (\(g_1\)), and the total gain (\(g_1 = g_{dc} + g_1\)) for the cases shown in Fig. 2 (solid curves) and in Fig. 4 (dashed curves). For comparison the total gain dependence for the incoherent case in which the signal waves do not interfere is added (dashed–dotted curve). The intensities of the counterpropagating signals are assumed to be equal. \(N_i\sigma_{sw} = 1\), \(\eta_s = 1.18\), \(I_{pt} = 1.7\).
gain by changing the spatial overlap between the pump and the signal interference patterns. The interference pattern of the signal waves is determined by the wavelength and the optical phases of the counterpropagating signal waves. Therefore the wave-mixing effect in a periodically pumped amplifier can be used for obtaining optical filters with a centered frequency that can be controlled by adjustment of the pump parameters. To analyze this filtering behavior we calculated the reflectivity of an optical filter (Fig. 5) that consists of a three-level optical amplifier attached to an ideal mirror, with reflectivity $R = 1$, similarly to the analysis in Ref. 12. Figure 6 shows the reflectivity of the signal wave ($I_{\text{out}}/I_{\text{in}}$) for the case when $k_p \approx 2k_s$ as a function of the phase difference, or the detuning, $\Delta kl$ between the pump and the signal interference patterns, where $\Delta kl = (2k_s - k_p)l$ and $l$ is the amplifier length. We calculated the reflectivity by numerically integrating the coupled-wave equations for the signal and the pump waves [Eqs. (4)–(7)]. The optical phases of the pump waves were chosen to be the values that give a maximum gain when $\Delta k = 0$. Figure 6 shows that the reflectivity of the signal wave strongly depends on its frequency. The periodicity of the reflectivity function and the bandwidth of its envelope are determined by the amplifier length. An increase of the detuning ($\Delta kl$) by $\pi/4$ changes the reflectivity from maximum to minimum. One can change the frequency where the maximum transmissivity of the filter is obtained by controlling the wavelength and the optical phases of the pump waves. In our calculation we neglected the dependence of the signal cross section ($\alpha_m$) on the frequency of the signal. Such an assumption is valid for an EDFA, for which the bandwidth of the gain (of the order of a terahertz) is much broader than the bandwidth of our filter ($\approx 10^9$ Hz).\(^7,12\)

A numerical analysis of the spectral behavior for the case when $\lambda_p/\lambda_s = m/n$ indicates that, as $m$ and $n$ are increased, the bandwidth of the filter decreases while the ratio between the maximum and the minimum reflectivity decreases. The increase of $m$ and $n$ causes the alteration of the wave-mixing effect along the amplifier: An increase of the gain in some regions of the amplifier and a

in the absorbing regions is partially canceled by the gain decrease in the amplifying regions. When the signal and the pump interference patterns become completely unsynchronized, the effect of the pump interference on the signal wave mixing vanishes. In this case, the wave mixing tends to decrease the overall gain when the active medium is pumped above threshold, as happens in a conventional saturable amplifier. We note that a detailed mathematical analysis for an amplifier that is uniformly pumped indicates that wave-mixing effects can also form a grating with a positive first-order Fourier component ($g_1 > 0$). However, such a grating is formed only when the signal intensity is much higher than the saturation intensity; thus the amplifier gain, as well as the gain increase that is due to the grating formation, is small.

The discussion above shows that one can control the signal gain in an amplifier with a spatially modulated

![Image 59x554 to 292x737](image)

**Fig. 4.** Wave-mixing effect in a periodically pumped amplifier for $\lambda_p = 2\lambda_s$, where the maxima of the signal interference pattern coincide with the maxima of the pumping intensity. $I_p$ and $I_s$ show the spatial profile of the pump and the signal interference pattern; $g_{\alpha m}$ is the gain for the undepleted case in which the signal intensity is much smaller than the saturation intensity ($I_s = 0.01$), and $g$ is the gain for a signal with high intensity ($I_s = 10$). $I_{p1} = 1.2, N_1, \alpha_{m1} = 1, \eta_s = 1.18$.

![Image 328x89 to 550x265](image)

**Fig. 5.** Schematic description of a nonlinear filter. $l$ is the length of the amplifier (Am), $M$ is an ideal mirror with reflectivity $R = 1$, $I_{\text{in}}$ and $I_{\text{out}}$ are the intensities of the input and the output signal waves, and $I_{\text{prop}}$ and $I_{\text{app}}$ are the intensities of the counterpropagating pump waves. One can adjust the wavelength for which the transmission of the filter is maximum by controlling the frequency and the optical phases of the pump waves.

**Fig. 6.** Reflectivity of the filter shown in Fig. 5 versus the detuning between the signal and the pump [$(2k_s - k_p)l$]. $I_{\text{in}} = 0.6, N_1, \alpha_{m1} = 4.6, R = 1, I_{\text{prop}} = I_{\text{app}} = 0.85, \eta_s = 1.18$.
and hence the bandwidth of the filter increases. causes the frequency dependence of the gain decreases, and $m$ incoherent. $N_t$ most curve) and for the case when the signal waves are mutually gating signals are mutually coherent and can interfere (uppermost curve) and for the case when the signal waves are mutually incoherent. $N_t/\sigma_{\text{int}} = 4.6$, $R = 1$, $\sigma_{\text{int}} = 0.85$, $\eta_s = 1.18$.

decrease of the gain at other regions. Therefore, as $m$ and $n$ are increased, the effective amplifier length that causes the frequency dependence of the gain decreases, and hence the bandwidth of the filter increases.

Previously we demonstrated theoretically and experimentally that a saturable absorber that is put inside a laser cavity can promote single-mode operation in an erbium-doped fiber laser because of the wave-mixing effect in the absorber. The measured bandwidth was less than 5 kHz; however, the lasing frequency was self-determined at the time when the laser was turned on. The behavior of the nonlinear filtering that was analyzed in Ref. 12 was similar to that obtained for a periodically pumped medium (shown in Fig. 6). Therefore the filtering caused by the periodic pumping can also be used for obtaining single-mode operation of lasers. Modes that are enhanced by the wave mixing [i.e., $\text{Re}(g_1) > 0$] will be promoted, whereas other modes with lower modal gain will be rejected. Figure 7 shows the reflectivity of the filter shown in Fig. 5 versus the normalized signal intensity for the case when the two counterpropagating signal waves are mutually coherent and interfere (uppermost curve) and for the case when the signal waves are not mutually coherent. The figure shows that the gain for the coherent case is higher than that for the incoherent case, and therefore we expect that the wave-mixing effect will yield single-mode operation of lasers, as obtained in Ref. 11. Note that in the research reported in Ref. 11 the rejection of modes with low intensities was caused by a grating that was induced by the lasing mode, whereas in periodically pumped medium the filtering behavior is caused by synchronization between the signal and the pump interference pattern. Therefore one can control the lasing frequency in a periodically pumped medium by changing the frequency and the phases of the pump waves.

In conclusion, we have analyzed new coupling and indirect wave-mixing effects in an amplifier that is optically pumped by two mutually coherent pump waves. The interference of the pump waves causes a spatial modulation of the gain along the active medium. In regions where the pumping is high the signal is amplified, whereas in regions where the pump intensity is low the signal is absorbed. The wave-mixing effect in such an amplifier has special filtering behavior, and it can increase or decrease the modal gain, depending on the synchronization between the interference patterns of the pump and the signal waves. This phenomenon can be used for filtering and for obtaining single-mode operation of lasers. One can determine the frequency of such a laser by controlling the wavelengths and the optical phases of the counterpropagating pump waves.

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