Congestion Control Through Input Rate Regulation

Moshe Sidi, *Senior Member, IEEE,* Wen-Zu Liu, Israel Cidon, *Senior Member, IEEE,* and Inder Gopal, *Fellow, IEEE*

***Abstract*—Traditional packet switching networks have typically employed window-based congestion control schemes in order to regulate traffic flow. In broadband networks, the high speed of the communication links and the varied nature of the carried traffic make such schemes inappropriate. Therefore, simpler and more efficient schemes have to be proposed to fully exploit the large available bandwidth.**

These schemes usually operate through input rate regulation. Typically, they force the information sources to limit their average input rate below some predefined rate while still allowing for a certain degree of burstiness. This ensures that no source will exceed for an extensive period of time the rate provided by the network during the call-setup procedure. The "leaky bucket" scheme is an example of an input rate regulation.

In this paper, input rate regulation schemes are extensively studied from the viewpoint of smoothing and regulating effects of the incoming traffic. The smoothing effect is characterized by the variance of the interdeparture time of the packet departure process from the input rate regulation mechanism. Under the assumption of Poisson arrivals the characteristics of this departure process are explicitly derived in terms of scheme's parameters and the tradeoff between the smoothness of the departure process and packets waiting time is studied. We present results for both finite and infinite buffer pool sizes.

I. Introduction

PACKET switched networks have changed considerably in recent years. One factor has been the dramatic increase in the capacity of the communication links. The advent of fiber optic media has pushed the transmission speed of communication links to over a Gigabit/s, representing a significant increase over typical links in today's packet switched networks [5]. A second factor is the altered nature of traffic transmitted through these networks. It is now accepted that packet switched networks [1], [5] (or variants of packet switching like ATM) will form the basis for multimedia high speed networks that will transmit voice, data and video through a common set of backbone nodes and links.

Both these factors have a significant impact on the design of the protocols and control procedures within the network. In particular, conventional mechanisms for controlling congestion within the network based on end-to-end windowing schemes [2]–[4], are unsuitable (we elaborate more on the reason below). In this paper we study an alternative approach to congestion control based on open-loop regulation of the input.

Window based mechanisms typically rely on end-to-end exchange of control messages in order to regulate traffic flow. The control messages (sometimes with additional congestion information added by the intermediate nodes) are used as feedback by the source node to regulate its traffic flow into the network. In high speed networks, the propagation delays across the network typically dominate the switching and buffering delays. Thus, the feedback from the network is usually outdated and any action the source takes is too late to resolve buffering or switching congestion. This argues for mechanisms that do not rely so heavily on network feedback. It is also important that the congestion control mechanism operates at the speed of communication link. For this reason, computationally intensive control schemes are less desirable than simple schemes that can be easily implemented in high-speed hardware.

The nature of traffic also affects the design of the congestion control. While today's data traffic can usually be slowed down in order to cope with network congestion, it is likely that the real-time nature of the traffic in broadband networks will require some level of bandwidth guarantee. Real-time traffic (e.g., voice, video, image) has an intrinsic rate determined by external factors that are outside the control of the network. Typically, this rate can be estimated by the network prior to the establishment of the connection. The ability to slow down such sources is usually very limited. Note, however, that the packet arrival process is stochastic so there is no guarantee that over short periods the source will keep to the specified average rate. In addition, the initial estimate of the rate may be incorrect.

The above factors suggest a simple congestion control mechanism that does not react dynamically to network conditions. Instead, it uses knowledge of the extrinsic parameters associated with the connection and controls the source by forcing it to conform to these parameters. We refer generically to such schemes as input rate regulation schemes. The "Leaky-Bucket" scheme proposed in [1] and the scheme used in PARIS [5] are examples of input rate regulation mechanisms.

The basic operation of such a scheme is simple. Input packets first enter a queue Q1. If the queue Q1 is full the packet is discarded at the source. In order for the packet at the head of the line to enter the network, it must obtain a token from a token-pool. Tokens are generated into this token-pool at fixed time intervals that correspond to the specified average rate of the connection. If a predefined maximum number of tokens (say, M) have collected in the token-pool the token

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M. Sidi was on leave to the IBM Thomas J. Watson Research Center, Yorktown Heights, NY 10598. He is now with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel.

W.-Z. Liu is with the Department of Electrical Engineering, Polytechnic University, Brooklyn, NY 11201.

I. Cidon was with the IBM Thomas J. Watson Research Center, Yorktown Heights, NY 10598. He is now with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel.

I. Gopal is with the IBM Thomas J. Watson Research Center, Yorktown Heights, NY 10598.

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generation process is shut-off. The operation of the scheme guarantees that during any interval of length T, the number of packets entering the network does not exceed the sum of a prespecified rate times the interval length T plus the constant M that is independent of T [6]. Thus, it is guaranteed that the long term average rate does not exceed the prespecified rate of the connection. However, over short periods, the scheme permits bursts of much higher rate. Essentially, the choice of M determines the burstiness of the transmission. A value of 1 maximizes the "smoothness" of the traffic. Typically, the choice of M should match the application being supported.

The original Leaky-Bucket scheme [1] does not provide an input queue (Q1). Thus, packets arriving at the system when a token is not available are discarded. An approximate analysis of the throughput of this system has been presented in [9]. In this paper we provide an exact analysis of this scheme as well as schemes providing an input queue. The motivation of having an input queue as suggested in [5] is to more effectively control the tradeoff between waiting times and loss probabilities.

Note that there are several parameters associated with these schemes (the size of Q1, M, the token generation rate). In addition, there are several measures of interest in evaluating the performance of the schemes. These include the "smoothness" of the packet stream offered to the network (measured by the variance of the interdeparture times), the waiting time at Q1, the number of packets discarded because of lack of room in Q1, etc. In this paper we analyze exactly input rate regulation schemes to obtain an understanding of the tradeoff between these parameters and measures. Specifically, for Poisson arrivals and for schemes with a finite or an infinite buffer we determine the Laplace transforms of the waiting time and the interdeparture time, the expected waiting time, the output rate (and thus the loss probability) and the variance of the interdeparture time.

II. Model and Analysis

A. The Queueing Model

The queueing model for an input rate regulation scheme is depicted in Fig. 1. A pool of tokens that can contain at most M tokens is available. The generation process of tokens is deterministic, i.e., each D seconds a new token is generated and stored in the pool if it contains fewer than M tokens. Otherwise, the newly generated token is discarded.

Packets arrive into a (finite or infinite) buffer (Q1) according to a Poisson process with rate λ. An arriving packet that finds the token pool nonempty, departs the system instantaneously and one token is removed from the token pool. An arriving packet that finds the token pool empty joins the queue (Q1) if the buffer is not full. When the queue (Q1) is not empty (the token pool must be empty in this case) and a token is generated, one packet departs the queue instantaneously (we assume a first-in-first-out order) and the token is removed from the pool. Note that the packet departure process from this system constitutes the input process to the network that should be regulated.

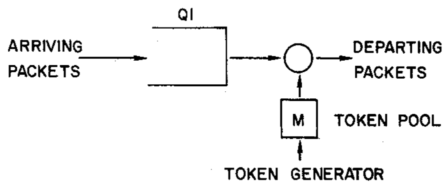

Fig. 1. Queueing model for an input rate regulation scheme.

B. Queue and Token Pool Occupancy: Embedded Joint Probability Distribution

Consider a slotted time axis where each slot is of length D and a new token is generated at each slot boundary. A generated token joins the token pool if it contains fewer than M tokens, otherwise, it is discarded. In this section we derive the steady-state joint probability distribution of the queue size and the token pool occupancy at slot boundaries, just prior to the generation of a new token.

Let $P^t(m,i)$ be the probability of having m tokens in the token pool and i packets in the buffer at time t ($t = 0, D, 2D, \cdots$), just prior to the token generation instances. Since the number of tokens cannot exceed M and packets wait in the queue if and only if there are no tokens in the token pool, we have for all t:

$$P^t(m,i) = 0 \qquad m > M; \qquad 1 \le m \le M \quad \text{and} \quad i \ge 1.$$

For convenience, let

$$p_i^t = \begin{cases} p^t(M-i, 0) & 0 \le i \le M \\ p^t(i-M, 0) & i \ge M. \end{cases} \tag{1}$$

Further, let p_i, $i \ge 0$ be the corresponding steady-state probability, i.e., $p_i = \lim_{t\to\infty} p_i^t$.

In the following a_i denotes the probability of i arrivals during a slot, i.e.,

$$a_i = \frac{e^{-\lambda D}(\lambda D)^i}{i!} \qquad i \ge 0.$$

1) Finite Buffer Size: When the buffer size is limited to K, a packet that arrives and finds the buffer full is discarded. The steady-state equations in this case are ($p_i = 0$ for $i > M+K$):

$$p_i = p_0 a_i + \sum_{j=0}^{i} p_{j+1} a_{i-j} \qquad 0 \le i \le M+K-1. \tag{2}$$

The solution of these equations is simple. One assumes a value for p_0. Then the p_i's ($1 \le i \le M+K$) are computed recursively via $p_i = (p_{i-1} - p_0 a_{i-1} - \sum_{j=0}^{i-2} p_{j+1} a_{i-1-j})/a_0$ and finally all quantities are normalized so that $\sum_{i=0}^{M+K} p_i = 1$.

The throughput of the system is

$$T = p_0 \left(\sum_{i=0}^{M+K} i a_i + (M+K)\overline{a}_{M+K} \right) + \sum_{j=1}^{M+K} p_j \left(\sum_{i=1}^{M+K-j+1} i a_i + (M+K-j+1) \cdot \overline{a}_{M+K-j+1} \right) \tag{3}$$

where $\bar{a}_j = 1 - \sum_{i=0}^{j} a_i$. The loss probability of a packet is given by

$$P_{\text{loss}} = 1 - \frac{T}{\lambda D}. \tag{4}$$

Note that when there are no buffers ($K = 0$), the scheme corresponds to the "Leaky-Bucket" scheme [1].

2) Infinite Buffer Size:

The steady-state equations in this case are the same as (2) except that they hold for all i,

$$p_i = p_0 a_i + \sum_{j=0}^{i} p_{j+1} a_{i-j} \qquad i \geq 0. \tag{5}$$

Let $G(z) = \sum_{i=0}^{\infty} p_i z^i$. Simple computation yields

$$G(z) = \frac{p_0 A_D(z)(z-1)}{z - A_D(z)} \tag{6}$$

where $A_u(z) = e^{\lambda u(z-1)}$ for all $u > 0$. The constant p_0 is simply determined from the normalization condition $G(z)|_{z=1} = 1$, and we have $p_0 = 1 - \lambda D$ and the condition for stability is $\lambda D < 1$. Note that since p_0 is known, other probabilities can be computed recursively from (5).

Note that $G(z)$ is the generating function of the joint probability distribution at embedded points, just prior to token generation instances. To obtain the distribution at an arbitrary epoch, the method described in the next section can be employed.

C. Waiting Time Distribution

In this section we derive the Laplace transform of the waiting time distribution of a packet when the input buffer is infinite. A similar procedure can be used in the finite buffer case as well.

Tag an arriving packet. Assume the packet arrives u units of time after the beginning of a slot and let q be the number of packets and m the number of tokens in the system just prior to the generation of a token at the beginning of that slot. Since the arrival process is Poisson, the random variable u is uniformly distributed in $(0, D)$. Given u, the number of packets arriving during the first u units of a slot, A_u, have a Poisson distribution with parameter λu. Let $\mathcal{W}_u$ be the waiting time of the tagged packet, given u. Then we have:

$$\mathcal{W}_u = \begin{cases} 0 & \text{if } q = 0,\ 0 \leq m \leq M-1,\ 0 \leq A_u \leq m \\ 0 & \text{if } q = 0,\ m = M,\ 0 \leq A_u \leq M-1 \\ D - u + (A_u - m - 1)D & \text{if } q = 0,\ 0 \leq m \leq M-1,\ A_u \geq m+1 \\ D - u + (A_u - M)D & \text{if } q = 0,\ m = M,\ A_u \geq M \\ D - u + (A_u + q - 1)D & \text{if } q \geq 1,\ m = 0,\ A_u \geq 0. \end{cases} \tag{7}$$

The explanation of (7) is as follows: the tagged packet departs immediately if no packets were in the system at the beginning of the slot and the packets that arrived before the tagged packet since the beginning of the slot did not consume all the tokens present in the token pool at the beginning of the slot. In any other case, the tagged packet waits until the beginning of the next slot $(D - u)$, and also waits an integer number of slots that correspond to departures of packets ahead of the tagged packet.

Using the fact that arrivals in u are independent of the state of the system at the beginning of the slot we obtain form (7) the Laplace transform of the waiting time, conditioned on u,

$$\begin{aligned} W_u^*(s) &= E[e^{-s\mathcal{W}_u}] \\ &= \sum_{m=0}^{M-1} \sum_{i=0}^{m} p_{M-m} a_i(u) + p_0 \sum_{i=1}^{M-1} a_i(u) \\ &\quad + \sum_{m=0}^{M-1} \sum_{i=m+1}^{\infty} e^{-s[(i-m)D-u]} p_{M-m} a_i(u) \\ &\quad + p_0 \sum_{i=M}^{\infty} e^{-s[D-u+(i-M)D]} a_i(u) \\ &\quad + \sum_{q=1}^{\infty} \sum_{i=0}^{\infty} e^{-s[(i+q)D-u]} p_{M+q} a_i(u) \end{aligned}$$

where $a_i(u) = e^{\lambda u} (\lambda u)^i / i!$. After some algebra, using (6), we obtain

$$\begin{aligned} W_u^*(s) &= \frac{p_0 (e^{-sD} - 1)}{e^{-sD} - A_D(e^{-sD})} e^{s[u+(M-1)D]} A_u(e^{-sD}) \\ &\quad + \sum_{m=0}^{M-1} \sum_{i=0}^{m} p_{M-m} a_i(u) \left(1 - e^{s[u+(m-i)D]}\right) \\ &\quad + p_0 \sum_{i=0}^{M-1} a_i(u) \left(1 - e^{s[u+(M-1-i)D]}\right). \end{aligned}$$

Using the fact that u is uniformly distributed in $(0, D)$, we obtain the Laplace transform of the waiting time $W^*(s) = E[e^{-s\mathcal{W}}]$

$$W^*(s) = \frac{1}{D} \int_0^D W_u^*(s)\, du.$$

The expected waiting time conditioned on u is obtained in a similar manner from (7)

$$\begin{aligned} E[\mathcal{W}_u] &= p_0 D + \sum_{m=0}^{M-1} \sum_{i=0}^{m} p_{M-m} a_i(u)((m-i)D + u) \\ &\quad + p_0 \sum_{i=0}^{M-1} a_i(u)((M-i)D - D + u) \\ &\quad + D\left(\lambda u - M + \lambda D + \frac{(\lambda D)^2}{2(1-\lambda D)}\right) - u \end{aligned}$$

and

$$\begin{aligned} E[W] &= \frac{1}{D}\int_0^D E[W_u]\,du \\ &= \frac{1}{\lambda}\sum_{m=0}^{M-1}\sum_{i=0}^{m} p_{M-m}\left((m-i)\overline{a}_i + \frac{i+1}{\lambda D}\overline{a}_{i+1}\right) \\ &\quad + \frac{1}{\lambda}p_0\sum_{i=0}^{M-1}\left((M-i-1)\overline{a}_i + \frac{i+1}{\lambda D}\overline{a}_{i+1}\right) \\ &\quad + D\left(\frac{1}{2(1-\lambda D)} - M\right) \end{aligned}$$

where we recall that $\overline{a}_i = 1 - \sum_{j=0}^{i} a_j$.

D. The Interdeparture process

In this section we derive the Laplace transform of the interdeparture time $\mathcal{V}$—the time between two successive departures of packets from the system. Since each departing packet is accompanied by a single token, the departure process of packets is the same as the process of departing tokens. Hence, we will look at the time between two successive token departures. We assume that tokens depart in a first-in–first-out order.

In the following $\mathcal{E}$ denotes interarrival time (exponentially distributed with mean $1/\lambda$. In addition, $\mathcal{R}_i^j$ $(i \geq 1,\ 1 \leq j \leq i)$ denotes the time between the jth arrival epoch in a slot until the end of the slot, given that exactly i packets arrived in that slot. The density function of $\mathcal{R}_i^j$ is given by [8]

$$f_{\mathcal{R}_i^j}(r) = \frac{i!}{(j-1)!\,(i-j)!}\left(1-\frac{r}{D}\right)^{j-1}\left(\frac{r}{D}\right)^{i-j}\frac{1}{D}$$
$$0 \leq r \leq D, \qquad 1 \leq j \leq i, \qquad i \geq 1$$

and the kth moment of $\mathcal{R}_i^j$ is

$$E\left[\left(\mathcal{R}_i^j\right)^k\right] = \frac{(i-j+1)(i-j+2)\cdots(i-j+k)}{(i+1)(i+2)\cdots(i+k)}D^k.$$

Consider an arbitrary token that arrives at the token pool at time t and tag it. In order that this token will ever depart the system, it must join the token pool (the probability of this event is $1 - p_0$). In the following we determine the Laplace transform of the time between the departure epoch of the tagged token and the subsequent departure. This is the interdeparture time $\mathcal{V}$.

Conditioned on the event that the tagged token joins the token pool, it may find the system in one of the following states.

1) There are q $(q \geq 2)$ packets and no tokens in the system. In this case the tagged token departs immediately (at time t) and the next departure will occur at time $t + D$, hence $\mathcal{V} = D$.

2) There is one packet and no tokens in the system. In this case the tagged token departs immediately (at time t) and the next departure will occur at time $t + D$ if at least one packet arrives during $(t, t + D)$, or it will occur upon the next arrival, i.e., at time $t + D + \mathcal{E}$ if no packets arrive in $(t, t + D)$. Hence, $\mathcal{V} = D$ with probability $1 - a_0$ and $\mathcal{V} = D + \mathcal{E}$ with probability a_0.

3) There are m $(0 \leq m \leq M-2)$ tokens and no packets in the system (note that when $M = 1$ this situation never occurs). If i $(0 \leq i \leq m)$ packets arrive during $(t, t+D)$ then $\mathcal{V} = \mathcal{E}$ since the tagged token will depart at some time after $t+D$ due to a packet arrival and the token that arrived at time $t + D$ will depart afterwards upon the next packet arrival. If $m + 1$ packets arrive during $(t, t + D)$, then the tagged token will depart upon the $(m+1)$st arrival and the time until the next token arrives is $\mathcal{R}_{m+1}^{m+1}$. When the next token arrives at time $t + D$, there are no packets in the system, hence it will depart upon the next packet arrival and therefore $\mathcal{V} = \mathcal{R}_{m+1}^{m+1} + \mathcal{E}$. If i $(i \geq m + 2)$ packets arrive during $(t, t + D)$, then $\mathcal{V} = \mathcal{R}_i^{m+1}$ since the tagged token departs upon the $(m+1)$st arrival and the next token departs at time $t + D$.

4) There are $M - 1$ tokens and no packets in the system. This case is similar to the above, except that until there is a slot in which at least one packet arrives the state of the system does not change since subsequent tokens cannot join the token pool. Conditioned on the event that at some slot at least one packet arrives we have that $\mathcal{V} = \mathcal{E}$ if i $(0 \leq i \leq M - 1)$ packets arrive during that slot; $\mathcal{V} = \mathcal{R}_M^M + \mathcal{E}$ if M packets arrive during that slot; and $\mathcal{V} = \mathcal{R}_i^M$ if i $(i \geq M+1)$ packets arrive during that slot.

In summary, the Laplace transform of the interdeparture time $V^*(s) = E\left[e^{-s\mathcal{V}}\right]$ is given by

$$\begin{aligned} (1-p_0)V^*(s) &= \sum_{q=2}^{\infty} p_{M+q}e^{-sD} \\ &\quad + p_{M+1}\left(a_0 E\left[e^{-s(D+\mathcal{E})}\right] + (1-a_0)e^{-sD}\right) \\ &\quad + \sum_{m=0}^{M-2} p_{M-m}\left(\sum_{i=0}^{m} a_i E\left[e^{-s\mathcal{E}}\right] \right. \\ &\qquad + a_{m+1}E\left[e^{-s(\mathcal{R}_{m+1}^{m+1}+\mathcal{E})}\right] \\ &\qquad \left. + \sum_{i=m+2}^{\infty} a_i E\left[e^{-s\mathcal{R}_i^{m+1}}\right]\right) \\ &\quad + \frac{p_1}{1-a_0}\left(\sum_{i=1}^{M-1} a_i E\left[e^{-s\mathcal{E}}\right] \right. \\ &\qquad + a_M E\left[e^{-s(\mathcal{R}_M^M+\mathcal{E})}\right] \\ &\qquad \left. + \sum_{i=M+1}^{\infty} a_i E\left[e^{-s\mathcal{R}_i^M}\right]\right) \end{aligned} \tag{8}$$

where an empty sum vanishes. Note that if the buffer size is finite $(K \geq 1)$, then we just replace ∞ by K in the first sum above. The case of no input buffers at all $(K = 0)$ should be treated separately, since, for instance, in case 3 above (and similarly in case 4) if i $(i \geq m + 2)$ packets arrive during $(t, t+D)$ then $\mathcal{V} = \mathcal{R}_i^{m+1} + \mathcal{E}$ since the tagged token departs upon the $(m+1)$st arrival and the next token departs upon the first arrival after $t + D$. In fact, when $K = 0$ the departure process corresponds to the departure process of a D/M/1 queueing system with a finite buffer of size m (see [7] for an analysis of this system).

After some algebra, using the fact that interarrival times are independent, we obtain from (8) that

$$\lambda D V^*(s) = e^{-sD}\left(1-\sum_{m=0}^{M} p_m\right) - p_{M+1}\frac{a_0 s}{s+\lambda}$$
$$+\sum_{m=0}^{M-2} p_{M-m}\left(\sum_{i=0}^{m}\frac{a_i\lambda}{s+\lambda}+\frac{a_{m+1}\lambda m}{D[s+\lambda]}\right.$$
$$\cdot\int_0^D e^{-sr}\left(1-\frac{r}{D}\right)^m dr + \lambda a_m$$
$$\cdot\int_0^D e^{-sr}\left(1-\frac{r}{D}\right)^m$$
$$\left.\cdot\left(e^{\lambda r}-1\right)dr\right)$$
$$+\frac{p_1}{1-a_0}\left(\sum_{i=1}^{M-1}\frac{a_i\lambda}{s+\lambda}+\frac{a_M\lambda(M-1)}{D[s+\lambda]}\right.$$
$$\cdot\int_0^D e^{-sr}\left(1-\frac{r}{D}\right)^{M-1} dr$$
$$+\lambda a_{M-1}\int_0^D e^{-sr}\left(1-\frac{r}{D}\right)^{M-1}$$
$$\left.+\left(e^{\lambda r}-1\right)dr\right).$$

The expected value of $\mathcal{V}$ is $1/\lambda$ when the buffer is infinite and $1/T$ when the buffer is finite, since the rate at which packets depart the system equals the input rate. The second moment of $\mathcal{V}$ is given by

$$\lambda D E[(\mathcal{V})^2] = \sum_{q=2}^{\infty} p_{M+q} D^2 + p_{M+1}$$
$$\cdot\left(a_0 E\left[(D+\mathcal{E})^2\right] + (1-a_0)D^2\right)$$
$$+\sum_{m=0}^{M-2} p_{M-m}\left(\sum_{i=0}^{m} a_i E\left[(\mathcal{E})^2\right]\right.$$
$$+a_{m+1}E\left[\left(\mathcal{R}_{m+1}^{m+1}+\mathcal{E}\right)^2\right]$$
$$\left.+\sum_{i=m+2}^{\infty} a_i E\left[\left(\mathcal{R}_i^{m+1}\right)^2\right]\right)$$
$$+\frac{p_1}{1-a_0}\left(\sum_{i=1}^{M-1} a_i E\left[(\mathcal{E})^2\right]\right.$$
$$\left.+a_M E\left[(\mathcal{R}_M^M+\mathcal{E})^2\right] + \sum_{i=M+1}^{\infty} a_i E\left[(\mathcal{R}_i^M)^2\right]\right)$$
$$= D^2\left[1-\sum_{m=0}^{M} p_m\right] + \frac{2}{\lambda^2} p_{M+1} a_0(\lambda D+1)$$
$$+\frac{1}{\lambda^2}\sum_{m=0}^{M-2} p_{M-m}\left(\sum_{i=0}^{m}[2a_i - a_{i+2}(m-i)\right.$$
$$\cdot(m-i-1)] + 2a_{m+1}\left[1+\frac{2\lambda D}{(m+2)(m+3)}\right]$$
$$+(\lambda D)^2 - 2(m+1)(\lambda D - a_1) + (m+1)$$
$$\left.\cdot(m+2)(1-a_0-a_1)\right)$$
$$+\frac{1}{\lambda^2}\frac{p_1}{1-a_0}\left(-2a_0+\sum_{i=0}^{M-1}[2a_i - a_{i+2}\right.$$
$$\cdot(M-i-1)(M-i-2)]$$
$$+2a_M\left[1+\frac{2\lambda D}{(M+1)(M+2)}\right]$$
$$+(\lambda D)^2 - 2M(\lambda D - a_1)$$
$$\left.+M(M+1)(1-a_0-a_1)\right)$$

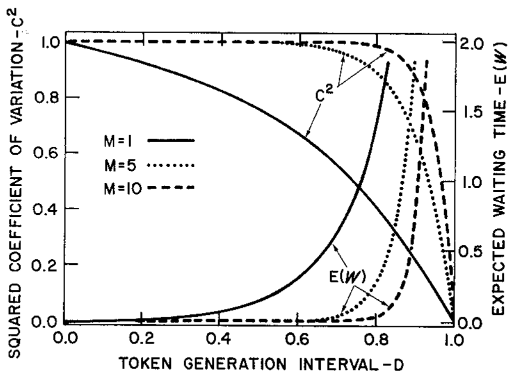

Fig. 2. The effect of D and M on the expected waiting time and the coefficient of variation.

The variance of the interdeparture time is $\mathrm{var}(\mathcal{V}) = E[(\mathcal{V})^2] - (1/\lambda)^2$ and the squared coefficient of variation is $C^2 = \lambda^2\,\mathrm{var}(\mathcal{V})$. The squared coefficient of variation is used as a measure for the smoothness of the output process.

Note that a similar method can be used in order to determine the joint distribution of two successive interdeparture times as we show in the Appendix.

III. Numerical Results and Discussion

In this section we present some numerical results to demonstrate the effects of the token generation time (D), the size of the the token pool (M), and the buffer size (K), on the performance of the input rate regulation schemes.

Consider first the case of an infinite input queue. In this case, the rate at which packets depart the system equals the arrival rate λ, so long as $\lambda < 1/D$ (when $\lambda \geq 1/D$ the departure rate is $1/D$). Yet, both the expected waiting time of a packet and the squared coefficient of variation (and hence the variance) of the interdeparture time of packets are greatly affected by M and D, and there is a clear tradeoff between these two quantities. An example is depicted in Fig. 2.

We observe that as D increases the expected waiting time increases while C^2 decreases. In the extreme case that tokens are generated very rapidly ($D \to 0$), the expected waiting time goes to zero, and $C^2 \to 1$ since each arriving packet finds an available token in the token pool. Clearly, in this case the output process is Poisson. In the other extreme case when

TABLE I
SUCCESSIVE INTERDEPARTURE TIMES

# of Packets q	# of Tokens m	Arrivals in $(t, t+D)$ i	Arrivals in $(t+D, t+2D)$ j	Interdeparture $\mathcal{V}_1$	Interdeparture $\mathcal{V}_2$
$q \geq 3$	$m = 0$	$i \geq 0$	$j \geq 0$	D	D
$q = 2$	$m = 0$	$i \geq 1$	$j \geq 0$	D	D
$q = 2$	$m = 0$	$i = 0$	$j \geq 1$	D	D
$q = 2$	$m = 0$	$i = 0$	$j = 0$	D	$D + \mathcal{E}$
$q = 1$	$m = 0$	$i \geq 2$	$j \geq 0$	D	D
$q = 1$	$m = 0$	$i = 1$	$j \geq 1$	D	D
$q = 1$	$m = 0$	$i = 1$	$j = 0$	D	$D + \mathcal{E}$
$q = 1$	$m = 0$	$i = 0$	$j \geq 2$	$2D - \mathcal{X}_j^1$	$\mathcal{X}_j^1$
$q = 1$	$m = 0$	$i = 0$	$j = 1$	$2D - \mathcal{X}_1^1$	$\mathcal{X}_1^1 + \mathcal{E}$
$q = 1$	$m = 0$	$i = 0$	$j = 0$	$2D + \mathcal{E}$	$\mathcal{E}$
$q = 0$	$0 \leq m \leq M - 1$[a]	$i \geq m + 3$	$j \geq 0$	$\mathcal{R}_i^{m+1}$	D
$q = 0$	$0 \leq m \leq M - 1$[a]	$i = m + 2$	$j \geq 1$	$\mathcal{R}_{m+2}^{m+1}$	D
$q = 0$	$0 \leq m \leq M - 1$[a]	$i = m + 2$	$j = 0$	$\mathcal{R}_{m+2}^{m+1}$	$D + \mathcal{E}$
$q = 0$	$0 \leq m \leq M - 1$[a]	$i = m + 1$	$j \geq 2$	$\mathcal{R}_{m+1}^{m+1} + D - \mathcal{X}_j^1$	$\mathcal{X}_j^1$
$q = 0$	$0 \leq m \leq M - 1$[a]	$i = m + 1$	$j = 1$	$\mathcal{R}_{m+1}^{m+1} + D - \mathcal{X}_1^1$	$\mathcal{X}_1^1 + \mathcal{E}$
$q = 0$	$0 \leq m \leq M - 1$[a]	$i = m + 1$	$j = 0$	$\mathcal{R}_{m+1}^{m+1} + D + \mathcal{E}$	$\mathcal{E}$
$q = 0$	$0 \leq m \leq M - 1$[a]	$0 \leq i \leq m$	$j \geq m - i + 3$	$\mathcal{X}_j^{m-i+1} - \mathcal{X}_j^{m-i+2}$	$\mathcal{X}_j^{m-i+2}$
$q = 0$	$0 \leq m \leq M - 1$[a]	$0 \leq i \leq m$	$j = m - i + 2$	$\mathcal{X}_j^{m-i+1} - \mathcal{X}_j^{m-i+2}$	$\mathcal{X}_j^{m-i+2} + \mathcal{E}$
$q = 0$	$0 \leq m \leq M - 1$[a]	$0 \leq i \leq m$	$j = m - i + 1$	$\mathcal{X}_j^{m-i+1} + \mathcal{E}$	$\mathcal{E}$
$q = 0$	$0 \leq m \leq M - 1$[a]	$0 \leq i \leq m$	$0 \leq j \leq m - i$	$\mathcal{E}$	$\mathcal{E}$

[a]For $m = M - 2$ and $i = 0$ we have to condition on $j \geq 1$. For $m = M - 1$ we have to condition on $i \geq 1$ and when $i = 1$ we have to condition on $j \geq 1$.

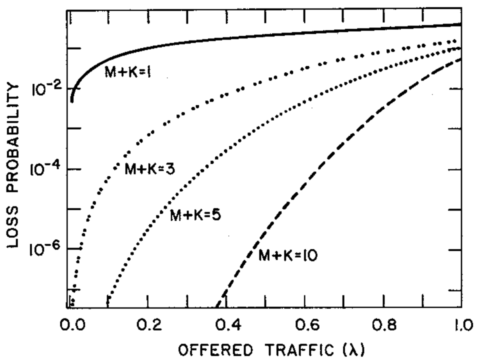

Fig. 3. Loss probability for a finite buffer.

tokens are generated very slowly ($D \to 1/\lambda$), the expected waiting time goes to infinity while $C^2 \to 0$ since each packet waits for a token to be generated and the output process approaches a deterministic process.

The effect of the size of the token pool is also demonstrated in Fig. 2. As M increases, the expected waiting time decreases while C^2 increases. It is interesting to note that even for relatively small values of M ($M \approx 10$), the token generation rate should be close to the arrival rate in order to have significant effect on the system performance. We also observe that in this range the system performance is very sensitive to changes in the token generation rate.

Next consider the case of a finite buffer for arriving packets. Examples of the loss probability behavior as a function of the arrival rate are depicted in Fig. 3. It should be obvious from (3) and (4) that the loss probability depends only on the sum of the number of tokens and the buffer size $M + K$.

APPENDIX

In this Appendix we indicate how to derive the joint distribution of two successive interdeparture times. To that end we use the notation of Section II-D, i.e., $\mathcal{E}$ denotes the interarrival time (exponentially distributed with mean $1/\lambda$), and $\mathcal{R}_i^j$ ($i \geq 1$, $1 \leq j \leq i$) denotes the time between the jth arrival epoch in a slot until the end of the slot, given that exactly i packets arrived in that slot. In addition, we define $\mathcal{X}_i^j$ similarly to $\mathcal{R}_i^j$, except that $\mathcal{X}_i^j$ corresponds to a different slot, so that $\mathcal{X}_i^j$ and $\mathcal{R}_l^k$ are independent for any i, j, l, and k.

As in Section II-D, we consider an arbitrary token that arrives at the token pool at time t and tag it. In order that this token will ever depart the system, it must join the token pool (the probability of this event is $1 - p_0$). Table I contains the various events the tagged token may encounter. For each event we indicate the two successive interdeparture times $\mathcal{V}_1$ (the time between the departure of the tagged token and the departure of the subsequent token) and $\mathcal{V}_2$ (the subsequent interdeparture time). The explanation of each entry in this table is similar to the explanations in Section II-D.

The joint distribution of $\mathcal{X}_i^j$ and $\mathcal{X}_i^{j+1}$ for $0 \leq x_2 \leq x_1 \leq D$, $1 \leq j < i$, $i \geq 2$ is given by

$$f_{\mathcal{X}_i^j, \mathcal{X}_i^{j+1}}(x_1, x_2) = \frac{i!}{(j-1)!\,(i-j-1)!}\left(1 - \frac{x_1}{D}\right)^{j-1} \cdot \left(\frac{x_2}{D}\right)^{i-j-1} \frac{1}{D^2} \tag{9}$$

and

$$E\left[\left(\mathcal{X}_i^j\right)^k\left(\mathcal{X}_i^{j+1}\right)^l\right] = \frac{(i-j)(i-j+1)\cdots(i-j+k+l)}{(i-j+l)(i+1)(i+2)\cdots(i+k+l)} D^{k+l}.$$

From Table I and (9) one can derive an explicit expression for the double Laplace transform of $\mathcal{V}_1$ and $\mathcal{V}_2$ $\left(E\left[e^{-s_1\mathcal{V}_1-s_2\mathcal{V}_2}\right]\right)$. The expression is very long and therefore omitted.

Addendum

The authors would like to comment that since the time the paper was written, many more papers in the area of leaky bucket analysis have been published.

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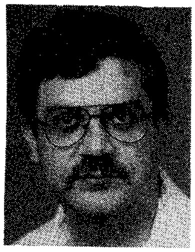
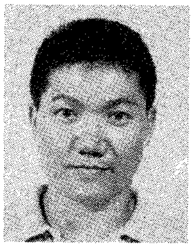


Moshe Sidi (S'77–M'82–SM'87) received the B.Sc., M.Sc., and the D.Sc. degrees from the Technion—Israel Institute of Technology, Haifa, Israel, in 1975, 1979, and 1982, respectively, all in electrical engineering.

From 1975 to 1981 he was a Teaching Assistant and a Teaching Instructor at the Technion in communication and data networks courses. In 1982 he joined the faculty of the Department of Electrical Engineering at the Technion. During the academic year 1983–1984 he was a Postdoctoral Associate at the Department of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology, Cambridge, MA. During 1986–1987 he was a visiting scientist at IBM, Thomas J. Watson Research Center, Yorktown Heights, NY. Currently, Dr. Sidi serves as the Editor for Communication Networks for the IEEE Transactions on Communications, the Associate Editor for Communication Networks and Computer Networks for the IEEE Transactions on Information Theory and as an Editor in the IEEE/ACM Transactions on Networking. His current research interests are in queueing systems and in the area of computer communications networks.

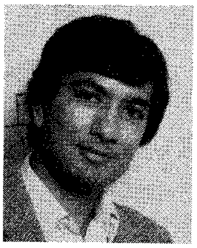


Wen-Zu Liu was born in In-Lin, Taiwan, in 1953. He received the B.S. degree in electronics engineering and the M.S. degree in electrical engineering from Chiao-Tung University, Hsinchu, Taiwan, in 1976 and 1978, respectively.

From 1978 to 1980 he served in the Navy of Taiwan as an electronics Ensign. From 1980 to 1986, he worked at Scientech Corp., Taiwan, in charge of large traffic monitoring system design and Fax modem development. In Fall 1986, he joined the Polytechnic University, Brooklyn, NY. Since September 1987, he has been a research assistant at Polytechnic University and is working on the Ph.D. degree in the area of congestion control in high speed networks. His current interests are in the area of high speed networks, queueing theory, and signal processing.

Israel Cidon (M'85–SM'91) received the B.Sc. (summa cum laude) and the D.Sc. degrees from the Technion—Israel Institute of Technology, Haifa, Israel, in 1980 and 1984, respectively, both in electrical engineering.

From 1980 to 1984, he was a Teaching Assistant and a Teaching Instructor at the Technion. From 1984 to 1985 he was on the faculty with the Department of Electrical Engineering at the Technion. In 1985 he joined the IBM Thomas J. Watson Research Center, Yorktown Heights, NY, where he was a Research Staff Member and manager of the Network Architectures and Algorithms group. Since 1990 he has been with the Department of Electrical Engineering at the Technion. His research interests are in high speed communication networks, distributed algorithms, and packet radio networks.



Inder Gopal received the B.A. degree in engineering science from Oxford University, Oxford, England, in 1977 and the M.S. and Ph.D. degrees in Electrical Engineering from Columbia University, New York, NY, in 1978 and 1982, respectively.

Since 1982, he has been a Research Staff Member at the IBM Thomas J. Watson Research Center, Yorktown Heights, NY. Currently he is a manager of the Broadband Networking Department. He is involved in the NSF/DARPA sponsored AURORA Gigabit testbed and in several other research projects in the area of high speed networking. His other research interests are in distributed algorithms and communication protocols. He has published extensively in these areas.

Dr. Gopal is currently an Area Editor for the IEEE Transactions on Communications. He has previously served as guest editor for *Algorithmica*, guest editor for IEEE Journal on Selected Areas in Communications, and technical editor for *IEEE Communications Magazine.* He is a Fellow of the IEEE.