

# Byzantine Agreement & SMR with Sub-Quadratic Communication

Idit Keidar, Technion

# Shout Out

*Not a COINcidence: Sub-Quadratic Asynchronous Byzantine Agreement WHP.*

Shir Cohen, Idit Keidar, and Alexander Spiegelman



*Expected Linear Round Synchronization: The Missing Link for Linear Byzantine SMR.*

Oded Naor and Idit Keidar



# Byzantine Agreement (BA)

- Consensus among  $n$  processes
- Up to  $f$  can be controlled by an adversary and act arbitrarily
- A building block for State Machine Replication (SMR)

# New Frontiers for BA & Byzantine SMR

- Permissioned blockchains – shared ledger
- Other FinTech infrastructures



ORBS



# BA Has Been Around for Four Decades

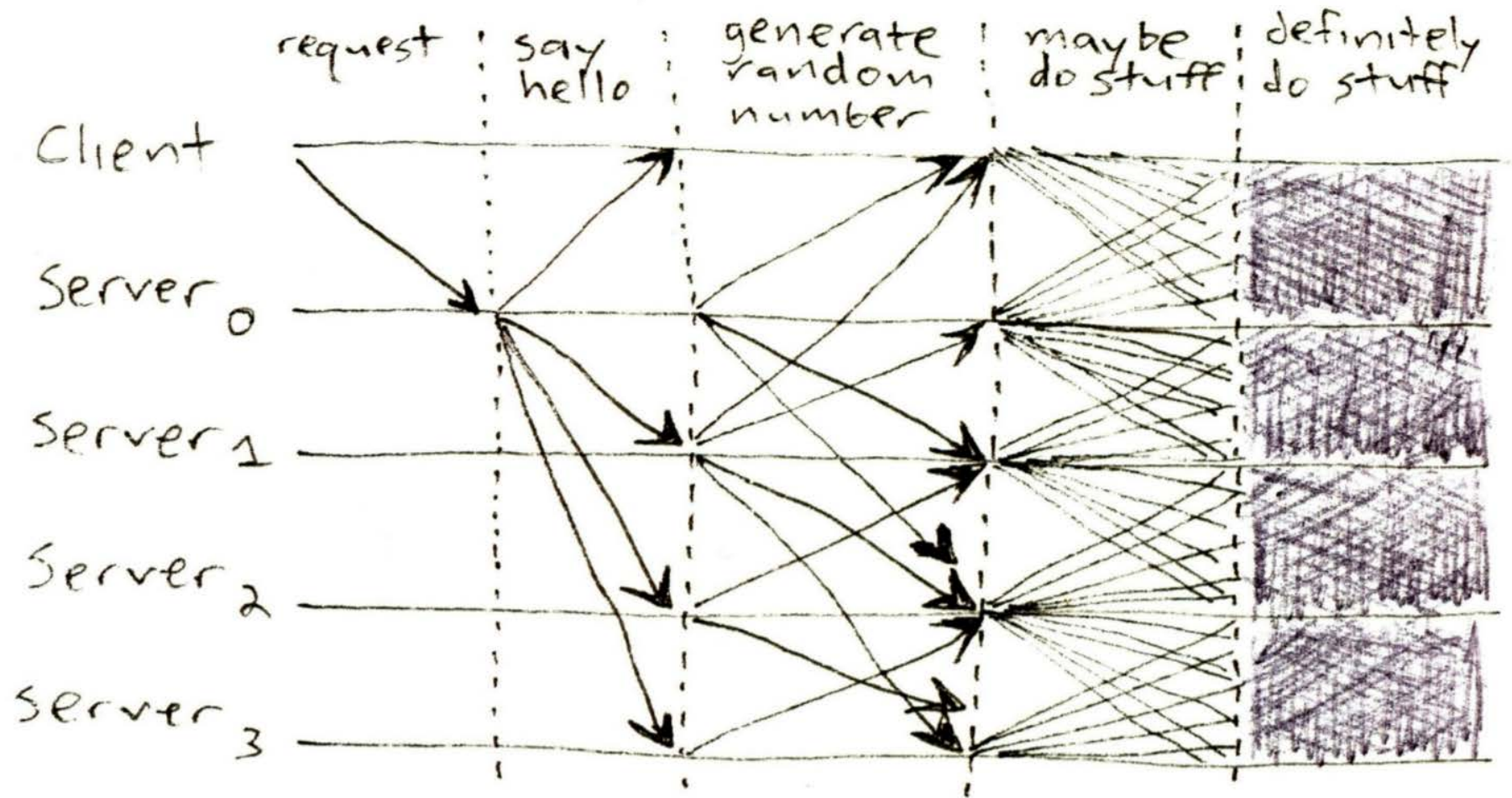
[Pease, Shostak, Lamport 1980], [Lamport, Pease, Shostak 1980]

- 2500+, 7000+ citations, resp.
- Traditional use-cases – a handful of processes

Will it scale?



# Traditional BFT According to James Mickens



**Figure 1:** Typical Figure 2 from Byzantine fault paper: Our network protocol

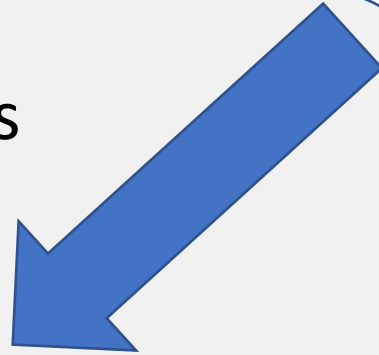
# Scalability Challenges

- Synchrony vs. asynchrony
  - Latency bounds defined in minutes
  - But deterministic fault-tolerant asynchronous consensus is impossible [Fisher, Lynch, Paterson 1985]
- Communication (word) complexity (of all processes together)
  - $\Omega(n^2)$  lower bound  
In the worst-case, in deterministic algorithms, regardless of synchrony [Dolev and Reischuk 1985]

# Making It Scale

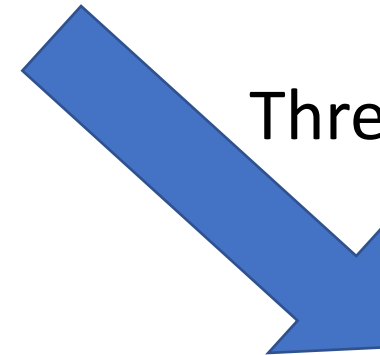


## VRFs



- Assume asynchrony
- Solve BA  
with high probability (WHP)  
(probability of being correct  
tends to 1 as  $n \rightarrow \infty$ )

## Threshold signatures



- Assume eventual synchrony
- Solve deterministic SMR
- Reduce *expected* complexity in  
some *optimistic* cases



# Not a COINcidence: Sub-Quadratic Asynchronous Byzantine Agreement WHP

Shir Cohen, Idit Keidar, Alexander Spiegelman

DISC 2020

# Contribution

## **The first sub-quadratic asynchronous BA WHP algorithm**

- $\tilde{O}(n)$  word complexity and  $O(1)$  expected time
  - Safety and Liveness properties are guaranteed WHP
  - Binary BA
- 
- Previous sub-quadratic works made synchrony assumptions [King and Saia 2011], Algorand [Gilad et al. 2017]

# Model

- Asynchronous
- $n$  processes (permissioned)
- Up to  $f$  Byzantine processes for  $n \approx 4.5f$
- Trusted PKI
  - Inherent for sub-quadratic algorithms  
[Abraham et al. 2019] [Blum et al. 2020] [Rambaud 2020]
- Delayed adaptive adversary:
  - Can use the contents of a message  $m$  sent by a correct process for scheduling a message  $m'$  only if  $m \rightarrow m'$

# Verifiable Random Function (VRF)

- A pseudorandom function that provides a proof of its correct computation
- For a secret key  $sk$  with a matching public key  $pk$ 
  - $VRF_{sk}(x)$  is a random value
  - Verifiable using  $pk$



random  
number  
+ proof

# Use VRFs for

1. Flipping a shared coin
  - First step:  $O(n^2)$  word complexity
2. Committee sampling
  - Cryptographic sortition
  - Reduces word complexity to  $O(n \log n)$

Following Algorand [Gilad et al. 2017]

# Shared Coin with Success Rate $\rho$

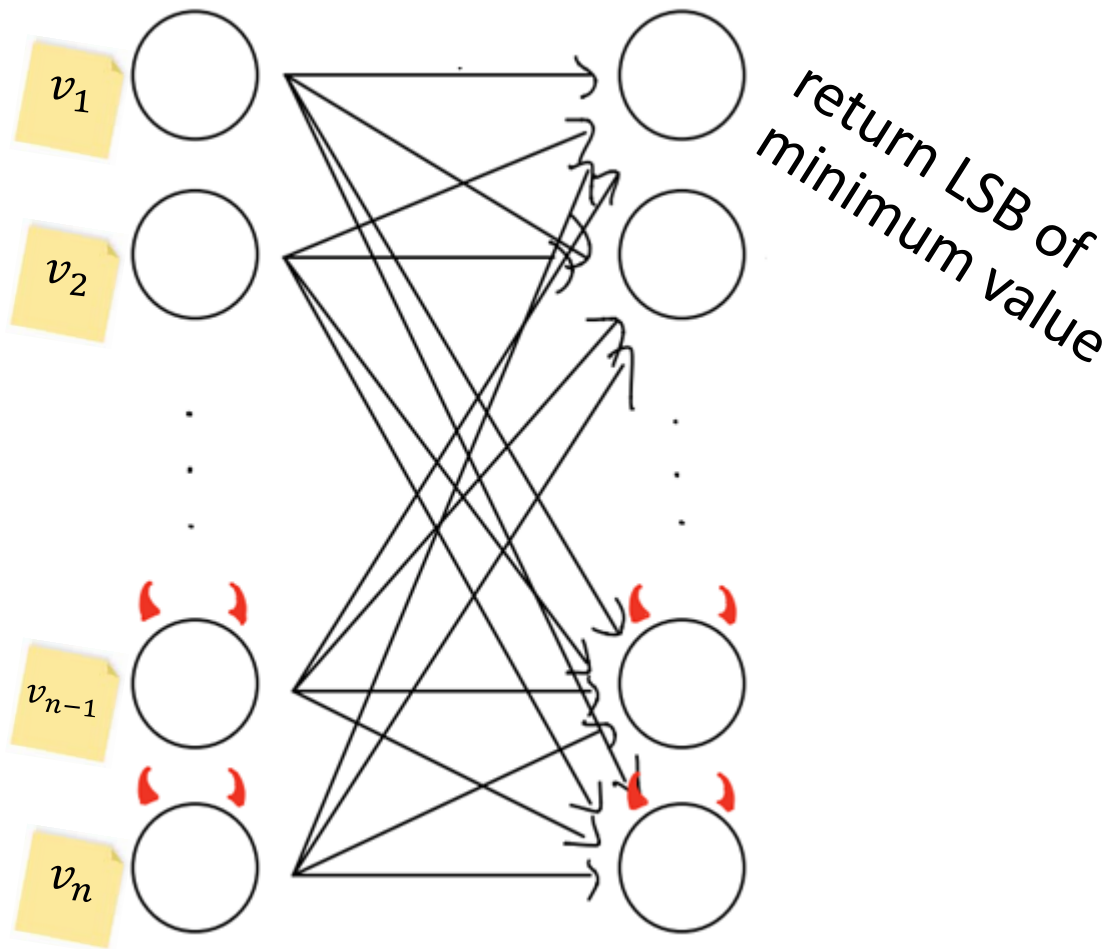
All correct processes output  $b$  with probability at least  $\rho$ , for any value  $b \in \{0,1\}$



# Shared Randomness



# Background: A Simple VRF-Based Shared Coin



- Synchronous  
[Micali 2017]
- If the minimum VRF is of a correct process, all agree
  - With probability  $\geq \frac{2}{3}$



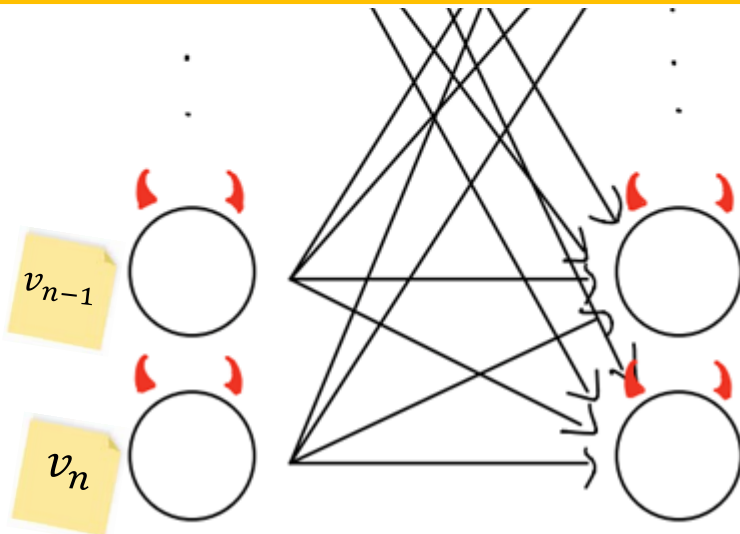
# Background: A Simple VRF-Based Shared Coin



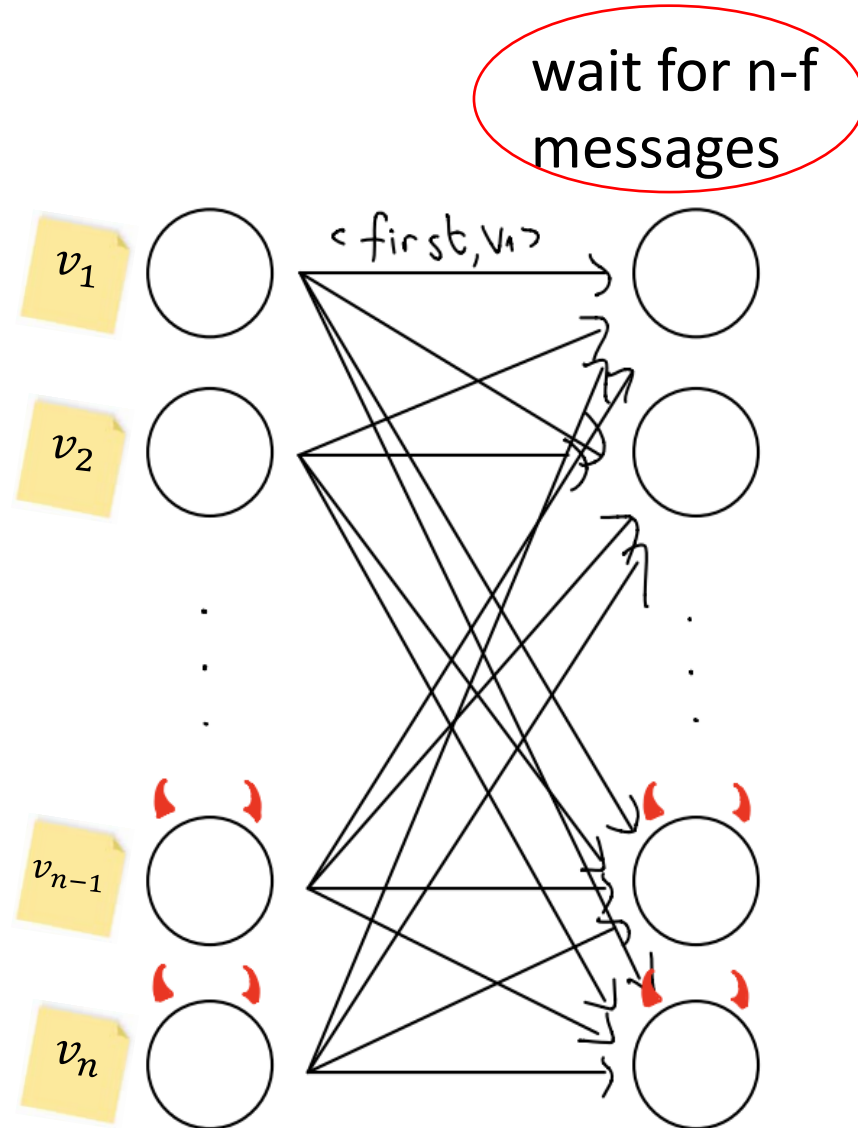
- Synchronous [Micali 2017]

- If the minimum VRF is of a

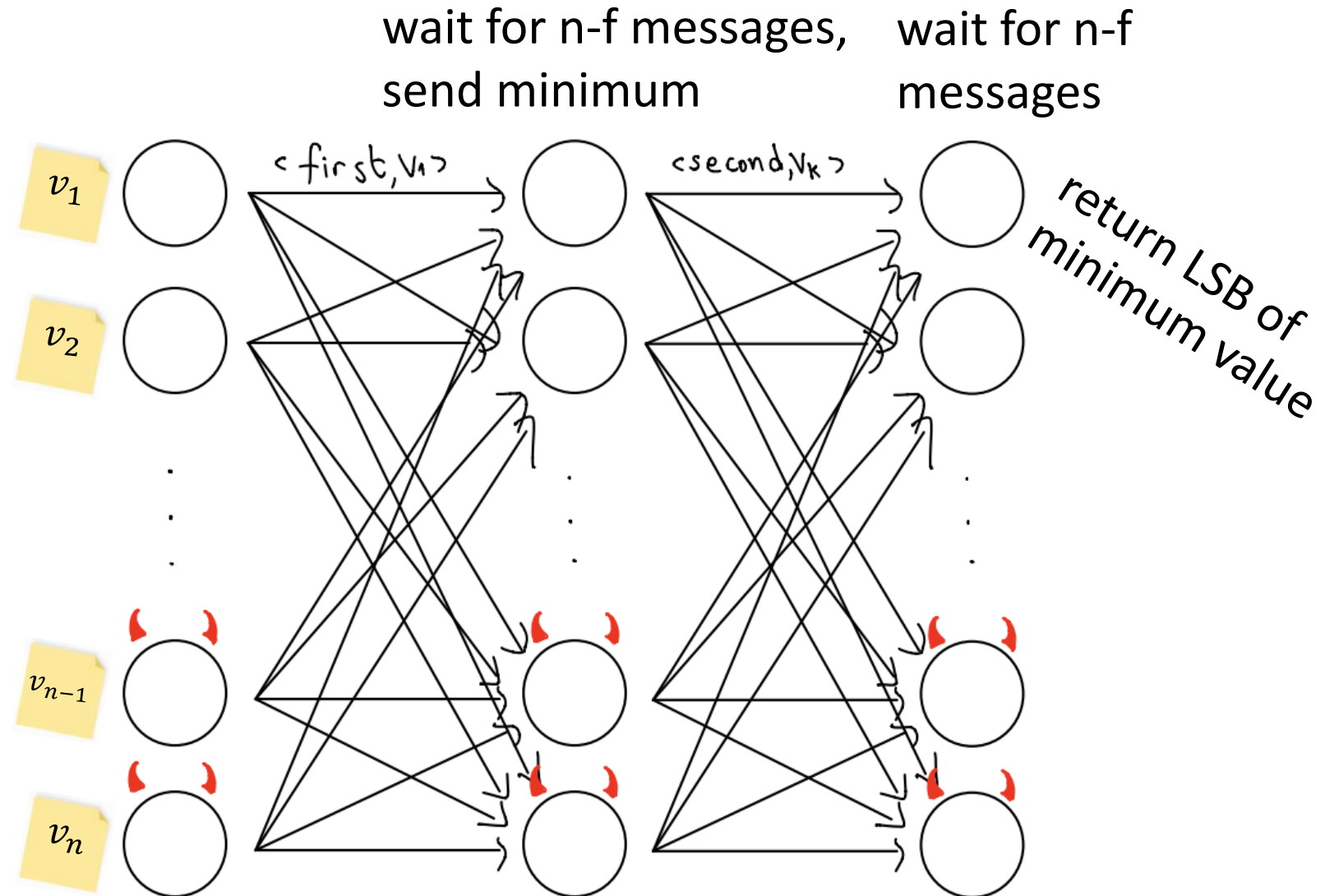
**Requires Synchrony**



# Asynchronous Shared Coin – Take 1



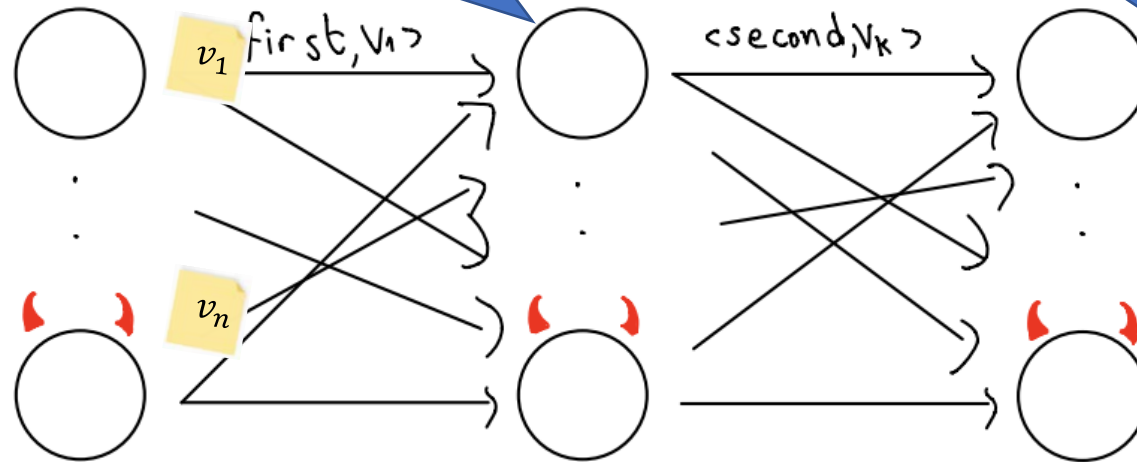
# Asynchronous Shared Coin – Take 1



# Asynchronous Shared Coin - Analysis

a value that reaches  $f + 1$  correct processes is *common*

common values reach all correct processes



- We prove:

- $\Omega(\epsilon)$  bound the number of common values
- our adversary “commits” to them in advance

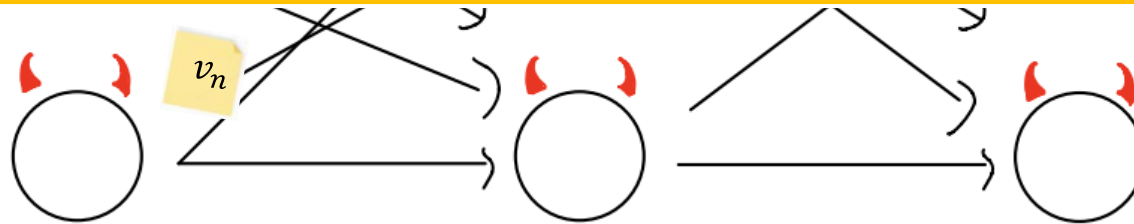
$\Rightarrow$  With a constant probability, the global minimum is common

# Asynchronous Shared Coin - Analysis

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common values reach all correct processes

Word complexity of  $O(n^2)$



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- $\Omega(\epsilon)$  bound the number of common values
- our adversary “commits” to them in advance

⇒ With a constant probability, the global minimum is common

# Use VRFs for

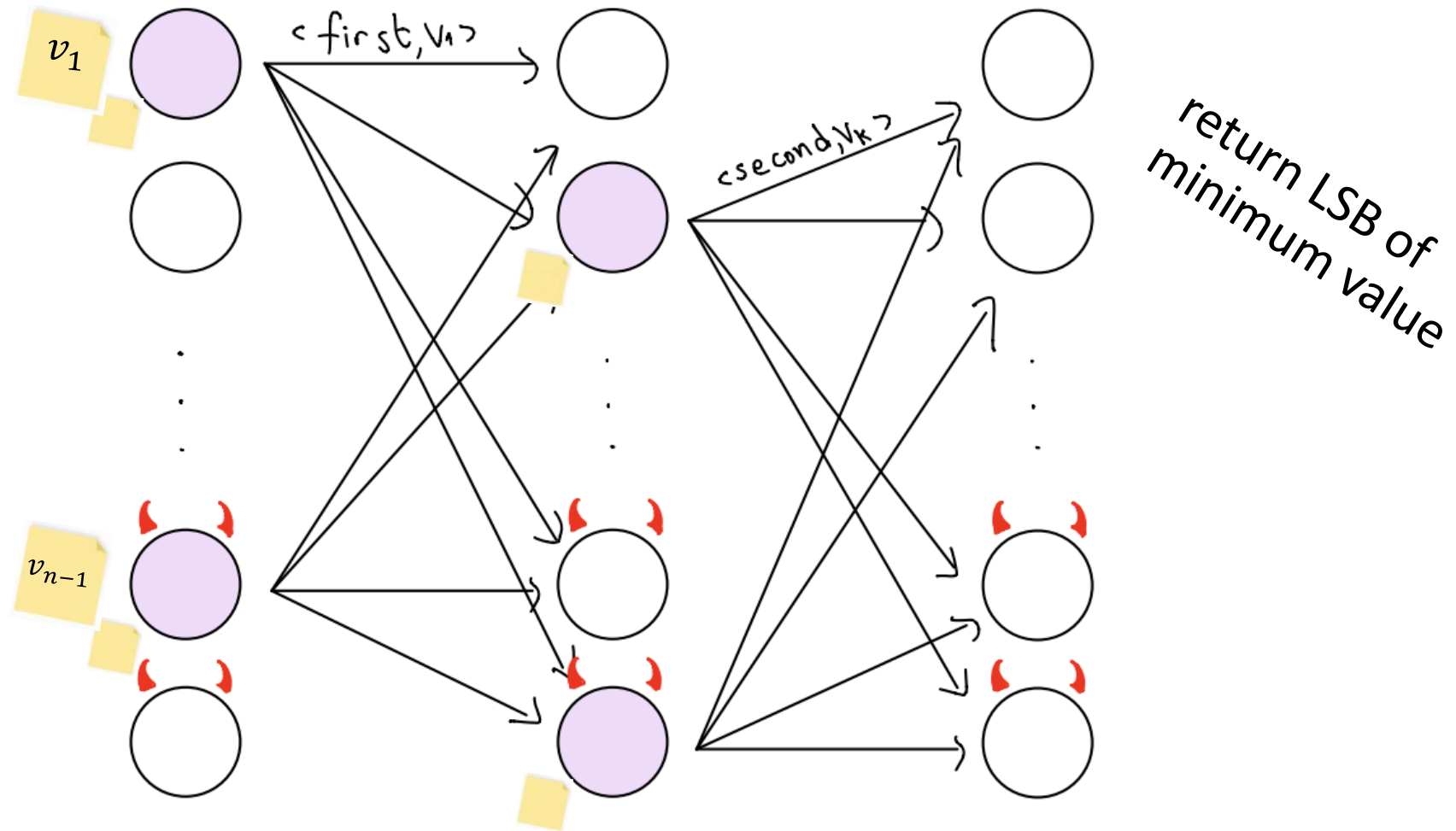
1. Flipping a shared coin
  - First step:  $O(n^2)$  word complexity
2. Committee sampling
  - Cryptographic sortition
  - Reduces word complexity to  $O(n \log n)$

Following Algorand [Gilad et al. 2017]

# Committee Sampling

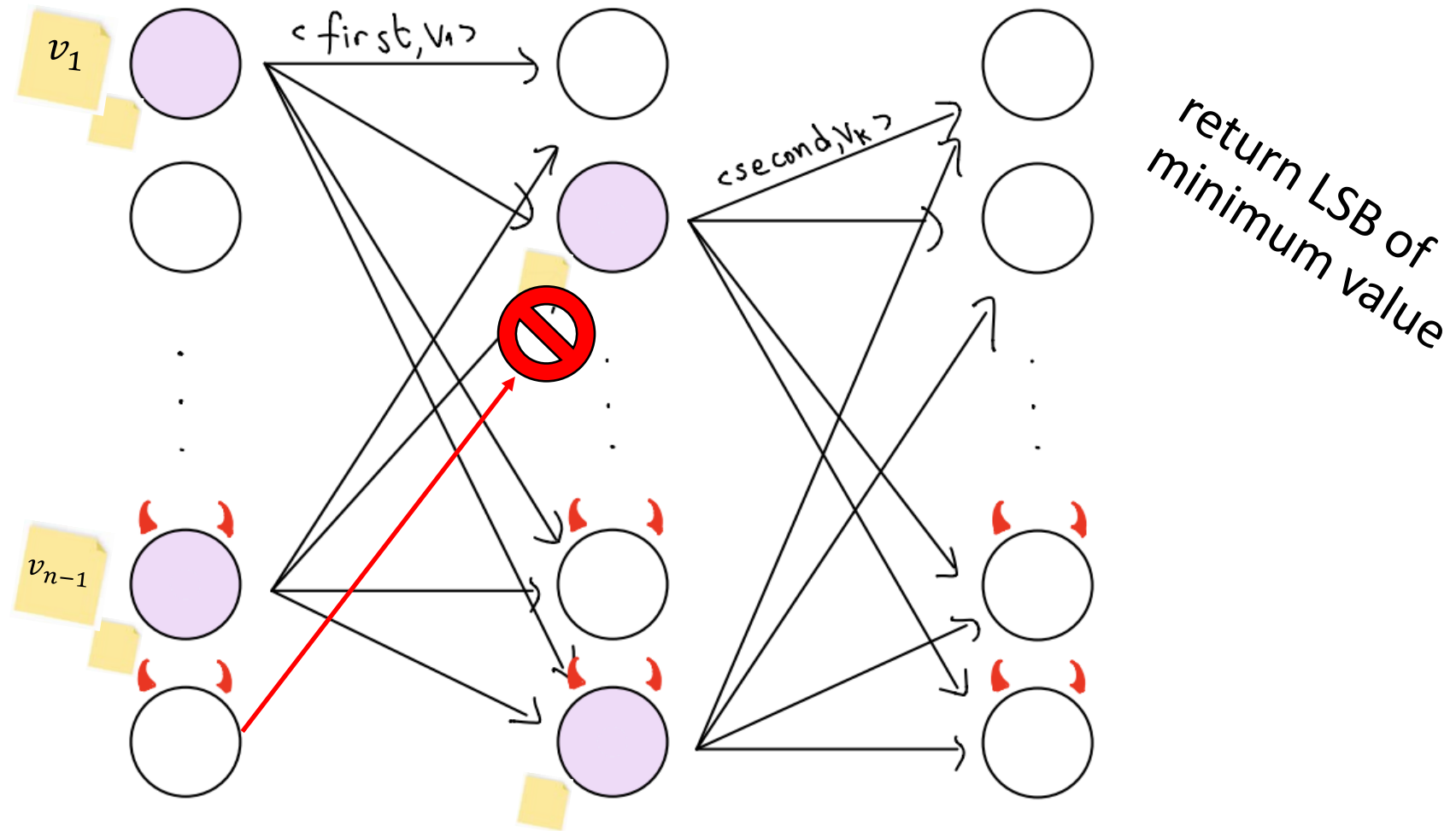
- Use the VRF to sample  $O(\log n)$  processes to a committee in each round
- Replace all-to-all rounds with committee-to-all rounds
- Evading the adversary:
  - Use a new committee in each round
  - Send to all since committees are unpredictable
  - By Chernoff bounds, “not too many” faulty processes in each committee

# Shared Coin – Take 2

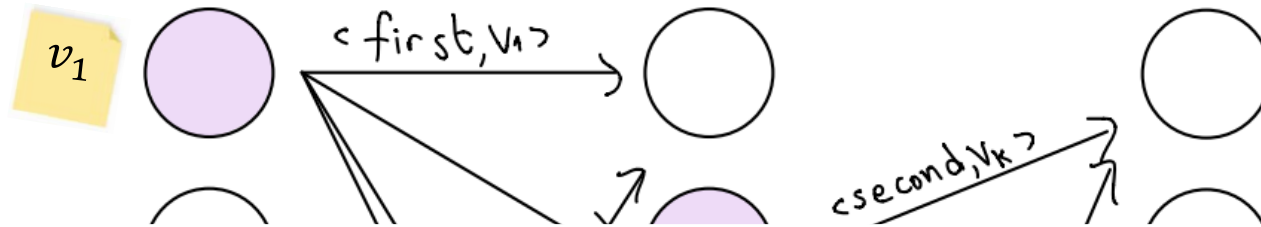




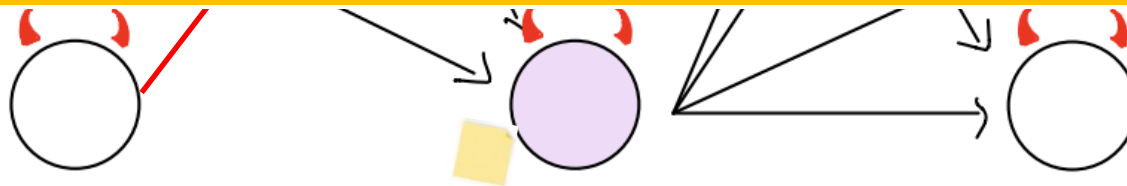
# Shared Coin – Take 2



## Shared Coin – Take 2



Word complexity of  $O(n \log n)$ ,  
but how many processes do we wait  
for?

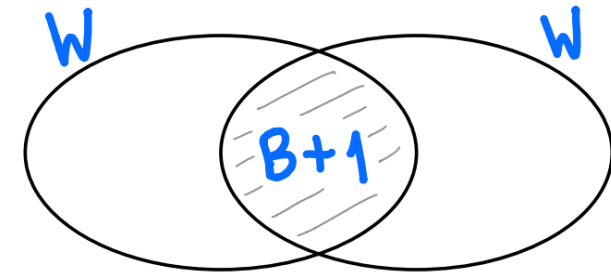


# Committee Sampling in Asynchronous Model

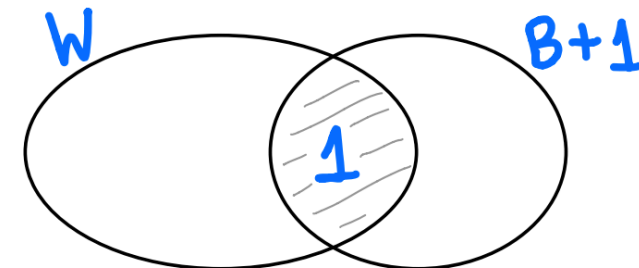
- Committee based protocols cannot wait for  $n - f$  processes. Instead, they wait for  $W$  processes.
- We choose  $W, B$  so that using Chernoff bounds, WHP:
  1. At least  $W$  processes in each committee are correct
  2. At most  $B$  processes in in each committee are Byzantine

# Committee Sampling in Asynchronous Model

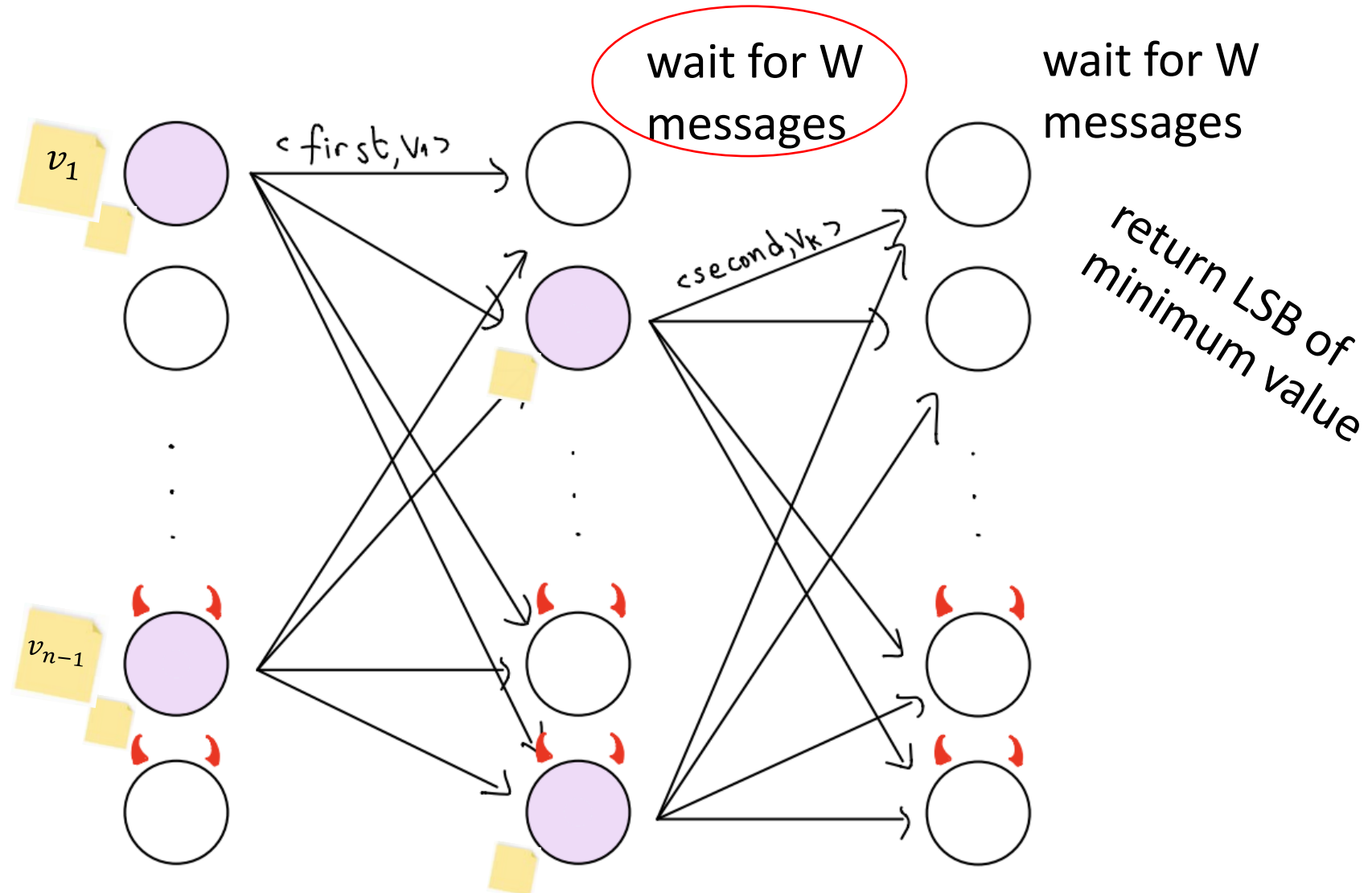
3. Every two subsets in a committee of size  $W$  intersect by at least  $B + 1$  processes



4. Every two subsets in a committee of size  $W$  and  $B + 1$  intersect by at least 1 process



# Shir Cohen's Shared Coin



# From Coin Flipping to (Binary) BA WHP



- Approver based on [Bracha 1987] – reliable broadcast
  - But with committee sampling
- BA based on [Mostefaoui et al. 2015]

# Approver

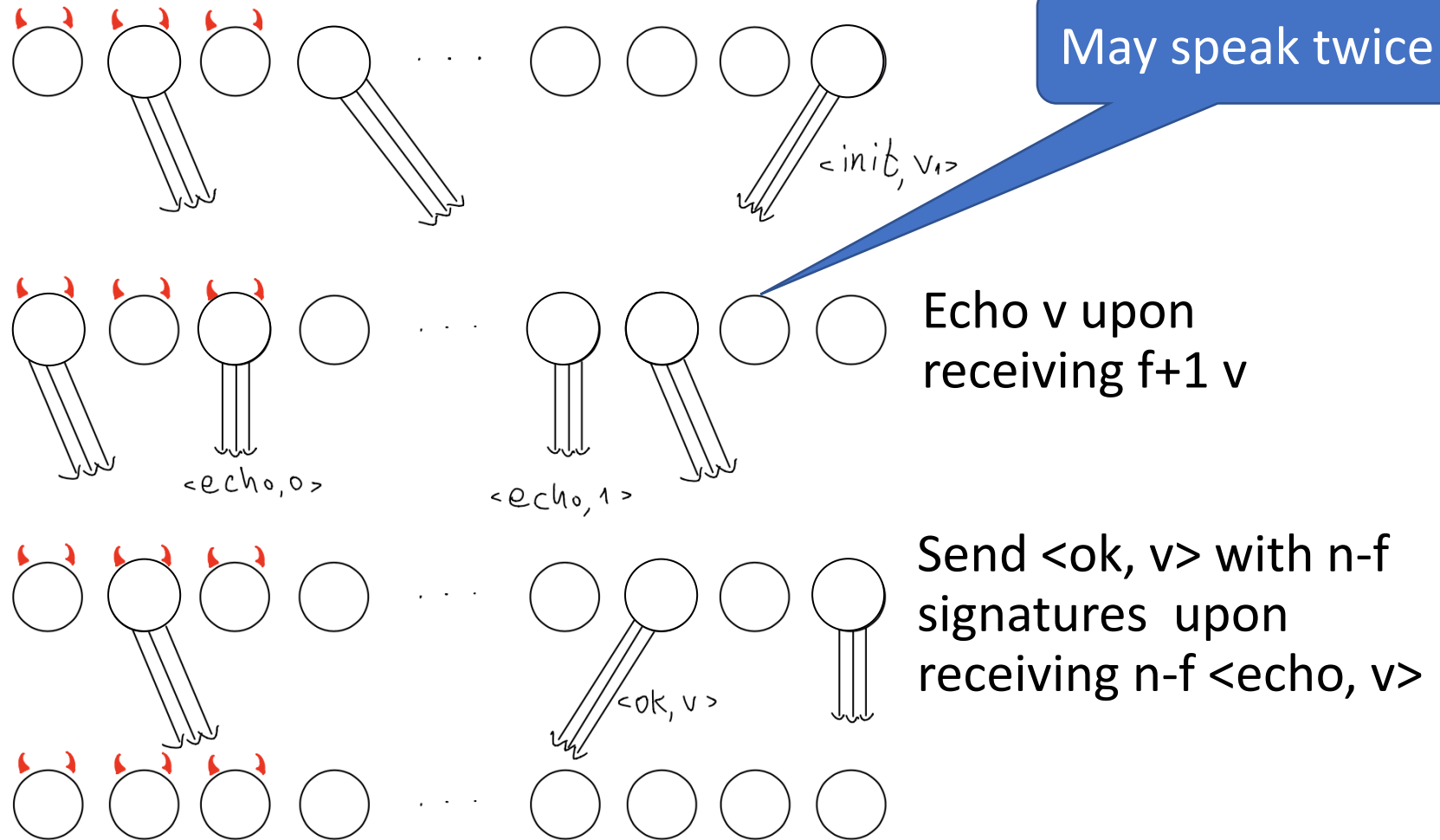
API:  $approve_i(v_i)$  returns a set of values

We assume *approve* is called with at most two different values

WHP the following hold:

- **Validity:** If all correct processes invoke  $approve(v)$  then the only possible return value of correct processes is  $\{v\}$
- **Graded agreement:** If correct processes return both  $\{v\}$  and  $\{w\}$  then  $v = w$
- **Termination:** If all correct processes invoke *approve* then it returns with a non-empty set at all of them

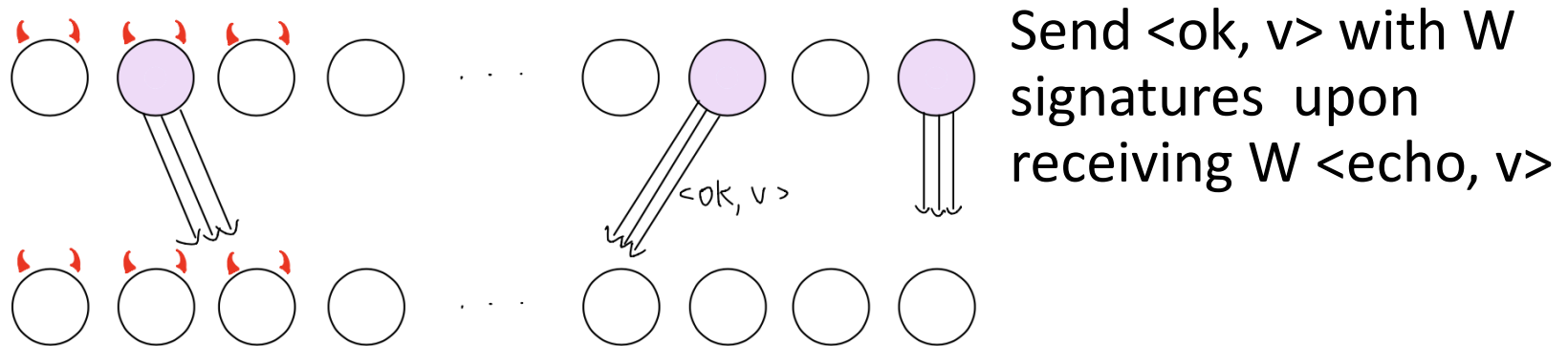
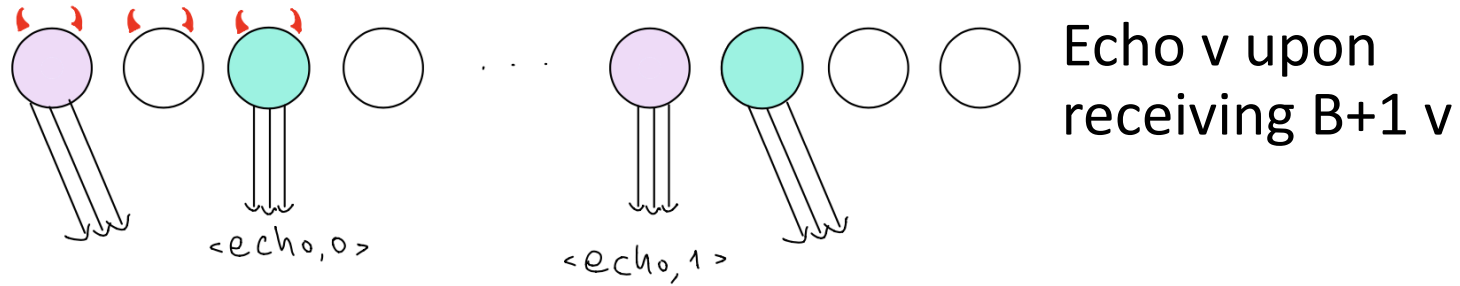
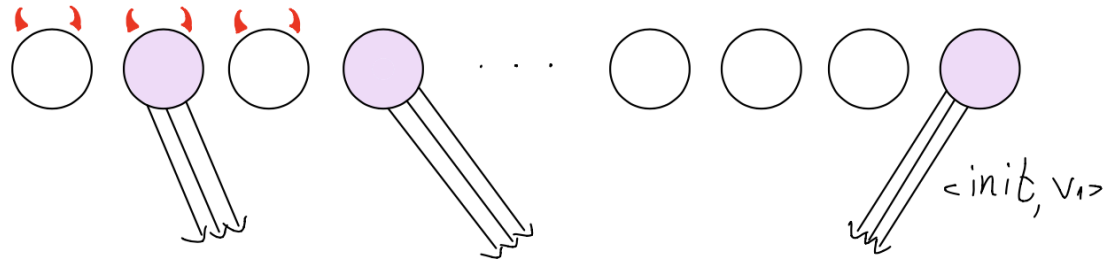
# Approver Without Sampling



Return the set of values in the first n-f ok messages

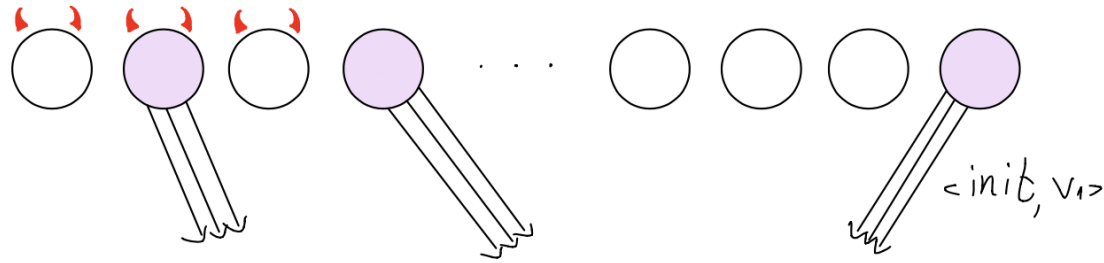


# Approver 👍 With Sampling

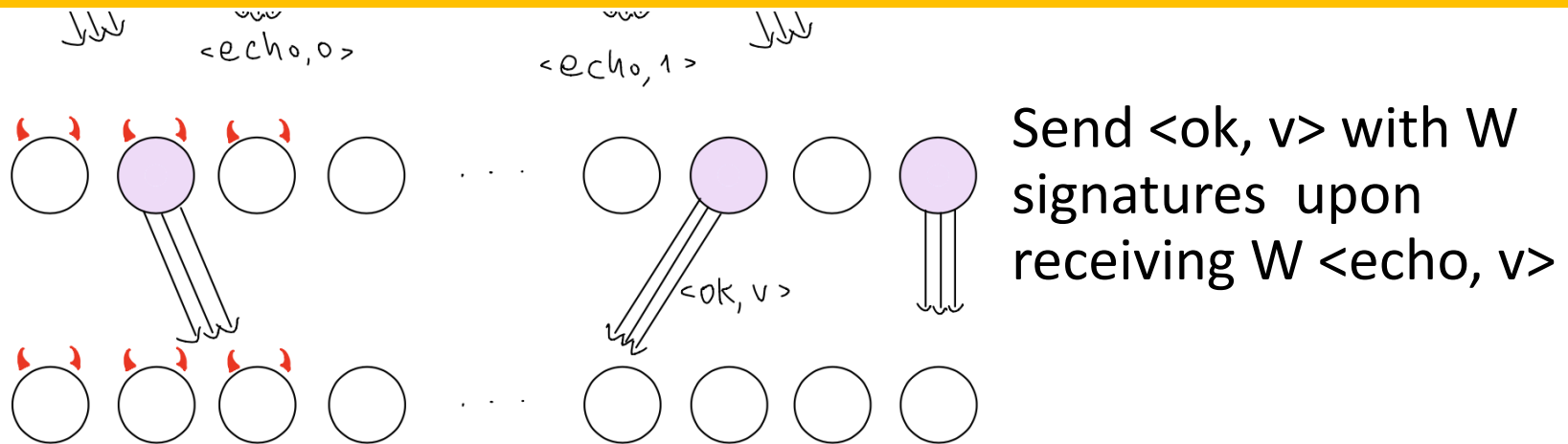


Return the set of values in the first  $W$  ok messages

# Approver With Sampling



Word complexity of  $O(n \log^2 n)$



Return the set of values in the first  $W$  ok messages

# From Coin Flipping to (Binary) BA WHP



- Approver based on [Bracha 1987] – reliable broadcast
  - But with committee sampling
- BA based on [Mostefaoui et al. 2015]

# BA WHP

```
1:  $est_i \leftarrow v_i$ 
2:  $decision_i \leftarrow \perp$ 

3: for  $r = 0, 1, \dots$  do
4:    $vals \leftarrow \text{approve}(est_i)$ 
5:   if  $vals = \{v\}$  for some  $v$  then
6:      $propose_i \leftarrow v$ 
7:   otherwise  $propose_i \leftarrow \perp$ 
8:    $c \leftarrow \text{whp\_coin}(r)$ 

9:    $props \leftarrow \text{approve}(propose_i)$ 
10:  if  $props = \{v\}$  for some  $v \neq \perp$  then
11:     $est_i \leftarrow v$ 
12:    if  $decision_i = \perp$  then
13:       $decision_i \leftarrow v$ 
14:  else
15:    if  $props = \{\perp\}$  then
16:       $est_i \leftarrow c$ 
17:    else  $\%props = \{v, \perp\}$ 
18:       $est_i \leftarrow v$ 
```

# BA WHP

```
1:  $est_i \leftarrow v_i$ 
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Word complexity of  $O(n \log^2 n)$

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5: if  $props = \{v\}$  for some  $v$  then  
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7: otherwise  $propose_i \leftarrow \perp$   
8:  $c \leftarrow \text{whp\_coin}(r)$ 
```

```
14: else  
15:   if  $props = \{\perp\}$  then  
16:      $est_i \leftarrow c$   
17:   else  $\%props = \{v, \perp\}$   
18:      $est_i \leftarrow v$ 
```

# Not a COINcidence Summary

- First formalization of randomly sampled committees using cryptography in asynchronous settings
- First sub-quadratic asynchronous shared coin and BA WHP algorithms
- Expected  $\tilde{O}(n)$  word complexity and  $O(1)$  expected time

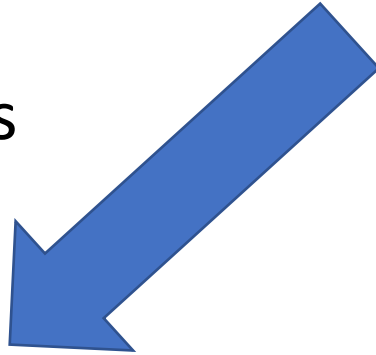
## Limitations:

- Binary consensus only
- Safety and liveness only WHP
- One-shot algorithm (not SMR)
- Non-optimal resilience – improved by [Blum et al. 2020]

# Making It Scale



VRFs



- Assume asynchrony
- Solve BA  
with high probability (WHP)  
(probability of being correct  
tends to 1 as  $n \rightarrow \infty$ )

Threshold signatures



- Assume eventual synchrony
- Solve deterministic SMR
- Reduce *expected* complexity in  
some *optimistic* cases



# Expected Linear Round Synchronization: The Missing Link for Linear Byzantine SMR

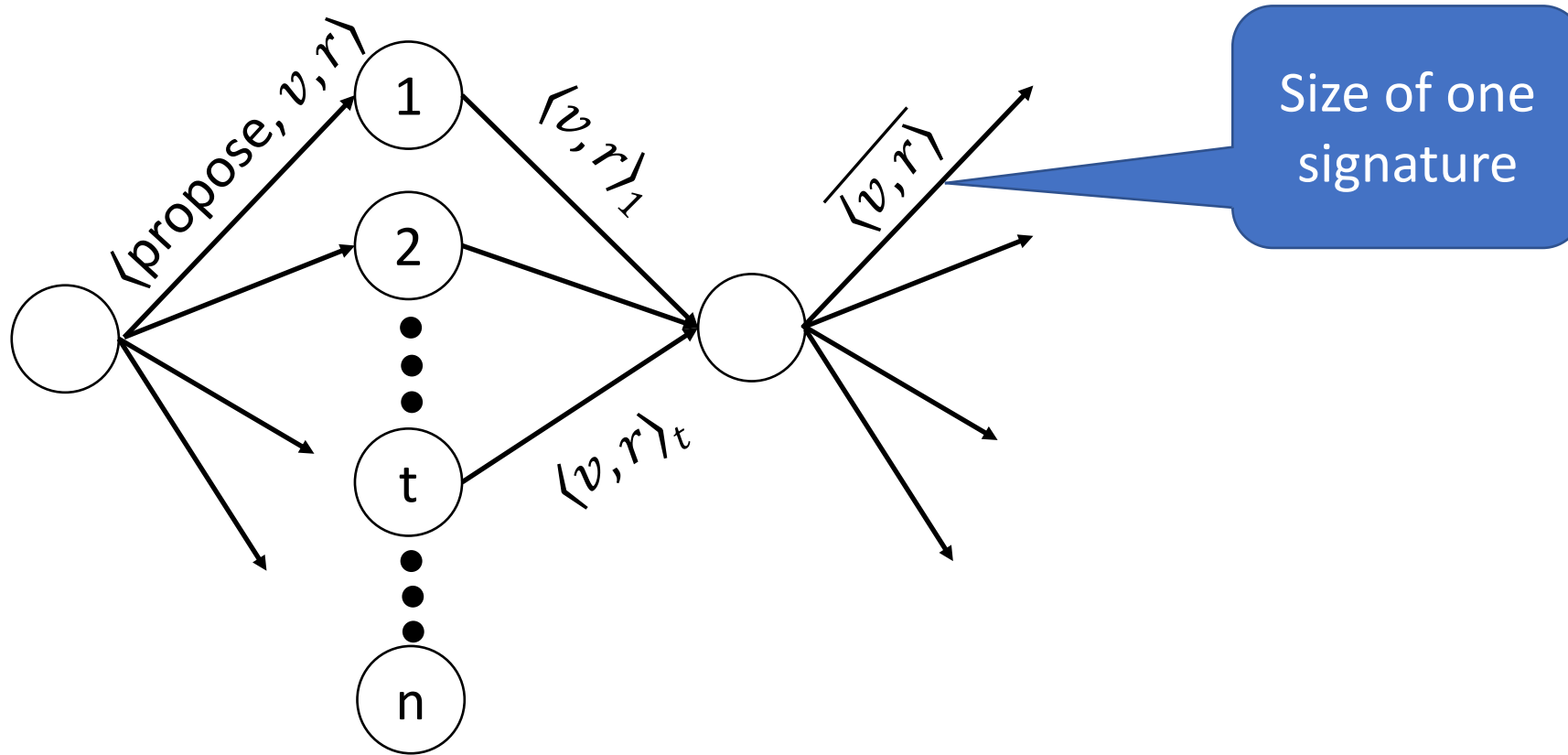
Oded Naor and Idit Keidar

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# Model

- Eventual synchrony
  - Initially asynchronous
  - Synchronous after *Global Stabilization Time (GST)*
  - With latency bound  $\delta$
- Optimal resilience:  $f < n/3$ 
  - For simplicity, assume  $n=3f+1$
- Crypto: threshold signatures, PKI
- Shared source of randomness

# Threshold Signatures Reduce Communication



# Byzantine SMR Communication Costs

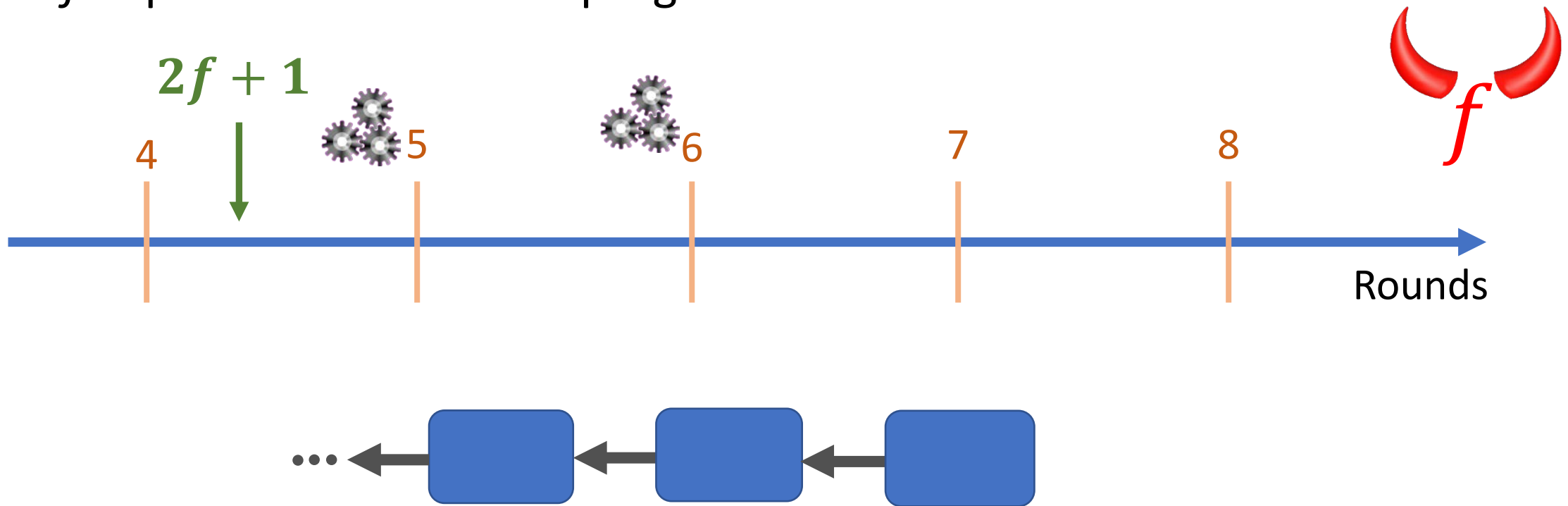
Year	Protocol	Word complexity to reach a decision
1988	DLS	$O(n^3)$
1999	PBFT	$O(n^2)$
2007	Zyzzzyva	$O(n^2)$
2016	Tendermint, Casper	$O(n)$
2017	Algorand	Committees
2018	HotStuff	$O(n)$
2019	LibraBFT	$O(n)$

$O(n)$  once  $2f+1$  correct processes follow a correct leader

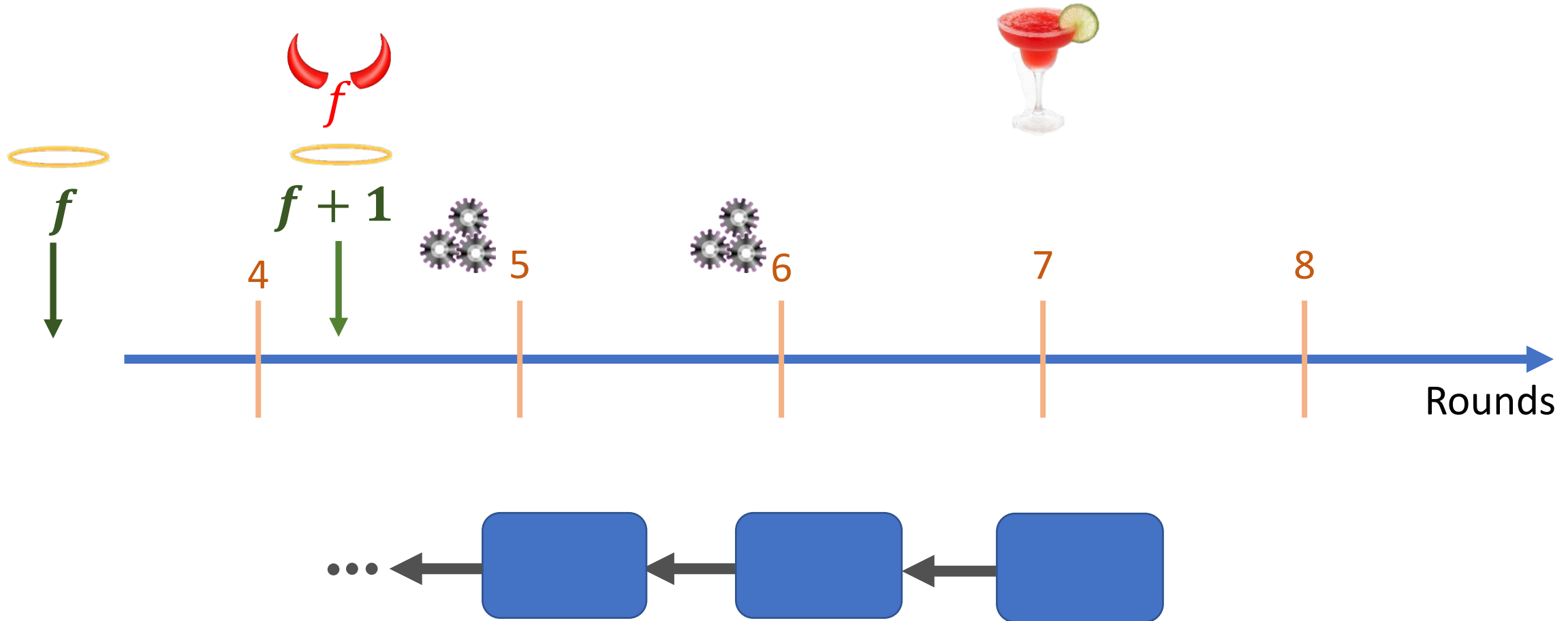


# Eventually Synchronous Byzantine SMR

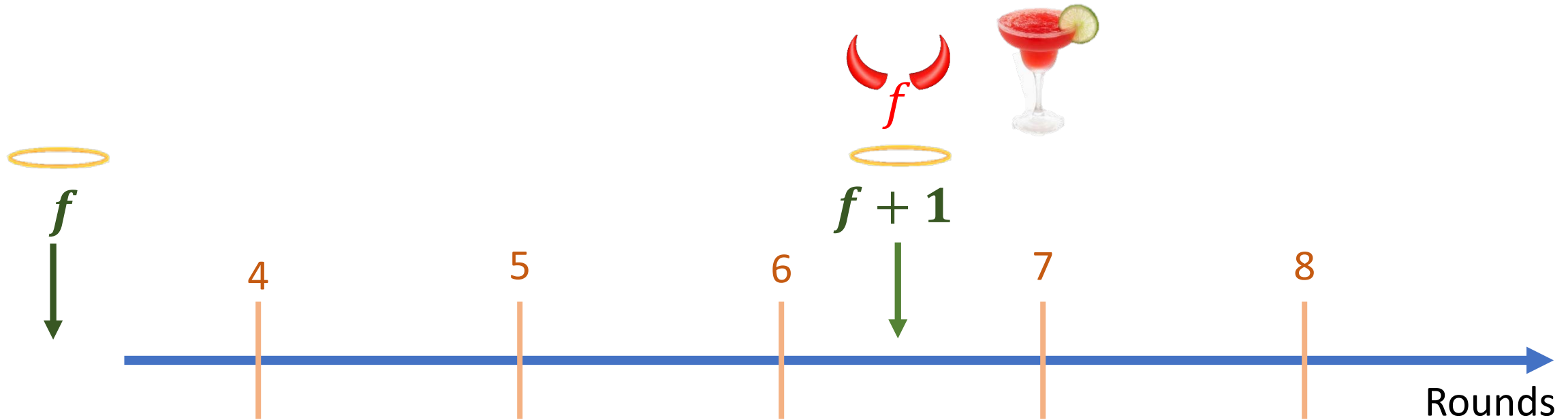
- Each process divides its time into rounds (aka views)
- $2f+1$  processes can make progress



# An Alternative Run



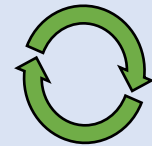
# Needed: Round Synchronization (RS)



# Round Synchronization Makes SMR Live

- Theorem 4 from HotStuff [Yin et al. 2019]:

“After GST, there exists a bounded time period  $T_f$  such that if all correct replicas remain in view  $v$  during  $T_f$  and the leader for view  $v$  is correct, then a decision is reached.”

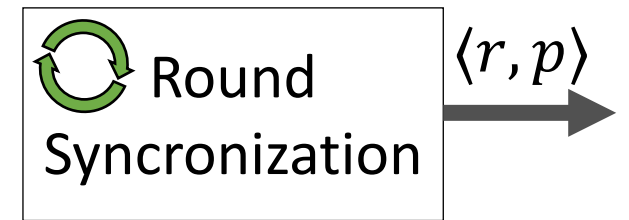


- Formulated and solved as a separate problem  
HotStuff Pacemaker, Cogsworth [Naor et al. 2020], [Bravo et al. 2020]



# The Round Synchronization Service

- Parametrized by a time period  $\Delta$  (e.g.,  $= 4\delta$ )
- Repeatedly outputs round-leader pairs  $\langle r, p \rangle$ 
  - Enter **round**  $r$  with **leader**  $p$
  - Rounds are monotonically increasing
  - Leaders are uniquely determined per round



- **Guarantee:**

For any time  $t$ , there is a **synchronization time**  $t_s \geq t$  so that all correct processes are in the same round with the same correct leader from time  $t_s$  for at least  $\Delta$

- The precondition needed for HotStuff's liveness theorem

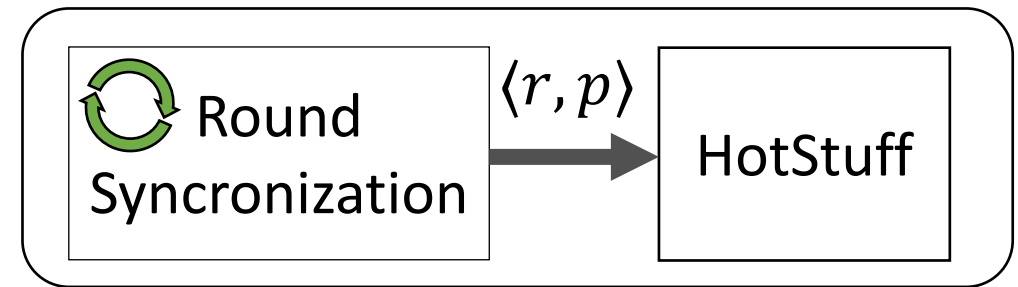
# RS is the Performance Bottleneck

- After round synchronization with a correct leader, we have deterministic SMR
  - $O(n)$  word complexity per decision
  - $O(1)$  time per decision

HotStuff [Yin et al. 2019]

Tendermint [Buchman et al. 2018]

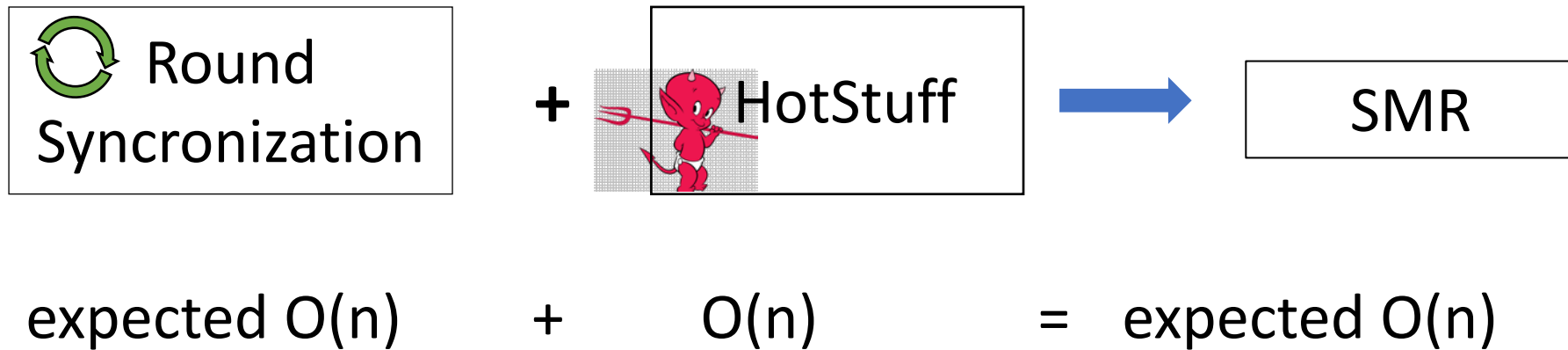
LibraBFT [Baudet et al. 2019]



SMR

- Our solution: RS with expected linear word complexity, constant time

# Fast RS is the Key to SMR Performance

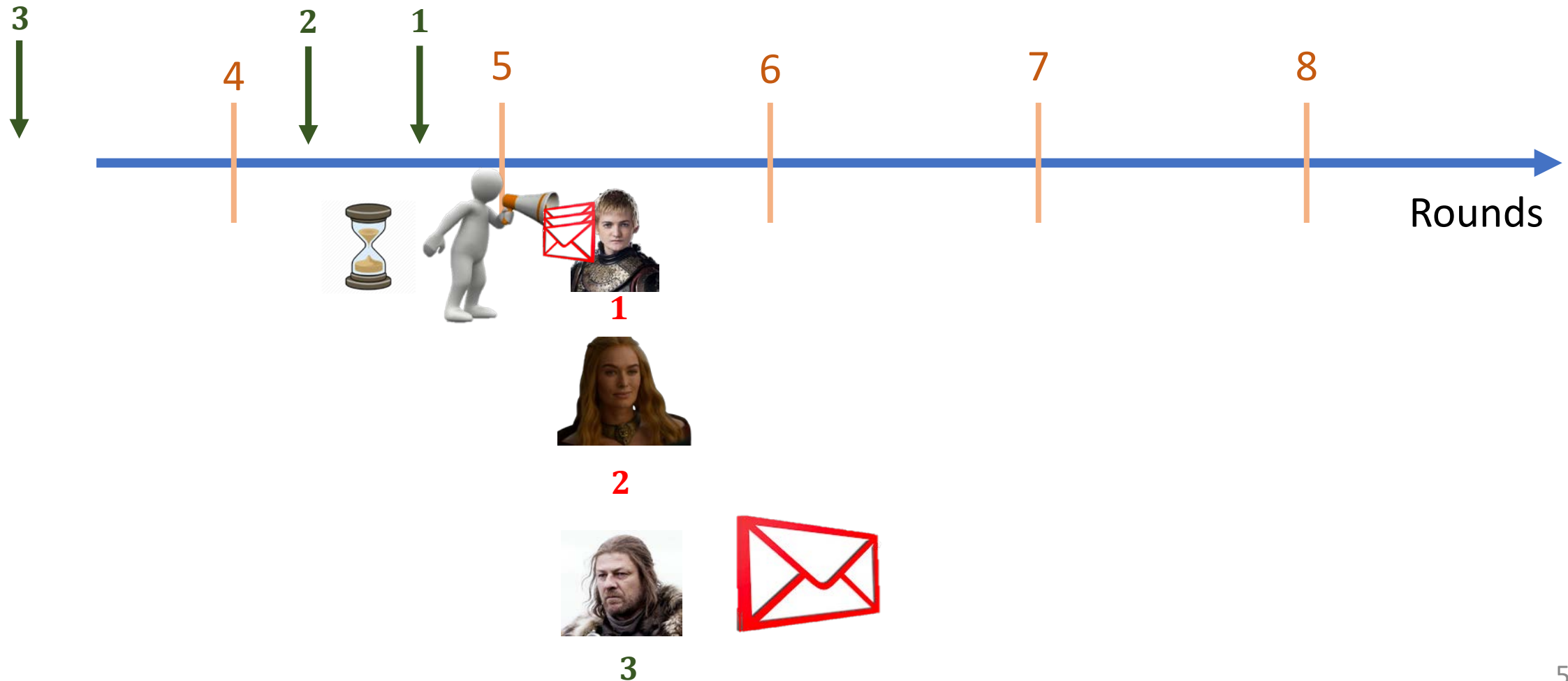


- We get: deterministic SMR, after GST, each decision with
  - Expected  $O(n)$  word complexity,  $O(n^3)$  worst-case
  - Expected  $O(1)$  time,  $O(n^2)$  worst case

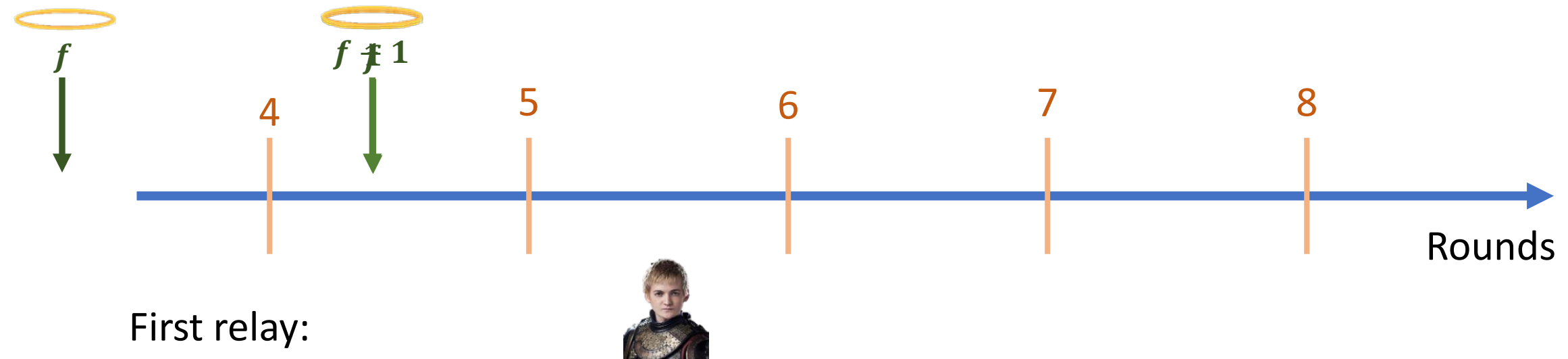
# Relay-Based Round Synchronization

- In each round  $r$ , a designated **relay** is responsible for synchronizing the processes to this round  $r$
- The relay collects **threshold signatures** to prove that enough processes proceed with it
- On **timeout**, switch to another relay
- **Randomly** permute relays in each round
  - In expected constant time, a correct relay is chosen

# Relay-Based Round Synchronization

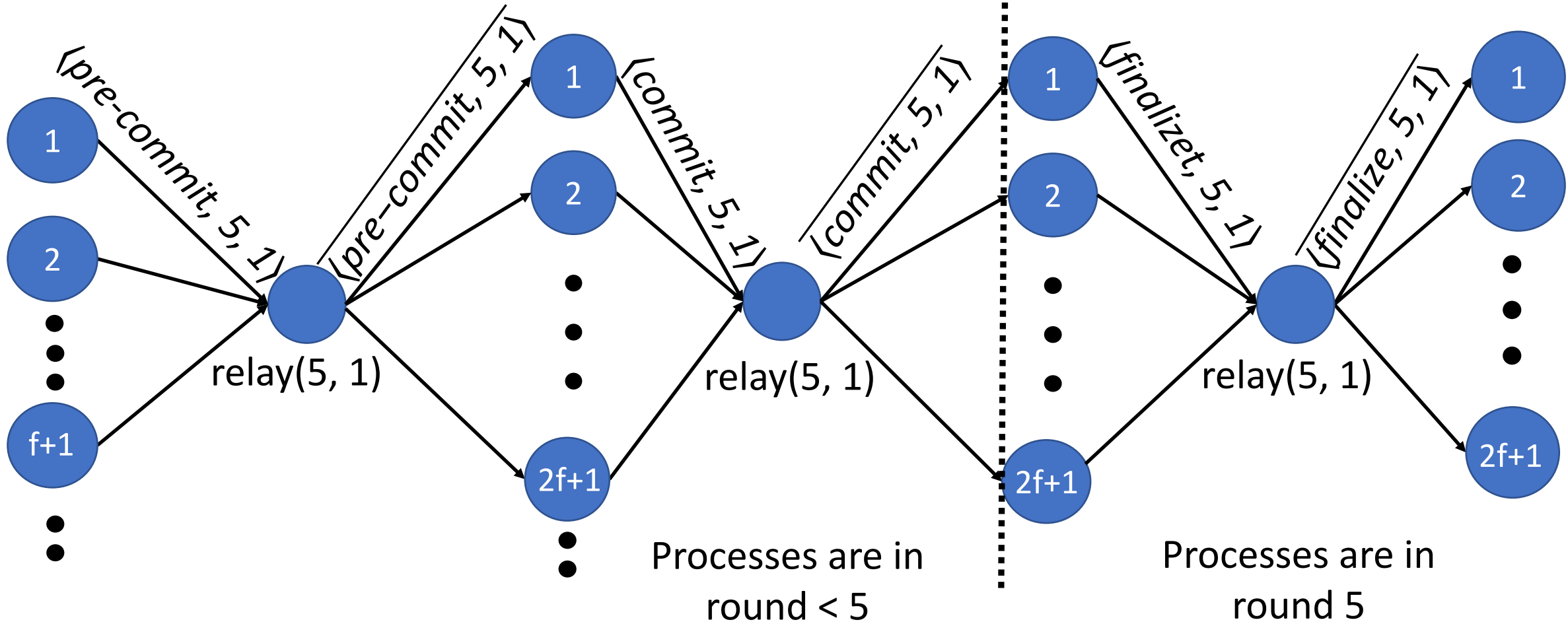


# Byzantine Relays Can Split the Good Guys



- Solved by adding another protocol phase - **finalize**

# Message Flow – Synchronize in Round 5



# Round Synchronization Summary

- Formalize RS abstraction
- Byzantine RS with
  - Expected linear word complexity
  - Expected constant latency
- The missing ingredient for Byzantine SMR with expected linear word complexity
  - Per decision
  - HotStuff, LibraBFT





# Conclusion

Sub-quadratic BA in two flavors:

1. Asynchronous, binary BA WHP
2. Eventually synchronous, multi-value SMR

Thank you!

Yes, it will scale!

