

Using Zodiacal Light For Spaceborne Calibration Of Polarimetric Imagers

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Abstract—We propose that spaceborne polarimetric imagers can be calibrated, or self-calibrated using zodiacal light (ZL). ZL is created by a cloud of interplanetary dust particles. It has a significant degree of polarization in a wide field of view. From space, ZL is unaffected by terrestrial disturbances. ZL is insensitive to the camera location, so it is suited for simultaneous cross-calibration of satellite constellations. ZL changes on a scale of months, thus being a quasi-constant target in realistic calibration sessions. We derive a forward model for polarimetric image formation. Based on it, we formulate an inverse problem for polarimetric calibration and self-calibration, as well as an algorithm for the solution. The methods here are demonstrated in simulations. Towards these simulations, we render polarized images of the sky, including ZL from space, polarimetric disturbances, and imaging noise.

Index Terms—Computational photography, Polarimetric calibration, Astronomy

1 INTRODUCTION

POLARIMETRIC cameras have various uses in computational photography [1], machine vision and scientific imaging. Examples include dynamic interferometry [2], image descattering [3], [4], [5] and lightfield imaging [6]. Polarization plays an important role in studying astronomical sources, a few outstanding examples of which are: Establishing a unified picture of obscured and unobscured sources near supermassive black holes using visible light [7], exploring the symmetry of relativistic radio jets [8], probing powerful magnetic fields in a γ -ray burst from an exploding star [9], and most recently, detecting the jet launching region from a black-hole accretion disk in X-rays [10].

As most scientific tools [11], [12], [13], [14], [15], [16], [17], [18], [19] polarimetric imagers require calibration. Polarization signals tend to be much subtler than radiance. Therefore, careful calibration is particularly needed and more challenging in polarimetry, relative to geometry and radiometry. Polarimetric calibration in a laboratory is well-established [20], [21], [22]. However, in some cases, calibration is required in the field. The reason is that outdoors, over time, instruments tend to degrade and shift away from their pre-calibrated settings. This problem is especially true for spaceborne instruments due to harsh conditions during launch and operation [23]: orbiting instruments experience extreme repetitive thermal changes in a vacuum, radiation damage, and exposure to reactive ions.

This paper focuses on polarimetric calibration and self-calibration of spaceborne imagers. Self calibration requires observing sources that have a significant degree of polarization and can cover the field of view. Polarimetric self-calibration on Earth [25], [26] focused on shiny specular objects. These are uncommon in space. Dedicated onboard calibration hardware [27], [28] requires resources that are



Fig. 1. Zodiacal light, photographed [24] at ESO's La Silla Observatory in Chile, September 2009.

limited or non-existent in some spacecraft. Specifically, on-board calibration requires moving parts, which may not be permitted in some satellites, due to kinetic implications and reliability risks. Some spacecraft calibrate polarimetric imagers by observing sunglint reflection on the ocean [29], [30], [31]. However, this signal is unreliable, being affected by random winds (that roughen the water) and unknown aerosol conditions. Variable aerosol conditions also affect the polarized signals reflecting from solar farms [32]. A calibrated instrument in one spacecraft can quantify a scene for *cross calibration* of another spaceborne instrument [33]. However, in optical wavelengths, currently, there are no such available systems.

We suggest solving the problem by harnessing the polarization of a common, wide source in the solar system: the faint zodiacal light (ZL), seen in Fig. 1. ZL is created by a dust cloud measuring several astronomical units (AU), which is much larger than Earth. So, for spaceborne systems, ZL is convenient to rely on and has several advantages:

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ZL is insensitive to the camera location, say, of a satellite in low Earth orbit (LEO) [34]. Hence, ZL can be used to simultaneously calibrate a constellation of orbiting satellites. From space, ZL is unaffected by atmospheric scattering or other terrestrial disturbances. ZL has a significant degree of linear polarization (DoLP) in a wide field of view. Its signal changes very slowly in time, on the scale of months, as the dust cloud orbits the sun. It is thus a quasi-constant target in realistic calibration sessions. Using ZL, allows for calibration far from Earth, in deep space missions to other planets or asteroids. We do not, however, recommend ZL as a calibration source on Earth, due to atmospheric interference.

This paper is the first to propose ZL as a source for polarimetric calibration. We derive a forward model for image formation. Based on the model, we formulate an inverse problem for polarimetric calibration and self-calibration in space. The paper then presents an algorithm for a solution.¹ The present work is timely, as in the coming years, several polarimetric imagers are expected to be launched to LEO [34], [35], [36]: Using these planned imagers, these methods can be demonstrated empirically. Meanwhile, the methods here are demonstrated in simulations. Toward these simulations, we create a simulator to render ad-hoc polarized images of the sky (including ZL) from space, accounting for noise and polarimetric disturbances.

2 THEORETICAL BACKGROUND

2.1 Interplanetary Dust

Zodiacal light (ZL) is seen Fig. 1. It is created by the scattering of sunlight and by thermal self-emission from interplanetary dust particles (IDPs). Sources of IDPs include comet dust, asteroidal dust, Kuiper belt dust, and interstellar dust. Interplanetary dust is significant for understanding the formation and evolution of our solar system and others [37].

IDPs have been studied by several space missions, including the Infrared Astronomical Satellite (IRAS) [38], the Cosmic Background Explorer (COBE) [39], and the Infrared Space Observatory (ISO) [40]. These and additional studies [41], [42], [43] advanced knowledge of the dust's spatial distribution of density, temperature and microphysics (albedo, shape, size distribution, refractive index, chemical content). IDPs are often made of silicate or carbonaceous materials such as graphite. Typical IDP sizes range from fractions of μm up to tens of μm .

IDPs mainly reside in a non-uniform cloud [44], concentrated in and near the ecliptic plane (which includes Earth's orbit). The Kelsall model [45] describes the IDP cloud, based on components having various spatial and dynamical properties. Based on this model, the dust density is illustrated in Fig. 2. The cloud is dynamic, as each IDP is subject to gravitational forces, radiation pressure, and Poynting–Robertson drag (a radiative process that causes an IDP to lose angular momentum [46]). From Earth, in the short-wave infrared range [46], the optical depth of the IDP cloud is $\sim 5 \cdot 10^{-8}$, for absorption and scattering.

1. Our code is available at <https://github.com/oravitan/zodiacal-polarized>

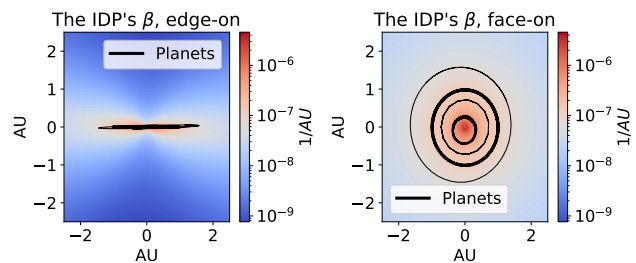


Fig. 2. Edge-on (left) and face-on (right) cross-sections of the extinction coefficient β of the IDPs, based on the Kelsall model.

2.2 Zodiacal Light

The IDP cloud is mainly concentrated between the sun and the asteroid belt. IDP density, irradiance, and temperature decrease with distance from the sun (the temperature is $\sim 250^\circ K$ near Earth [46]). Hence, ZL is brighter near the sun. As a result, from Earth, ZL can be seen by the naked eye near the horizon, around the ecliptic, for a short time after sunset and before dawn (See Fig. 1). It is very faint, thus observations require being away from stray light and atmospheric pollution sources, especially towards the horizon. Using long exposures, ZL is measured at night from ground stations [47]. The best way to observe it, of course, is to be in space, away from any light pollution created by atmospheric scattering and devoid of atmospheric attenuation. This paper uses spaceborne imaging of ZL, thus the optical models we use have no atmospheric contribution.

Thermal self-emission is unpolarized and it is dominant only in long-wave infrared. The visible and near-infrared spectrum of ZL is similar to that of sunlight, which IDPs scatter. This scattering is partially polarized (ZL DoLP is as high as 20%), which benefits our calibration goal. The DoLP is typically higher in viewing directions where the ZL is fainter [37].

2.3 Zodiacal Light Model

Due to the low optical depth of the IDP cloud, the model of ZL radiance resembles a haze model as in [48], [49], having single scattering of sunlight and background of planets and stars, in addition to thermal self-emission.

Let \mathbf{X} denote location in 3D space. Let λ denote wavelength. The spectral radiance from a region around \mathbf{X} is $L_{\text{scene}}(\mathbf{X}, \lambda)$. The IDP cloud is comprised of various components, created by various sources (comets, asteroids, etc.). Index a component by $\kappa = 1 \dots N_{\text{comp}}$. The cloud is optically very thin, hence components practically do not attenuate or scatter each other. Hence their contribution to the spectral radiance of the scene is additive:

$$L_{\text{scene}}(\mathbf{X}, \lambda) = \sum_{\kappa=1}^{N_{\text{comp}}} L_{\kappa}(\mathbf{X}, \lambda) \left[\frac{W}{m^2 \cdot sr \cdot \mu m \cdot AU} \right] \cdot (1)$$

We now describe how each component is expressed. Each component has spatially varying, wavelength-dependent particle albedo $A_{\kappa, \mathbf{X}, \lambda}$, phase function $\Phi_{\kappa, \mathbf{X}, \lambda}$ [sr^{-1}] and temperature $T_{\kappa, \mathbf{X}, \lambda}$. The extinction coefficient (including both scattering and absorption) $\beta_{\kappa, \mathbf{X}, \lambda}$ has units of AU^{-1} . IDPs are illuminated by solar flux density $F_{\mathbf{X}, \lambda}$ in units

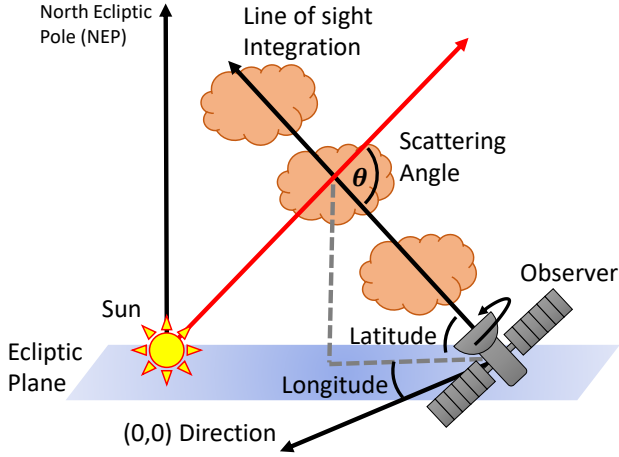


Fig. 3. Line of sight scattering integration.

of $\frac{W}{m^2 \cdot \mu m}$. Let h, c, k^B denote Planck's constant, the speed of light in vacuum, and Boltzmann's constant, respectively. Thermally emitted radiance by IDPs is approximately described by black body radiation, proportional to

$$B_{\kappa, \mathbf{X}, \lambda} = \frac{2hc^2}{\lambda^5 [\exp(hc/k^B \lambda T_{\kappa, \mathbf{X}, \lambda}) - 1]} \left[\frac{W}{m^2 \cdot sr \cdot \mu m} \right]. \quad (2)$$

Deviations from the black body model are expressed [45] by a unitless emissivity $E_{\kappa, \lambda}$.

Accounting both for scattering and self-emission [45], the volumetric spectral radiance of component κ from a region around \mathbf{X} and wavelength λ is

$$L_{\kappa}(\mathbf{X}, \lambda) = \beta_{\kappa} [A_{\kappa} F \Phi_{\kappa} + (1 - A_{\kappa}) E_{\kappa} B_{\kappa}], \quad (3)$$

where we omitted the \mathbf{X}, λ subscripts on the right-hand side, for brevity. Due to the very low optical depth of the IDP cloud, neglecting the attenuation is valid along any line of sight (LOS) \mathcal{L} , and between \mathbf{X} and the sun. The ZL is obtained by LOS integration of $L_{\text{scene}}(\mathbf{X}, \lambda)$ along \mathcal{L} :

$$I_{ZL}(\mathcal{L}, \lambda) = \int_{\mathbf{X} \in \mathcal{L}} L_{\text{scene}}(\mathbf{X}, \lambda) d\mathbf{X} \left[\frac{W}{m^2 \cdot sr \cdot \mu m} \right]. \quad (4)$$

This is illustrated in Fig. 3.

Kelsall's Model

The Kelsall model [45] sets the parameters above, for short-wave infra-red bands. The model is based on IRAS, COBE, and ISO data, and is designed to yield unpolarized ZL radiance expressed in Eq. (4). The IDP density stemming from this model is illustrated in Fig. 2. Let \mathbf{X}_{\odot} be the location of the Sun and \mathbf{X}_C be the location of the camera. The direction along \mathcal{L} is $\hat{\mathcal{L}} = [\mathbf{X} - \mathbf{X}_C] / \|\mathbf{X} - \mathbf{X}_C\|$. Solar illumination at a voxel is $\hat{\mathbf{d}}_{\odot} = [\mathbf{X} - \mathbf{X}_{\odot}] / \|\mathbf{X} - \mathbf{X}_{\odot}\|$. Then, the scattering angle is

$$\theta(\mathbf{X}) = \arccos(-\hat{\mathcal{L}} \cdot \hat{\mathbf{d}}_{\odot}). \quad (5)$$

In the Kelsall model, the unpolarized phase function for scattering at angle θ is parameterized by

$$\Phi_{\kappa, \lambda}^{\text{kelsall}}(\theta) = C_{\kappa, \lambda}^{(0)} + C_{\kappa, \lambda}^{(1)} \theta + \exp(C_{\kappa, \lambda}^{(2)} \theta). \quad (6)$$

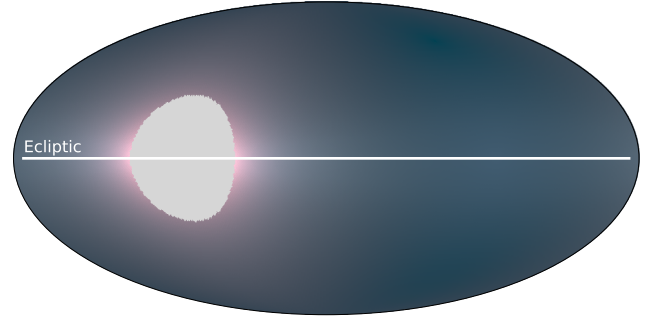


Fig. 4. Zodiacal light full-sky image as seen from earth, based on the Kelsall model, false-colored: $\lambda = 1.25 \mu m, 2.2 \mu m, 3.5 \mu m$ are represented by RGB (channels stretched each to $[0, 1]$, then gamma-corrected with $\gamma = 0.25$), simulated for June 14th, 2022, in Mollweide projection. The area around the sun is excluded (gray).

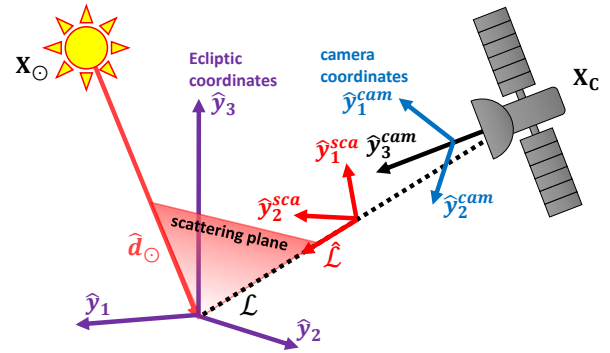


Fig. 5. The three dimensional coordinate systems.

The parameters $C_{\kappa, \lambda}^{(0)}, C_{\kappa, \lambda}^{(1)}, C_{\kappa, \lambda}^{(2)}$ are fitted to data that had been sensed at $\lambda = 1.25 \mu m, 2.2 \mu m, 3.5 \mu m$, while the phase function is normalized. This modeled phase function is consistent with measurements of the ZL [37].

To demonstrate this, we ran the model of Eqs. (1,2,3,4,6). Full-sky image results are shown in Fig. 4. The visualization here is in the Mollweide projection, where the central point is at (lon, lat) = (0, 0) in ecliptic coordinates, directed towards the vernal equinox.

3 FORWARD MODEL

3.1 Three-dimensional (3D) Coordinate systems

We use several 3D right-handed coordinate systems [50] (See Fig. 5).

- The *ecliptic* system has axes $\hat{\mathbf{y}}_1 \perp \hat{\mathbf{y}}_2 \perp \hat{\mathbf{y}}_3$. Axis $\hat{\mathbf{y}}_1$ is in the ecliptic plane, pointing to the vernal equinox. Axis $\hat{\mathbf{y}}_3$ is perpendicular to the ecliptic plane, pointing to the north ecliptic pole.
- In the *camera* system, axis $\hat{\mathbf{y}}_3^{\text{cam}}$ aligns with the optical axis of the camera. Axes $\hat{\mathbf{y}}_1^{\text{cam}}, \hat{\mathbf{y}}_2^{\text{cam}}$ are orthogonal to $\hat{\mathbf{y}}_3^{\text{cam}}$ and align with the sensor rows and columns, respectively.
- The *scattering* system is set by the *scattering plane* which contains the sun's projection $[\mathbf{X}_C - \mathbf{X}_{\odot}] / \|\mathbf{X}_C - \mathbf{X}_{\odot}\|$ and \mathcal{L} . The system axes are $\hat{\mathcal{L}}$ and

$$\hat{\mathbf{y}}_1^{\text{sca}} = \hat{\mathcal{L}} \times \frac{\mathbf{X}_C - \mathbf{X}_{\odot}}{\|\mathbf{X}_C - \mathbf{X}_{\odot}\|}, \quad \hat{\mathbf{y}}_2^{\text{sca}} = \hat{\mathcal{L}} \times \hat{\mathbf{y}}_1^{\text{sca}}. \quad (7)$$

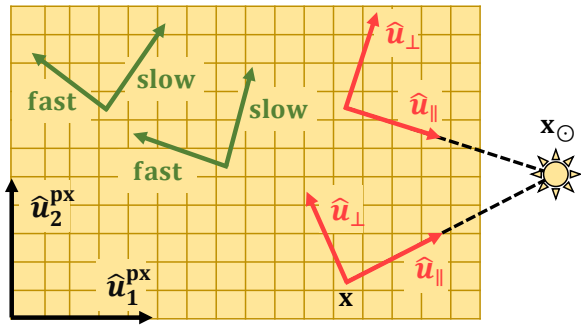


Fig. 6. The two dimensional coordinate systems.

Rotation between 3D coordinate systems is described by a 3×3 rotation matrix \mathbf{G} . The matrix \mathbf{G}_{cam} transforms directions in the ecliptic system, where astronomical data are defined, to the camera system, where images are projected. For more details on 3D coordinate systems in conjunction to polarimetric sensor arrays, see [50].

3.2 Two-dimensional (2D) Coordinate systems

We use 2D coordinate systems (see Fig. 6):

- The *pixel* system, defined by $\hat{\mathbf{u}}_1^{\text{px}} \perp \hat{\mathbf{u}}_2^{\text{px}}$, which aligns with the rows and columns of the pixel array.
- The *projected scattering* system, defined by $\hat{\mathbf{u}}_{\parallel} \perp \hat{\mathbf{u}}_{\perp}$. The vector $\hat{\mathbf{u}}_{\parallel}$ aligns with the projection of the *scattering plane* to the image plane. The scattering plane includes the sun, which projects to 2D location \mathbf{x}_{\odot} on the image plane. LOS \mathcal{L} projects to a pixel at \mathbf{x} . Hence, $\hat{\mathbf{u}}_{\parallel} = [\mathbf{x}_{\odot} - \mathbf{x}] / \|\mathbf{x}_{\odot} - \mathbf{x}\|$. The angle between the *projected scattering* and *pixel* coordinate systems is α^{cam} . As seen in Fig. 6, α^{cam} is a function of the pixel location \mathbf{x} .
- A *retarder* coordinate system, which is useful for describing birefringence. The system is defined by two orthogonal axes: light polarized in one axis experiences a higher refractive index (slow) than light polarized in the other (fast) [51] axis. The fast component has angle α_{br} relative to the *pixel* coordinate system. As seen in Fig. 6, α_{br} is generally a smooth function of the pixel location \mathbf{x} .

Light that is partially linearly polarized is expressed by a three-element Stokes vector. Let \top denote transposition. The scene Stokes vector projected by LOS \mathcal{L} is

$$\mathbf{s}_{\mathcal{L}}(\lambda) = [I_{\mathcal{L}}(\lambda) \ Q_{\mathcal{L}}(\lambda) \ U_{\mathcal{L}}(\lambda)]^{\top}. \quad (8)$$

Here the scene's radiance is $I_{\mathcal{L}}(\lambda)$, while its angle of polarization is associated with the ratio of $Q_{\mathcal{L}}$ and $U_{\mathcal{L}}$. In ZL, the angle of polarization is set by the scattering plane. We then represent this angle (thus the ratio of $Q_{\mathcal{L}}$ and $U_{\mathcal{L}}$) in the *projected scattering* coordinate system.

Transforming a Stokes vector from any 2D coordinate system to another is a rotation by angle α , expressed by a Mueller rotation matrix

$$\mathbf{R}(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\alpha) & \sin(2\alpha) \\ 0 & -\sin(2\alpha) & \cos(2\alpha) \end{bmatrix}. \quad (9)$$

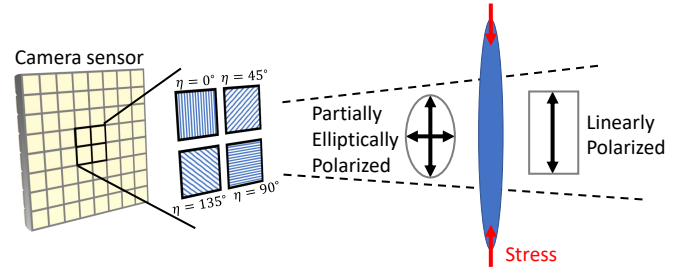


Fig. 7. Imaging optics may induce depolarization, particularly due to birefringence created by stress, that is exacerbated by thermal changes. This leads to partial mixing of polarization components. Due to the system's point spread function, light along LOS \mathcal{L} is spread over a set of sensor pixels. Each sensor pixel is covered by a small polarizing filter. The filter angle η depends (and is known) on the sensor pixel location.

3.3 Polarimetric Imaging

Propagating through the camera's optical elements, as shown in Fig. 7, light polarization may be affected by birefringence [20]. This is a particular concern in LEO, as mentioned in Sec. 1. Generally, a LEO satellite is periodically either heated strongly by direct solar illumination or exposed to extremely low temperatures of space, in Earth's shadow. Thermal variations lead to temporal changes of stress on the optical system [52]. These, in turn, may cause birefringence that varies gradually in orbit. Birefringence mixes polarization components and is typically modeled as a linear retarder. A retarder has phase retardance δ between the fast and slow components (Sec. 3.2). In the *retarder coordinate system* the birefringence Mueller matrix is

$$\mathbf{M}_{\text{br}}(\delta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \cos \delta \end{bmatrix}. \quad (10)$$

In the *pixel coordinate system*, a birefringence Muller matrix is

$$\mathbf{B}(\delta, \alpha_{\text{br}}) = \mathbf{R}^{-1}(\alpha_{\text{br}}) \mathbf{M}_{\text{br}}(\delta) \mathbf{R}(\alpha_{\text{br}}). \quad (11)$$

Let

$$\mathbf{a} = \cos^2(2\alpha_{\text{br}}) + \cos \delta \sin^2(2\alpha_{\text{br}}), \quad (12)$$

$$\mathbf{b} = \cos(2\alpha_{\text{br}}) \sin(2\alpha_{\text{br}}) (1 - \cos \delta), \quad (13)$$

$$\mathbf{c} = \sin^2(2\alpha_{\text{br}}) + \cos \delta \cos^2(2\alpha_{\text{br}}). \quad (14)$$

From Eqs. (9,10,12,13,14),

$$\mathbf{B}(\delta, \alpha_{\text{br}}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathbf{a} & \mathbf{b} \\ 0 & \mathbf{b} & \mathbf{c} \end{bmatrix}_{3 \times 3}. \quad (15)$$

Recall that $\mathbf{s}_{\mathcal{L}}(\lambda)$ in Eq. (8) is expressed in the *projected scattering* coordinate system. Transferring $\mathbf{s}_{\mathcal{L}}(\lambda)$ to the pixel coordinate system and then passing through optical imaging (lens) elements yields

$$\mathbf{s}_{\text{pre}}(\mathcal{L}, \lambda) = \mathbf{B}(\delta, \alpha_{\text{br}}) \mathbf{R}(\alpha^{\text{cam}}) \mathbf{s}_{\mathcal{L}}(\lambda), \quad (16)$$

based on Eqs. (8,9,10,11). This is the Stokes vector of light heading towards the detector (sensor) array. Consider cameras where each sensor pixel has an attached linear polarizer [2], [50]. Let $\mathbf{x}(\mathcal{L})$ be a pixel on the sensor array, to which

LOS \mathcal{L} projects. A pixel is covered by a tiny polarizing filter at angle η relative to the rows of the pixel array. The filter has polarizance $P \leq 1$. After light passes the filter, it yields a signal equivalent to radiance

$$I_{\text{cam}}[\mathbf{x}(\mathcal{L}), \lambda, \eta] = \left[\frac{1}{2} \frac{P \cos(2\eta)}{2} \frac{P \sin(2\eta)}{2} \right] \mathbf{s}_{\text{pre}}(\mathcal{L}, \lambda), \quad (17)$$

in units of $\frac{W}{m^2 \cdot sr \cdot \mu m}$. The optical system's point spread function spans several sensor pixels, thus the signal of \mathcal{L} spreads (not spatially resolved) over several nearby pixels. Term *super-pixel* a 2×2 set of adjacent sensor pixels. The polarizer angles in this set are $\eta = 0^\circ, 45^\circ, 90^\circ, 135^\circ$. Their polarizance is P . Because the signal irradiating the sensor pixels is the same within the super-pixel, we assign the whole super-pixel the location \mathbf{x} . Then, from Eq. (17).

$$\begin{bmatrix} I_{\text{cam}}[\mathbf{x}(\mathcal{L}), \lambda, 0^\circ] \\ I_{\text{cam}}[\mathbf{x}(\mathcal{L}), \lambda, 45^\circ] \\ I_{\text{cam}}[\mathbf{x}(\mathcal{L}), \lambda, 90^\circ] \\ I_{\text{cam}}[\mathbf{x}(\mathcal{L}), \lambda, 135^\circ] \end{bmatrix} = \mathbf{V} \mathbf{s}_{\text{pre}}(\mathcal{L}, \lambda), \quad (18)$$

where

$$\mathbf{V} = \frac{1}{2} \begin{bmatrix} 1 & P & 0 \\ 1 & 0 & P \\ 1 & -P & 0 \\ 1 & 0 & -P \end{bmatrix}_{4 \times 3}. \quad (19)$$

3.4 Light to Electrons

The radiance obtained by Eq. (17) is converted to model an expected signal $\bar{N}[\mathbf{x}(\mathcal{L})]$ and noise in photoelectrons, as in [50]. A light detector is sensitive to a narrow spectral band Λ . Let $p, \Delta t, D, f$ be the camera's pixel length, exposure time, lens diameter, and focal length. The optical train has transmittance τ_λ . This accounts also for imperfect transmissivity by a polarizer aligned with the polarization of an incoming field. A pixel has quantum efficiency QE_λ . Radiance $I_{\text{cam}}(\mathbf{x}, \lambda, \eta)$ corresponding to \mathbf{x} yields an expected number of photo-electrons

$$\bar{N}(\mathbf{x}, \eta) = \int_{\Lambda} \Gamma_\lambda I_{\text{cam}}(\mathbf{x}, \lambda, \eta) d\lambda, \quad (20)$$

where

$$\Gamma_\lambda = \pi \Delta t \tau_\lambda \left(\frac{D}{2f} \right)^2 \text{QE}_\lambda \frac{\lambda}{hc} p^2 \left[\frac{m^2 sr}{J} \right]. \quad (21)$$

In a practical camera, Γ_λ expresses specifications that are known following *unpolarized* calibration of radiometry [53], [54] and geometry [55], [56]. These methods are well established, specifically from LEO. The polarized parameters constitute a set $\Psi_{\mathbf{x}} \equiv \{\mathbf{a}, \mathbf{b}, \mathbf{c}, P\}$. A vector of the expected number of photo-electrons from the four-pixel elements in a super-pixel is

$$\bar{\mathbf{n}}(\mathbf{x}) = [\bar{N}(\mathbf{x}, 0^\circ), \bar{N}(\mathbf{x}, 45^\circ), \bar{N}(\mathbf{x}, 90^\circ), \bar{N}(\mathbf{x}, 135^\circ)]^\top. \quad (22)$$

A scene is a *map* $\mathcal{S} \equiv \{\mathbf{s}_{\mathcal{L}}(\lambda)\}_{\forall \mathcal{L}, \lambda}$ of all Stokes vectors of the scene, in all potential viewing directions and relevant wavelengths. A measurement at a state of the spacecraft is indexed by k . There are K measurements, each yielding a 4-element vector $\bar{\mathbf{n}}(\mathbf{x})$ per \mathbf{x} , as in Eq. (22). Per k , the camera is oriented differently, thus a different LOS $\mathcal{L}(k)$ projects to

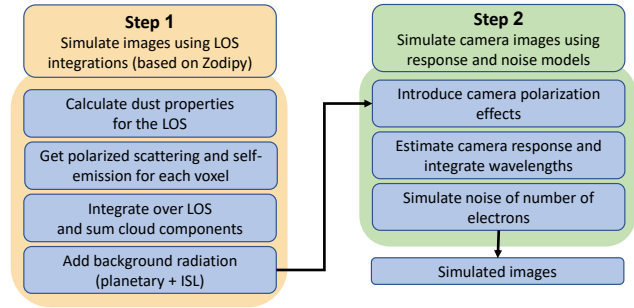


Fig. 8. Rendering partially polarized sky images, including ZL.

super-pixel \mathbf{x} . Denote this projection by $\mathcal{L}(k) \rightarrow \mathbf{x}$. Then, super-pixel \mathbf{x} measures per k a Stokes vector

$$\mathbf{s}_k(\mathbf{x}) = \mathbf{s}\{\mathcal{L}(k) \rightarrow \mathbf{x}\}. \quad (23)$$

Using Eqs. (16,17,18,19,20,22,23), the forward model is

$$\bar{\mathbf{n}}_k = \int_{\Lambda} \Gamma_\lambda \mathbf{VBR}_k \mathbf{s}_k(\lambda) d\lambda \quad [\text{electrons}], \quad (24)$$

where $\mathbf{R}_k = \mathbf{R}(\alpha_k^{\text{cam}})$, and $\bar{\mathbf{n}}_k$ is a 4-element column vector. Define a Stokes vector aggregated over the spectral band as

$$\tilde{\mathbf{s}}_k = \begin{bmatrix} \tilde{I}_k \\ \tilde{Q}_k \\ \tilde{U}_k \end{bmatrix} \equiv \int_{\Lambda} \Gamma_\lambda \mathbf{s}_k(\lambda) d\lambda \quad [\text{electrons}]. \quad (25)$$

We assume $\Psi_{\mathbf{x}}$ is insensitive to λ within Λ . From Eq. (24,25),

$$\bar{\mathbf{n}}_k = \mathbf{VBR}_k \tilde{\mathbf{s}}_k \quad [\text{electrons}]. \quad (26)$$

4 RENDERING THE LEO POLARIZED SKY

The main goal of this paper is to solve an inverse problem based on data acquired in space, rather than rendering. However, to test some of our hypotheses using simulations, we prefer to have a rendering model. This can serve us in simulating celestial polarization images as if taken by a spaceborne camera pointing away from Earth. We stress that this section is not a part of our principle and calibration methods. A reader uninterested in rendering, but seeks solely calibration, can skip directly to Sec. 5.

Some rendering models [57] use average ZL maps from 1968, oblivious to spatiotemporal ZL variability. Our rendering flow is summarized in Fig. 8. Secs. 3.3 and 3.4 convert a spectral Stokes vector along a scene LOS $\mathbf{s}_{\mathcal{L}}(\lambda)$, to a camera signal. This section models $\mathbf{s}_{\mathcal{L}}(\lambda)$ in an ad-hoc manner.

From Eq. (8), the first component in $\mathbf{s}_{\mathcal{L}}(\lambda)$ is the scene intensity $I_{\mathcal{L}}(\lambda)$. Eq. (4) expresses the ZL intensity along \mathcal{L} . However, a LOS \mathcal{L} also observes background planets, stars, and other objects, e.g., galaxies. Extra-solar objects are accounted for by integrated starlight (ISL), $I_{\text{ISL}}(\mathcal{L})$. In our solar system, each planet p has radiance $I_p^{\text{planet}}(\lambda)$. Overall,² the radiance reaching the camera in units of $\frac{W}{m^2 \cdot sr \cdot \mu m}$ is

$$I_{\mathcal{L}}(\lambda) = I_{\text{ZL}}(\mathcal{L}, \lambda) + I_{\text{ISL}}(\mathcal{L}, \lambda) + \sum_{p \in \mathcal{L}} I_p^{\text{planet}}(\lambda). \quad (27)$$

2. Additional background radiation sources exist, but they can be neglected in our context.

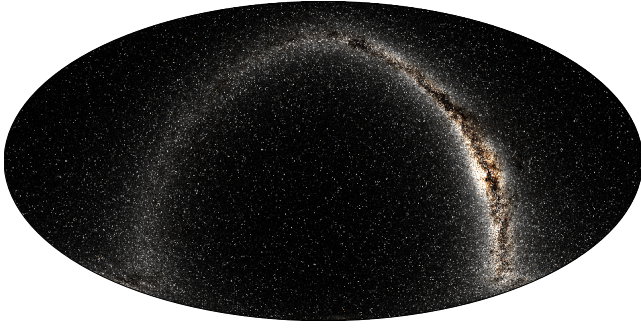


Fig. 9. The integrated starlight model in full-sky view (real-color in logarithmic scale), simulated for June 14th, 2022, in Mollweide projection.

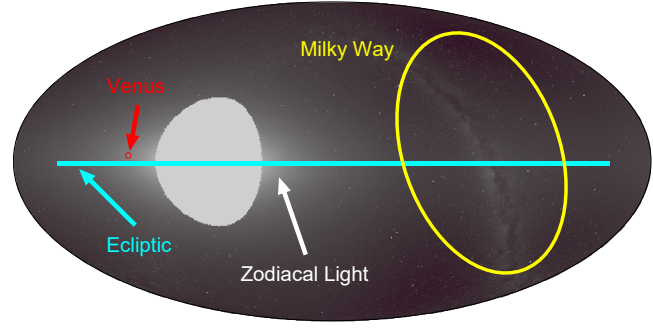


Fig. 10. Full Sky simulated color image (gamma corrected with $\gamma = 0.4$), simulated for June 1st, 2023, in Mollweide projection.

Eq. (4) requires $L_{\text{scene}}(\mathbf{X}, \lambda)$. However, the Kelsall model is only given in short-wave infrared, not visible light. Our literature search did not find a satisfactory model for visible light. We found several ad-hoc approximations, which we use. First, in Kelsall's model, the extinction coefficient is insensitive to the wavelength, and indeed, many long exposure images of ZL show it rather colorless. Hence, we use $\beta_{\kappa, \mathbf{X}}$ as in the Kelsall model. Second, the unpolarized phase function is extrapolated in the wavelength domain. LOS integration over dust properties is performed by the Zodipy library [58].

We model the non-ZL components similarly to [42]. For the ISL, object locations and multi-spectral magnitudes are found by the Two Micron All Sky Survey (2MASS) [59] catalog using AstroQuery [60]. The multi-spectral magnitudes are fit to Eq. (2), yielding star temperatures and emissions of visible light. Afterward, we sample these bodies in every direction by Monte-Carlo simulation and integration. This yields $I_{\text{ISL}}(\mathcal{L}, \lambda)$, as shown in Fig. 9.

Per date, the location \mathbf{X}_p of planet p is obtained using Ephemeris from Astropy [61]. A planet has radius R_p . The Bond albedo K_p is the fraction of incident power that is scattered back to space. The solar spectral flux density at planet p is $F_{p, \lambda}$, as in Eq. (3). The scattering angle between the vector of solar irradiance and the direction from the planet to the camera is θ_p . Then [62],

$$I_p^{\text{planet}}(\lambda) = \frac{2K_p}{3} \frac{R_p^2}{\|\mathbf{X}_p - \mathbf{X}_c\|_2^2} F_{p, \lambda} [\sin \theta_p - \theta_p \cos \theta_p]. \quad (28)$$

An example for the overall $I_{\mathcal{L}}$ (Eq. 27) is shown in Fig. 10.

Next, we assign the polarized components $Q_{\mathcal{L}}(\lambda), U_{\mathcal{L}}(\lambda)$ of Eq. (8). ISL is assumed to have negligible³ polarization [42]. Planetary light can be polarized, but we use the planets only for geometric calibration, hence we do not model planetary polarization. Thus, $Q_{\mathcal{L}}(\lambda), U_{\mathcal{L}}(\lambda)$ depend only on the ZL. For ZL, the angle of polarization $\text{AoP}_{\mathcal{L}}$ is known: it is perpendicular to the scattering plane, per \mathcal{L} . Hence, in the *projected scattering* coordinate system

$$s_{\mathcal{L}} = [I_{\mathcal{L}}(\lambda) \ Q_{\mathcal{L}}(\lambda) \ 0]^T. \quad (29)$$

3. Resolvable stars are essentially point sources that cover a tiny fraction of the field of view. Stars are intrinsically unpolarized. Some star observations sense $\text{DoLP} \sim [1 - 2\%]$, due to iron grains in interstellar dust that align with interstellar magnetic fields [63], [64], [65]. This small effect is seen mainly in the galactic plane, that is, in the bright band of the Milky Way. If we wish to avoid (rather than use) this effect, we may direct our imager away from this bright band.

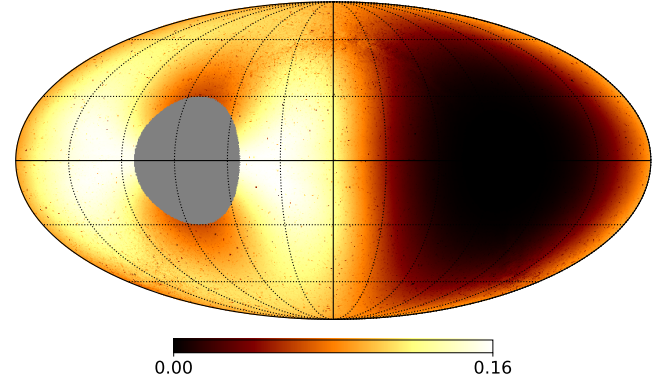


Fig. 11. Full sky DoLP image (red channel), simulated for June 14th, 2022, in Mollweide projection.

The component $Q_{\mathcal{L}}(\lambda)$ is associated with the corresponding ZL component, which is modeled analogously to Eqs. (1,4):

$$Q_{\mathcal{L}}(\lambda) \sim Q_{\text{ZL}}(\mathcal{L}, \lambda) = \int_{\mathbf{X} \in \mathcal{L}} \sum_{\kappa=1}^{N_{\text{comp}}} L_{\kappa}^{\text{Q}}(\mathbf{X}, \lambda) d\mathbf{X}. \quad (30)$$

Here $L_{\kappa}^{\text{Q}}(\mathbf{X}, \lambda)$ is the polarized component of scattered light at \mathbf{X} . It is derived in analogy⁴ to Eq. (3)

$$L_{\kappa}^{\text{Q}}(\mathbf{X}, \lambda) = \beta_{\kappa, \mathbf{X}, \lambda} A_{\kappa, \mathbf{X}, \lambda} F_{\mathbf{X}, \lambda} \Phi_{\kappa, \lambda} \text{DoLP}_{\text{IDP}}[\theta(\mathbf{X})], \quad (31)$$

where $\text{DoLP}_{\text{IDP}}[\theta(\mathbf{X})]$ is the degree of linear polarization of light scattered at \mathbf{X} , and $\theta(\mathbf{X})$ is defined in Eq. (5). Note that $\text{DoLP}_{\text{IDP}}[\theta(\mathbf{X})]$ depends on the IDPs in voxel \mathbf{X} . An approximation that fits empirical visible-range data [66] is

$$\text{DoLP}_{\text{IDP}}[\theta] = 0.33 \sin^5 \theta. \quad (32)$$

A camera senses an integral over a LOS (Eq. 30). The DoLP of light reaching the camera is $Q_{\mathcal{L}}(\lambda)/I_{\mathcal{L}}(\lambda)$. Eqs. (30,31,32) yield the polarized component of the Stokes vector.

5 POLARIMETRIC CALIBRATION BY ZL

Equation (26) provides a forward model for imaging. This paper seeks an estimation of the camera's polarization set of parameters $\Psi_{\mathbf{x}}, \forall \mathbf{x}$. The main question is whether \mathcal{S} is known, or at least modeled well enough. If it is, then we can solve the problem of polarimetric calibration using ZL. If not, then we can seek *self-calibration* of some parameters.

4. Thermal emission is unpolarized, hence does not affect $L_{\kappa}^{\text{Q}}(\mathbf{X}, \lambda)$.

5.1 Polarimetric Calibration

A known ZL scene \mathcal{S} is observed by a camera. Polarization signals depend on the axial orientation of the camera relative to the scene, so it is beneficial to sample a set of orientations. Moreover, a set of multiple measurements helps to suppress the consequence of random measurement noise and random inaccuracies in \mathcal{S} . Rotating the instrument around a LOS \mathcal{L} yields different values of α^{cam} , which through Eq. (16) alters the signal filtered by the camera optics and the sensor-mounted polarizers at $\mathbf{x}(\mathcal{L})$. A general 3D rotation (see Sec. 3.1) varies \mathbf{G}_{cam} , thus the LOSs that correspond to \mathbf{x} . Then, pixel \mathbf{x} observes a different part of the known \mathcal{S} , diversifying the measurements.

The locations of observable stars and planets are known and saved in a look-up table. Matching these observed points with the look-up table determines orientation, using onboard star trackers [23]. Star trackers are not always mounted in nano-satellites, due to resource limitations. Nevertheless, we do not need a star-tracker to sense \mathbf{G}_{cam} , because, by definition, we have an observational camera that happens to sense these point sources, along with ZL. Hence \mathbf{G}_{cam} is determined from the image data. Noisy measurements at \mathbf{x} constitute a column vector $\mathbf{n}_k^{\text{measure}}(\mathbf{x})$. Define a fitting error

$$E(\Psi_{\mathbf{x}}, \mathcal{S}) = \sum_{\mathbf{x}} \sum_{k=1}^K |\mathbf{n}_k^{\text{measure}}(\mathbf{x}) - \bar{\mathbf{n}}(\mathbf{x}, \Psi_{\mathbf{x}}, \mathcal{S}, \mathbf{G}_{\text{cam}})|^2. \quad (33)$$

The sum over \mathbf{x} excludes pixels corresponding to planets and observable stars, known from the 2MASS catalog. Then, assuming \mathcal{S} is modeled and known, calibration is performed by solving the optimization problem

$$\hat{\Psi}_{\mathbf{x}} = \underset{\Psi_{\mathbf{x}}}{\text{argmin}} E(\Psi_{\mathbf{x}}, \mathcal{S}). \quad (34)$$

Note that the entire forward model is differentiable. Moreover, there are some parameters for which the forward model is linear. Hence, optimization can be done efficiently.

We estimate $\Psi_{\mathbf{x}}$ per camera pixel \mathbf{x} using alternating minimization. We initialize P, \mathbf{B} using corresponding values $P^{\text{prev}}, \mathbf{B}^{\text{prev}}$ from a previous calibration, done for example pre-launch in a lab, or from previous spaceborne sessions. Then, we iterate two steps: {1} Estimate P assuming \mathbf{B} and \mathcal{S} are known; {2} Estimate \mathbf{B} assuming P and \mathcal{S} are known.

Define the $4K \times 1$ column-stacked data vector

$$\mathbf{N}^{\text{measure}} = \left[(\mathbf{n}_1^{\text{measure}})^{\top}, (\mathbf{n}_2^{\text{measure}})^{\top} \dots (\mathbf{n}_K^{\text{measure}})^{\top} \right]^{\top}. \quad (35)$$

Define the Stokes vector that arrives at the pixel polarizers after the camera optics,

$$\tilde{\mathbf{s}}_k^P \equiv \begin{bmatrix} \tilde{I}_k^P \\ \tilde{Q}_k^P \\ \tilde{U}_k^P \end{bmatrix} = \mathbf{B} \mathbf{R}_k \tilde{\mathbf{s}}_k. \quad (36)$$

The matrix \mathbf{B} is assumed to be known in step {1}. So, define known column vectors of length $4K$,

$$\mathbf{S}_P = \frac{1}{2} \left[\tilde{Q}_1^P, \tilde{U}_1^P, -\tilde{Q}_1^P, -\tilde{U}_1^P \dots \tilde{Q}_K^P, \tilde{U}_K^P, -\tilde{Q}_K^P, -\tilde{U}_K^P \right]^{\top}. \quad (37)$$

$$\mathbf{I}_P = \left[\tilde{I}_1^P, \tilde{I}_1^P, \tilde{I}_1^P, \tilde{I}_1^P \dots \tilde{I}_K^P, \tilde{I}_K^P, \tilde{I}_K^P, \tilde{I}_K^P \right]^{\top}. \quad (38)$$

From Eqs. (26,33,35,37,38), the forward model is

$$\mathbf{N}^{\text{measure}} = \frac{1}{2} \mathbf{I}_P + \mathbf{S}_P P. \quad (39)$$

Because \mathbf{S}_P stems from independent measurements, we use a pseudo-inverse

$$\hat{P} = \left(\mathbf{S}_P^{\top} \mathbf{S}_P \right)^{-1} \mathbf{S}_P^{\top} (\mathbf{N}^{\text{measure}} - \mathbf{I}_P/2). \quad (40)$$

The elements of $\tilde{\mathbf{s}}_k = [\tilde{I}_k, \tilde{Q}_k, \tilde{U}_k]^{\top}$ defined in Eq. (25) are known here. Then, step {2} estimates $\mathbf{a}, \mathbf{b}, \mathbf{c}$ as follows. Define the known terms

$$\mathbf{g}_k = \tilde{Q}_k \cos 2\alpha_k^{\text{cam}} - \tilde{U}_k \sin 2\alpha_k^{\text{cam}}, \quad (41)$$

$$\mathbf{f}_k = \tilde{Q}_k \sin 2\alpha_k^{\text{cam}} + \tilde{U}_k \cos 2\alpha_k^{\text{cam}}. \quad (42)$$

Define a known $4K \times 3$ matrix \mathbf{F}_B and a $4K \times 1$ vector \mathbf{I}_B :

$$\mathbf{F}_B = \frac{P}{2} \begin{bmatrix} \mathbf{g}_1 & 0 & -\mathbf{g}_1 & 0 & \dots & -\mathbf{g}_K & 0 \\ \mathbf{f}_1 & \mathbf{g}_1 & -\mathbf{f}_1 & -\mathbf{g}_1 & \dots & -\mathbf{f}_K & -\mathbf{g}_K \\ 0 & \mathbf{f}_1 & 0 & -\mathbf{f}_1 & \dots & 0 & -\mathbf{f}_K \end{bmatrix}^{\top}, \quad (43)$$

$$\mathbf{I}_B = \left[\tilde{I}_1, \tilde{I}_1, \tilde{I}_1, \tilde{I}_1 \dots \tilde{I}_K, \tilde{I}_K, \tilde{I}_K, \tilde{I}_K \right]^{\top}. \quad (44)$$

From Eqs. (19,26,33,35,43,44)

$$\mathbf{N}^{\text{measure}} = \frac{1}{2} \mathbf{I}_B + \mathbf{F}_B [\mathbf{a} \ \mathbf{b} \ \mathbf{c}]^{\top}. \quad (45)$$

Then, pseudo-inverse yields

$$\begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \\ \hat{\mathbf{c}} \end{bmatrix} = \left(\mathbf{F}_B^{\top} \mathbf{F}_B \right)^{-1} \mathbf{F}_B^{\top} (\mathbf{N}^{\text{measure}} - \mathbf{I}_B/2). \quad (46)$$

In typical optical systems, birefringence is generally a smooth function, thus changing slowly across the field of view. Therefore, we smooth the pixel-based results $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$. We use a spatial mean having a support of 5×5 superpixels.

5.2 Polarimetric Self-Calibration

Current ZL models are not yet good enough for calibration. We then suggest self-calibration, motivated by [25]. Here, a camera observes an unknown scene \mathcal{S} . We need to solve

$$\hat{\Psi}_{\mathbf{x}}, \hat{\mathcal{S}} = \underset{\Psi_{\mathbf{x}}, \mathcal{S}}{\text{argmin}} E(\Psi_{\mathbf{x}}, \mathcal{S}). \quad (47)$$

The polarizance P of the imager pixels changes slowly in time. Birefringence, on the other hand, is a fast effect, caused by thermal variations in a harsh environment. At the beginning of a mission, the effects of \mathbf{B} can be studied, assuming P is as measured in the lab. Over time, P can be re-estimated as well. We assume that over the entire camera, some of the pixels are not degraded since a prior measurement. They can anchor P .

We generalize Sec. 5.1, by iterating three steps:

(i) Estimate \mathcal{S} assuming P and \mathbf{B} are known. Initially, we use P^{prev} and \mathbf{B}^{prev} .

(ii) Estimate P assuming \mathbf{B} and \mathcal{S} are known.

(iii) Estimate \mathbf{B} assuming P and \mathcal{S} are known.

We iterate until convergence. Steps (ii) and (iii) are the same as described in Sec. 5.1. In this section, we describe step (i).

The estimate of the Stokes vector of LOS \mathcal{L} is $\tilde{\mathbf{s}}_c$. This LOS is sequentially projected to a set of superpixels

$\{\mathbf{x}_k\}_{k=1}^K$. We align the images using stars and planets, and then associate \mathcal{L} to superpixels by nearest-neighbor interpolation. When \mathcal{L} is projected to \mathbf{x}_k , the measured numbers of electrons are represented by the 4×1 vector

$$\mathbf{n}_{\mathcal{L},k}^{\text{measure}} = [N_{\mathcal{L},k}^{\text{measure}}(0^\circ) \dots N_{\mathcal{L},k}^{\text{measure}}(135^\circ)]^\top. \quad (48)$$

Let $\mathbf{n}_{\mathcal{L}}^{\text{measure}}$ be the $4K \times 1$ vector of measurements corresponding to \mathcal{L} , measured at $\{\mathbf{x}_k\}_{k=1}^K$:

$$\mathbf{n}_{\mathcal{L}}^{\text{measure}} = [(\mathbf{n}_{\mathcal{L},1}^{\text{measure}})^\top \dots (\mathbf{n}_{\mathcal{L},K}^{\text{measure}})^\top]^\top. \quad (49)$$

Each super-pixel \mathbf{x}_k has a different set $\Psi_{\mathbf{x}_k}$, thus corresponding matrices $\mathbf{V}_{\mathbf{x}_k}, \mathbf{B}_{\mathbf{x}_k}$. Let $\mathbf{0}_{n \times m}$ be an $n \times m$ zero matrix. Using Eq. (19), define the matrices

$$\tilde{\mathbf{V}}_{\mathcal{L}} = \begin{bmatrix} \mathbf{V}_{\mathbf{x}_1} & \mathbf{0}_{4 \times 3} & \dots & \mathbf{0}_{4 \times 3} \\ \mathbf{0}_{4 \times 3} & \mathbf{V}_{\mathbf{x}_2} & \dots & \mathbf{0}_{4 \times 3} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{4 \times 3} & \mathbf{0}_{4 \times 3} & \dots & \mathbf{V}_{\mathbf{x}_K} \end{bmatrix}_{4K \times 3K}, \quad (50)$$

$$\tilde{\mathbf{B}}_{\mathcal{L}} = \begin{bmatrix} \mathbf{B}_{\mathbf{x}_1} & \mathbf{0}_{3 \times 3} & \dots & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{B}_{\mathbf{x}_2} & \dots & \mathbf{0}_{3 \times 3} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \dots & \mathbf{B}_{\mathbf{x}_K} \end{bmatrix}_{3K \times 3K}. \quad (51)$$

Using the corresponding rotation Muller matrices $\{\mathbf{R}_k\}_{k=1}^K$, define the $3K \times 3$ matrix

$$\tilde{\mathbf{R}} = [\mathbf{R}_1^\top, \mathbf{R}_2^\top \dots \mathbf{R}_K^\top]^\top. \quad (52)$$

From Eqs. (26,49,50,51,52),

$$\mathbf{n}_{\mathcal{L}}^{\text{measure}} = \tilde{\mathbf{V}}_{\mathcal{L}} \tilde{\mathbf{B}}_{\mathcal{L}} \tilde{\mathbf{R}} \tilde{\mathbf{s}}_{\mathcal{L}} \equiv \mathbf{F}_s \tilde{\mathbf{s}}_{\mathcal{L}}, \quad (53)$$

where $\mathbf{F}_s = \tilde{\mathbf{V}}_{\mathcal{L}} \tilde{\mathbf{B}}_{\mathcal{L}} \tilde{\mathbf{R}}$. Using a pseudo-inverse of Eq. (53),

$$\tilde{\mathbf{s}}_{\mathcal{L}} = (\mathbf{F}_s^\top \mathbf{F}_s)^{-1} \mathbf{F}_s^\top \mathbf{n}_{\mathcal{L}}^{\text{measure}}. \quad (54)$$

5.3 Constraints

Pseudo-inverse steps do not impose natural constraints on the variables. Moreover, there is an ambiguity that needs to be resolved. These matters are handled in this section.

5.3.1 Birefringence

Recall the form of matrix \mathbf{B} from Eqs. (10,11,15). Define the matrices

$$\mathbf{M}_{QU} = \begin{bmatrix} 1 & 0 \\ 0 & \cos \delta \end{bmatrix}, \quad \mathbf{R}_{QU} = \begin{bmatrix} \cos(2\alpha_{\text{br}}) & \sin(2\alpha_{\text{br}}) \\ -\sin(2\alpha_{\text{br}}) & \cos(2\alpha_{\text{br}}) \end{bmatrix}. \quad (55)$$

Then,

$$\mathbf{B}_{QU} \equiv \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{b} & \mathbf{c} \end{bmatrix} = \mathbf{R}_{QU}^{-1} \mathbf{M}_{QU} \mathbf{R}_{QU}. \quad (56)$$

Consequently, the eigenvalues of \mathbf{B}_{QU} must be $\{1, \cos \delta\}$. This is a constraint. Section 5.1 yields $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$. These values form a matrix that is diagonalized as follows:

$$\hat{\mathbf{B}}_{QU} = \begin{bmatrix} \hat{\mathbf{a}} & \hat{\mathbf{b}} \\ \hat{\mathbf{b}} & \hat{\mathbf{c}} \end{bmatrix} = \mathbf{J}^\top \begin{bmatrix} \mathbf{e}_1 & 0 \\ 0 & \mathbf{e}_2 \end{bmatrix} \mathbf{J}. \quad (57)$$

Here \mathbf{J} is a rotation matrix made of orthonormal eigenvectors, while $\{\mathbf{e}_1, \mathbf{e}_2\}$ are the eigenvalues of $\hat{\mathbf{B}}_{QU}$. The matrix

$$\hat{\mathbf{B}}'_{QU} = \mathbf{J}^\top \begin{bmatrix} \mathbf{e}_1 / \max(\mathbf{e}_1, \mathbf{e}_2) & 0 \\ 0 & \mathbf{e}_2 / \max(\mathbf{e}_1, \mathbf{e}_2) \end{bmatrix} \mathbf{J} \quad (58)$$

satisfies the eigenvalue constraint. Based on $\hat{\mathbf{B}}'_{QU}$, the constrained values of $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$ are extracted, in matrix elements corresponding to Eqs. (56,57). This is done in each iteration.

5.3.2 Polarizance

Because $0 \leq P(\mathbf{x}) \leq 1$, then $\hat{P}(\mathbf{x})$ is clipped to this range in each iteration. Still, self-calibration has an inherent ambiguity, as the scene is unknown. Consider Eqs. (8,9,10,11,16,17). Suppose we scale the polarization components of the scene by a factor ξ , such that $Q_{\mathcal{L}}(\lambda) \rightarrow \xi Q_{\mathcal{L}}(\lambda)$ and $U_{\mathcal{L}}(\lambda) \rightarrow \xi U_{\mathcal{L}}(\lambda)$. Simultaneously, suppose that the polarizance is scaled as $P \rightarrow P/\xi$. Then, the measurement I_{cam} in Eq. (17) is invariant to this scale. Hence, when using only polarimetric data without a prior, there is a fundamental scale ambiguity in estimating P .

Our disambiguation uses two priors: $\{\mathbf{i}\}$ Over time, the polarizance $P(\mathbf{x})$ is monotonically not-increasing. $\{\mathbf{ii}\}$ Pixels degrade at different rates. Some pixels are least degraded: their polarizance is very close, or equal to the corresponding value $\hat{P}^{\text{prev}}(\mathbf{x})$ measured previously (pre-launch in the lab, or in a previous spaceborne calibration).

Consequently, define $\chi(\mathbf{x}) = [\hat{P}(\mathbf{x})/\hat{P}^{\text{prev}}(\mathbf{x})]$. Consider a re-scaled polarizance $\hat{P}'(\mathbf{x}) = \hat{P}(\mathbf{x})[\max_{\mathbf{x}} \chi(\mathbf{x})]^{-1}$. The map $\hat{P}'(\mathbf{x})$ is resistant to scale ambiguity. The least-degraded pixel yields $\hat{P}'(\mathbf{x}) = \hat{P}^{\text{prev}}(\mathbf{x})$, correctly when intact. However, a maximum is not robust to noise. Hence, sort $\chi(\mathbf{x})$. Let $\mathbf{x}_{95\%}$ be the pixel corresponding to the 95th percentile of $\chi(\mathbf{x})$. We then use $\hat{P}'(\mathbf{x}) = \hat{P}(\mathbf{x})[\chi(\mathbf{x}_{95\%})]^{-1}$.

6 SIMULATION

We simulated observations of ZL as in Sec. 4. The simulated imager field of view spans 5° , having a resolution of 200×300 pixels. The imager parameters are set to $p^2 = (7\mu\text{m})^2$, $D = 16.6\text{mm}$, $f = 24\text{mm}$, $\tau_\lambda = 0.96$, a red spectral band, 100nm wide and $\text{QE}_\lambda \approx 0.8$. The imager has spatially varying ground-truth polarization parameters $\Psi_{\mathbf{x}}^{\text{true}}$. Spatial maps of the corresponding ground-truth $P^{\text{true}}, \mathbf{a}^{\text{true}}, \mathbf{b}^{\text{true}}, \mathbf{c}^{\text{true}}$ appear in Fig. 12.

The simulation was done for June 14th, 2022. As a baseline (which we deviate from), the imager is directed 17° away from the sun, on the ecliptic plane, 65° from the vernal equinox. An example of simulated image data is shown in Fig. 13. Based on ZL characteristics and specifications of the simulated camera, we calculated the expected signals. As a baseline (which we deviate from in an ablation study), we use exposure time $\Delta t = 10\text{s}$ and $K = 30$ measurements, each at a different camera orientation. To achieve a signal in the range of thousands of photoelectrons, a long exposure time of around $\Delta t = 10\text{s}$ is required in this system. This induces significant dark noise. To reduce noise variance, in each measurement k , we capture 20 images (in 3.3 minutes) and temporally average them. ZL and the background remain effectively static during this time.

6.1 Simulation Noise

The actual measured number of electrons is random due to noise. Photoelectrons are Poisson distributed as

$$\tilde{N}(\mathbf{x}) \sim \text{Poisson}[\bar{N}(\mathbf{x})]. \quad (59)$$

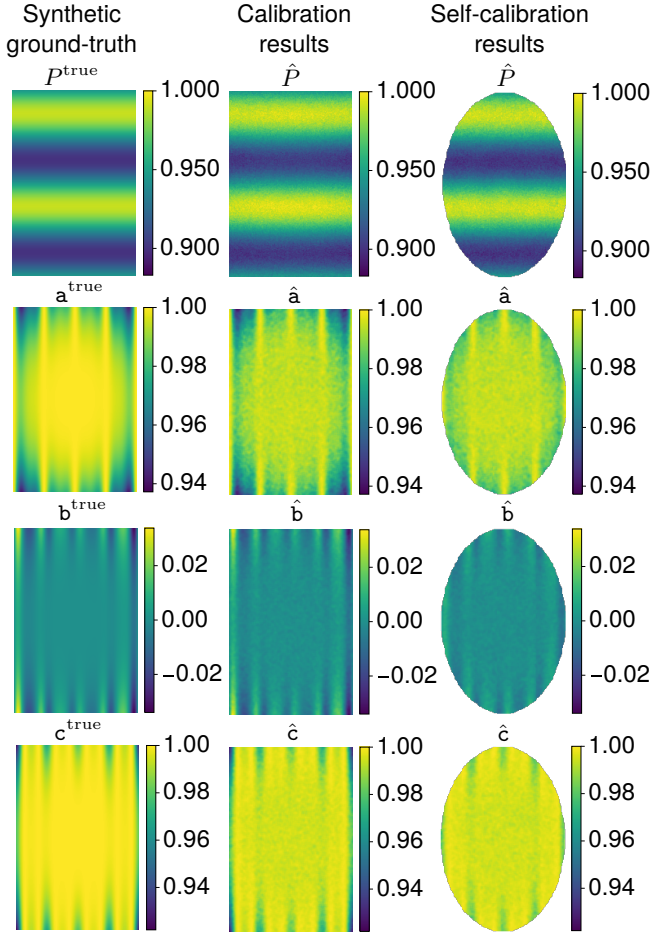


Fig. 12. (Left) Synthetic ground-truth spatial maps P^{true} , a^{true} , b^{true} , c^{true} . (Center) Calibrated maps. (Right) Self-calibrated maps.

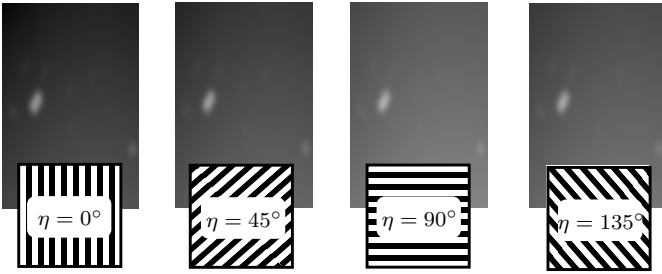


Fig. 13. Simulated sky images (per polarizer angle η), including ZL and background stars. Simulated for June 14th, 2022, directed 37° away from the sun, on the ecliptic plane, 45° from the vernal equinox. For display clarity only, the images are presented gamma corrected with $\gamma = 0.6$.

A camera has dark current \mathcal{B}_T (in units of electrons per second), which depends on the sensor temperature T . It yields dark noise $\zeta_{\text{dark}} \sim \mathcal{N}(\mathcal{B}_T \Delta t, \mathcal{B}_T \Delta t)$. The readout electronics have read-noise $\zeta_{\text{read}} \sim \mathcal{N}(0, \sigma_{\text{read}}^2)$ having a standard deviation of σ_{read} electrons. A pixel full well has N_{full} electrons. The measurement is quantized by b bits. So, a model [50] for a noisy measurement of photo-electrons is

$$\tilde{N}^{\text{measured}}(\mathbf{x}) = \frac{N_{\text{full}}}{2^b} \left[\frac{2^b}{N_{\text{full}}} \left\{ \tilde{N}(\mathbf{x}) + \zeta_{\text{read}} + \zeta_{\text{dark}} \right\} \right]. \quad (60)$$

Values of $\tilde{N}^{\text{measured}}$ higher than N_{full} are clipped. The simulation uses noise parameters $\mathcal{B}_T = 3.51 s^{-1}$, $\sigma_{\text{read}} = 2.31$, $N_{\text{full}} = 10500$, and $b = 10$.

Camera jitter causes a random motion path, averaged during the exposure time. We set a jitter [23] amplitude $\sim 0.1^\circ$. Based on the camera specifications above, this corresponds to blur by a Gaussian kernel having a standard deviation of 4 pixels, and blur by rotation around the optical axis (RAOA). Blur by RAOA is simulated by a weighted sum of rotated images, each indexed i and rotated by angle α_i . Geometric rotation maps to \mathbf{x} a pixel $\mathbf{x}_i = \mathbf{G}_{\alpha_i} \mathbf{x}$, where \mathbf{G}_{α_i} is a rotation matrix in the image plane. As in Eq. (23), this mapping leads \mathbf{x} to sense light along a projected LOS $\mathcal{L}(i) \rightarrow \mathbf{x}_i$. RAOA also rotates the Stokes vector. This perturbation is defined by a Muller rotation matrix \mathbf{R}_{α_i} . Combining these two RAOA effects, a Stokes vector that is axially rotated and averaged during Δt is simulated by

$$\mathbf{s}_{\mathbf{x}}^{\text{blur}} = \sum_i w_i \mathbf{R}_{\alpha_i} \mathbf{s}\{\mathcal{L}(i) \rightarrow \mathbf{x}_i\}, \quad (61)$$

using the weight $w_i \sim \exp(-[\alpha_i/0.1^\circ]^2/2)$. A sensitivity study we performed shows that jitter having a directional amplitude of up to 0.2° does not affect the results.

6.2 Simulation results

In pre-processing, we remove the bias caused by dark noise, $N_i^{\text{measured}} = \tilde{N}_i^{\text{measured}} - \mathcal{B}_T \Delta t$. We assume that calibration is done occasionally during the mission lifetime, so we have rough previous estimates \hat{P}^{prev} , $\hat{\mathbf{B}}^{\text{prev}}$ that need to be refined. These values thus initialize the optimization, and are likely not far from the true values. Hence, we set the initial values as noisy versions of the ground truth. Let $z_P \sim \mathcal{N}(0.02, 0.01^2)$ and $z_a, z_b, z_c \sim \mathcal{N}(0, 0.02^2)$. Then, we set $\hat{P}^{\text{prev}} = P^{\text{true}} + z_P$, $\hat{a}^{\text{prev}} = a^{\text{true}} + z_a$, $\hat{b}^{\text{prev}} = b^{\text{true}} + z_b$, $\hat{c}^{\text{prev}} = c^{\text{true}} + z_c$ and clip the values to the valid domains of P and \mathbf{B} . The matrix $\hat{\mathbf{B}}^{\text{prev}}$ is formed based on Eq. (15) using \hat{a}^{prev} , \hat{b}^{prev} , \hat{c}^{prev} .

We quantify results using root mean squared errors (RMSE). Let $\|\cdot\|_2$ and $\|\cdot\|_F$ be respectively the ℓ_2 norm and Frobenius norm of arrays. Here the arrays span estimations at all pixels \mathbf{x} and sampled examples. Then,

$$\text{RMSE}(P) = \|\mathbf{P}^{\text{true}}(\mathbf{x}) - \hat{P}(\mathbf{x})\|_2. \quad (62)$$

$$\text{RMSE}(\mathbf{B}) = \|\mathbf{B}^{\text{true}}(\mathbf{x}) - \hat{\mathbf{B}}(\mathbf{x})\|_F. \quad (63)$$

Prior to calibration, $\text{RMSE}(P) \approx 2\%$ and $\text{RMSE}(\mathbf{B}) \approx 2\%$. Calibration optimization converges within 3-4 iterations. After convergence, calibration reached $\text{RMSE}(P) \approx 0.6\%$ and $\text{RMSE}(\mathbf{B}) \approx 0.3\%$. Results are shown in Fig. 12.

In self-calibration, observations were taken by rotating the camera around its axis. So, self-calibration estimations are valid in pixels within an ellipse around the center of the sensor array. Within the ellipse, all pixels are exposed in all measurements. Self-calibration achieved $\text{RMSE}(P) \approx 0.5\%$ and $\text{RMSE}(\mathbf{B}) \approx 0.3\%$. Self-calibration results are shown in Fig. 12. Calibration and self-calibration results are assessed in scatter plots (Fig. 14).

Sec. 5.1 mentions spatial smoothing of \hat{a} , \hat{b} , \hat{c} after each iteration. We made an ablation test. Introducing smoothing by kernels of sizes 3×3 , 5×5 and 9×9 improve $\text{RMSE}(\mathbf{B})$ by $\times 1.3$, $\times 2$, $\times 2$, respectively. So, we settle for a 5×5 kernel.

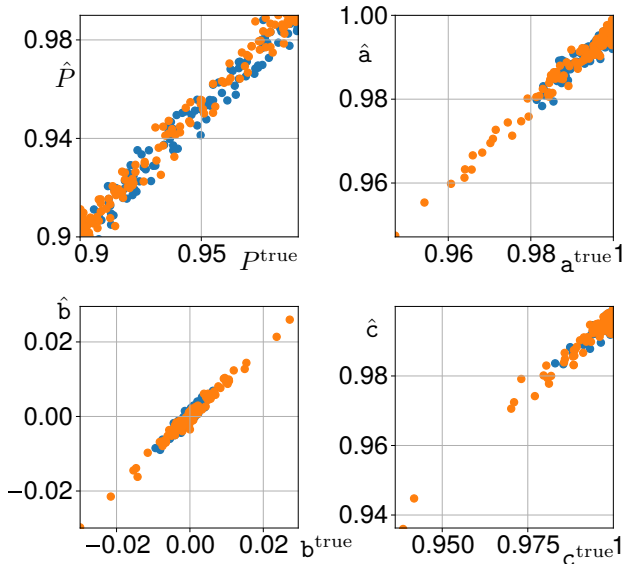


Fig. 14. Scatter plots of simulated calibrated (orange) and self-calibrated (blue) P , a , b , c results. Here $\Delta t = 10s$ and $K = 30$. Displayed points are 150 randomly sampled representative super-pixels out of the total array.

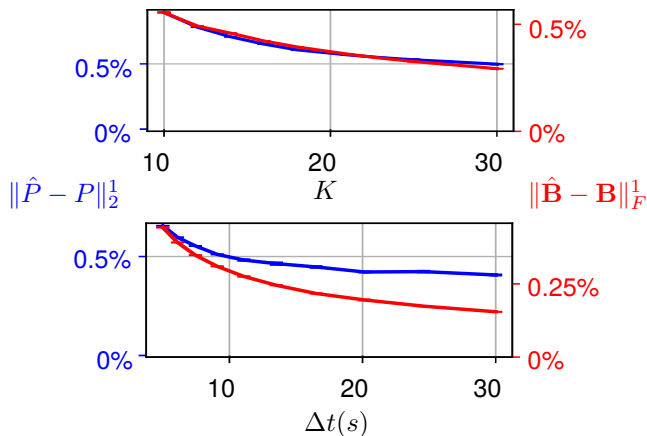


Fig. 15. Self-calibration $RMSE(P)$ and $RMSE(B)$ as a function of K with $\Delta t = 10s$ [Top], and as a function of Δt with $K = 30$ [Bottom]. Fifteen simulations were done, to assess RMSE mean and standard deviation. Without self-calibration, P and B have RMSE of 2%.

An ablation study tests the effects of K and Δt . One test uses $K = 30$. Then, increasing Δt from 5s to 30s, improves the final calibration $RMSE(P)$ and $RMSE(B)$ by $\times 1.6$ and $\times 2.8$, respectively. Another test uses $\Delta t = 10s$. Then, increasing K from 10 to 30, improves the final calibration $RMSE(P)$ and $RMSE(B)$ by $\times 1.8$ and $\times 1.9$ respectively. Such trends also appear in tests of self-calibration, as shown in Fig. 15. From this figure, the product $K\Delta t$ is 5 minutes. Since we averaged 20 images per observation, this means ≈ 1.6 hours total acquisition time. Increasing the acquisition time improves the results.

The acquisition time relates to D and the intensity of ZL. Recall that the baseline test points 17° away from the sun. To avoid lens-flare [67], [68], however, such an angle requires the sun to be eclipsed. This can be achieved by being at Earth's night side while the sun is 17° beyond the horizon. This limits the timing of calibration sessions.

Directions farther from the sun have lower ZL radiance. So, to achieve similar signals, directing the imager 27° or 37° away from the sun requires, respectively, $\Delta t = 30s$ and $\Delta t = 60s$. The long acquisition time stems from the noise parameters listed in Sec. 6.1. Larger optics (D) or lower B_T can yield a high signal-to-noise ratio using shorter Δt and fewer images.

7 DISCUSSION

Computational imaging contributes to astrophysics, as evident by [69]. We suggest spaceborne polarimetric camera calibration and self-calibration using ZL. This principle exploits a natural phenomenon that occurs at a relatively large distance at a wide angle. There is wide tolerance to pose deviations. A natural phenomenon does not require using special equipment onboard or outside a spacecraft. ZL changes very slowly and is relatively predictable.

The approach is not limited to LEO. We believe it can also be helpful for deep space probes, observing ZL, or other reliable polarized sources. Future research should hopefully demonstrate these methods using spaceborne data taken by future missions. The paper already includes many sources of disturbances: spatially varying birefringence and polarization, integrated starlight, thermal and photon noise, motion blur, and rotational perturbation to the Mueller matrix by platform jitter. Real measurements may be affected by additional sources. Further real-world discrepancies, so far unaccounted for, can be due to the entry of a comet into the inner solar system, which adds IDPs in its path. New comets are tracked. Thus, during calibration, the imager can point to a sky region that has no overlap with a new comet's path.

The optimization can explicitly include a prior of spatial regularity (smoothness) of Ψ_x . Thus far, computations of calibration or self-calibration are fast. It takes us less than a minute on a common laptop to perform 10 alternating iterations. The process is expected to be offline, once in a while during a space mission's lifetime. So, the time and computation resources needed are small.

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