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Abstract

We consider networking systems with messages that consist of blocks of *consecutive* (fixed length) cells. A message can be generated at a single instant of time as a batch or it can be dispersed over time. In this paper we focus on the model of dispersed generation processes which naturally arises in packet switched networks such as ATM. The main difficulty in the analysis of message related quantities is due to the correlation between the system states observed by different cells of the same message.

The following important quantities are analyzed in this paper: (1) The message delay process, defined as the time elapsing between the arrival epoch of the first cell of the message to the system until after the transmission of the last cell of that message is completed. In many systems the message delay, and not the individual cell delay, is the measure of interest for the network designer. (2) The maximum delay of a cell in a message. (3) Number of cells in a message whose delays exceed a pre-specified time threshold. The latter two quantities are important for the proper design of playback algorithms and time-out mechanisms for retransmissions.

We analyze the probability distribution of these quantities. In particular, we present a new analytical approach that yields efficient recursions for the computation of the probability distribution of each quantity. Numerical examples are provided to compare this distribution with the distribution obtained by using an independence assumption on the cell delays. These examples show that the correlation between cell delays of the same message has a strong effect on each of these quantities. A simulation of an 8-node tandem queueing model of a virtual connection is provided to show that the general phenomena observed for the single node system hold for a network environment as well.

Keywords: Discrete-time queues; Dispersed messages; Delay; Jitter; ATM

1. Introduction

We consider networking systems with messages that consist of blocks of consecutive (fixed length) cells. In these systems the integral transmitted quantity is a cell and the time axis is slotted with slot duration equal

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to the transmission time of a cell. (We shall use the terms cell and packet alternately throughout the paper.) The message corresponds to a higher layer protocol data unit. In general, one can distinguish between two types of message generation processes. The message can be generated at a single instant of time as a batch, i.e., all the cells that compose the message arrive to the system at the same time slot (which corresponds to the well-known batch arrival model), or it can be dispersed over time, i.e., the cells that compose the message arrive to the system at different time slots. In this paper we focus on the new model of dispersed generation processes which naturally arises in packet switched networks such as ATM [5]. Individual cell delay distributions are usually insufficient for proper understanding of the system behavior. Therefore, there is a need to analyze message-related quantities. The main difficulty in the analysis of such quantities is due to the correlation between the system states observed by different cells of the same message. For instance, the delays of two consecutive cells are strongly correlated, i.e., the delay of the second cell conditioned on the event that the first cell delay is large (small) is larger (smaller) than the delay of an arbitrary cell.

In this paper we analyze the following message-related quantities. The *message delay process*, defined as the time elapsing between the arrival epoch of the first cell of the message to the system until after the transmission of the last cell of that message is completed. In many systems the message delay, and not the packet delay, is the measure of interest for the network designer. This is due to the fact that packets are data units which are only meaningful at lower layers and are created because of the network data unit size limitations. The ATM [5], TCP/IP [11] and TDMA based systems [23] are examples in which the application message is segmented into bounded size packets (cells) which are then transmitted through the network. At the receiving end, the transport protocol (or the adaptation layer) reassembles these cells back into a message before the delivery to higher layers. In some applications message delay is not the result of segmentation at the network layer but of the nature of data partitioning in the storage. A file can be composed of multiple records which are stored at different locations in the disk. These records are read individually and may be transmitted as separate packets. However, the entire file transfer delay is the measure of interest for the overall file transfer application. We provide numerical examples to show that the variance of the message delay may be over-estimated by the independence assumption, i.e., the assumption that the delays of cells are independent from cell to cell, for a wide range of message sizes. Most prior analyses of the message delay were associated with batch arrival processes [17,23], i.e., each batch corresponds to a message. In this case, the message delay coincides with the delay of the last packet of the message (batch). This fact facilitates the analysis of the message delay distribution. However, in packet switched networks, packets which belong to the same message may arrive at different instants of times (be dispersed). The message delay process for the dispersed generation model was first analyzed in [13] for continuous time systems such as M/M/1 and M/G/1 systems. Closed form expressions were obtained in [4].

The second message related quantity analyzed in this paper is the *maximum delay of a cell in a message*. One of the central problems in the support of real-time applications (voice, video) within a packet switching network is the existence of delay jitter. Similarly to loss, the jitter is an inherent problem of packet switched networks. The delay time of a packet in these networks is composed of a fixed component of propagation delay and a variable component caused by the waiting time in the buffers of the network. The jitter problem is solved by the playback algorithm at the destination node by delaying the first cell of a message for a time which represents the worst-case delay (or variable delay). This delay depends on the maximum delay of a packet in a message (or, a video frame), see [19], or on the distribution of the message delay jitter defined as the difference between the maximum and the minimum delay of a packet in a message, see [9]. Another design problem in which the distribution of the maximum delay of a packet in a message is important appears in the design of time-out mechanisms for data retransmissions. Time-out mechanisms along with acknowledgment packets are used in data link protocols and end-to-end protocols to ensure reliable communication between source and destination nodes (see [6]). The time-out is equal to the round trip delay between the source and the destination and is composed of the fixed part of the propagation and processing delays and the variable part of queueing delay which is equal to the maximum delay of a packet in a message. The design of a proper time-out value affects the performance of the system, for example, the number or rate of false retransmissions. In addition, in a window flow control mechanism it is desirable that the window size be less than the time-out in order to achieve high throughput. We prove that the independence assumption yields an upper bound (in the sense of stochastically larger) on the maximum delay of a cell in a message, and show that it converges to $\alpha^{-1}\,\mathrm{Ln}\,n$ (in probability) as n becomes very large, where n is the number of cells in a message, and α is the smallest positive pole of the probability generating function of the cell delay time.

The third message related quantity analyzed in this paper is the *number of cells in a message whose delays exceed a pre-specified threshold*. This quantity has a practical importance for the design of playback algorithms and it can also be used to estimate the number of false retransmissions in systems that employ time-out mechanisms. In real-time applications, a packet that exceeds some delay limit is obsolete and hence lost. The number of lost cells in a message is an important measure for data and real-time applications as shown in [12]. Queueing systems in which the waiting time (or delay) of a packet is limited to a pre-specified delay limit were studied in the past, see, e.g., [10,14,16]. In these systems, packets whose waiting times (or delays) exceed some pre-specified delay limit are lost. For these systems the packet loss probability and the delay time of a packet which completed service were analyzed. Overload control problems with FIFO-timeout were analyzed in [14]. Here, an infinite buffer is assumed and all packets are accepted to the system. If after time period T the packet is still in the system it is dropped (actually, the decision to drop a packet can be taken at the arrival time of the packet). In [14], an M/M/1 system with a FIFO-timeout dropping policy was analyzed using level crossing arguments. In [16], an M/M/m system with waiting times (or delay times) limited to fixed or random times (exponentially distributed) was analyzed using birth–death processes. In [10], an M/G/1 system with waiting times (or delay times) limited to fixed or random times (exponentially distributed) was analyzed using generating functions approach. In contrast to the above studies which consider a single packet, we investigate the effect of the correlation between the delays of a block of consecutive packets on the performance of the system. We provide numerical examples to compare the probability distribution of the number of cells whose delays exceed a pre-specified threshold with the same probability distribution obtained by using the independence assumption on the cell delays. We show that the independence assumption may yield overly optimistic results, and that correlation exists not only for the cell loss process (assuming that a cell is lost if its delay exceeds the threshold) but also for the no-loss process.

In this paper we analyze the probability distribution of the above three quantities. In particular, we present a new analytical approach that yields efficient recursions for the computation of the probability distribution of each quantity. Numerical examples are provided to compare this distribution with the distribution obtained by using an independence assumption on the cell delays. These examples show that the correlation between cell delays of the same message has a strong effect on each of these quantities in all cases explored. A simulation of an 8-node tandem queueing model of a virtual connection is provided to show that the general phenomena observed for the single node system hold for a network environment as well.

The paper is structured as follows. In Section 2 we present the queueing model and the notations used throughout the paper. Here we use a discrete-time, single source, single server with an infinite buffer system.

Every n consecutive cells belong to one message, i.e., the cell stream originated at the source is assumed to obey a message structure with every consecutive n cells belonging to one message. This assumption is motivated by the fact that these cells are flow controlled at the entrance to the network which tends to smooth the arrival of cells to the network (for example the Leaky–Bucket mechanism adopted by the ATM Forum [5]). Also, the message structure originally defined at the source is disturbed by the multiplexing and interaction with other sources inside the network. In Section 3 we analyze the message delay. In Section 4 we analyze the maximum delay of a cell in a message. We prove that the independence assumption yields an upper bound on this quantity and obtain an asymptotic result for the upper bound. We introduce the *message delay jitter* and describe a procedure to compute its distribution. In Section 5 we analyze the number of cells in a message whose delays exceed a pre-specified threshold. We introduce an important measure for real-time sessions, the *cell overload period*, and compute its distribution. In Section 6 we provide numerical examples for each of the above quantities. In Section 7 we provide a simulation of an 8-node tandem queueing model of a virtual connection for each of the above quantities.

It is interesting to note that the analysis of the above measures (delay, jitter and threshold crossing) can be extended to the case of an ON–OFF source (with geometrically distributed ON and OFF periods), where each ON period corresponds to a single message. This can be done using similar techniques to those used in [13]. Similarly, one can use these techniques to analyze the message delay process in a system with multiple sources. However, the analysis of the jitter and the threshold crossing processes for multiple sources is significantly more complicated and is still an open problem.

2. Model and notations

The time axis is slotted and each slot corresponds to the transmission time of a single cell. Cells arrive randomly to the system from a single source. The arrival process is assumed to be independent and identically distributed from slot to slot. The cells are stored in a buffer that can accommodate an infinite number of cells, and are transmitted according to the First-Come-First-Serve (FCFS) policy (assuming that no more than one cell may arrive instantaneously to the system).

Assume that departures take place at the beginning of slots, and arrivals within slots. Let $q_{\rm t}$ be the queue length at the end of the tth slot, $\Pr\{q=i\} \stackrel{\Delta}{=} \lim_{t\to\infty} \Pr\{q_{\rm t}=i\}$ and $Q(z) \stackrel{\Delta}{=} E[z^q]$. Let b be a generic random variable (r.v.) of the number of cells that arrive within a slot, and define $b_i \stackrel{\Delta}{=} \Pr[b=i]$, $i \ge 0$, $\overline{b} \stackrel{\Delta}{=} E[b]$ and $B(z) \stackrel{\Delta}{=} E[z^b]$. Assume that $\overline{b} < 1$ so that the system is ergodic. Consider an arbitrary cell arriving to the system and denote by $b^{\rm b}$ and $b^{\rm a}$ the number of cells that arrive before and after (including) this cell, respectively, within its arrival slot. Note that, $b^{\rm b}$ and $b^{\rm a}$ are the backward and the residual recurrence times in the (discrete time) renewal process whose inter-event time distribution is given by the r.v. b. Thus, the distributions of $b^{\rm b}$ and $b^{\rm a}$ are given by

$$b_k^{\rm b} \stackrel{\Delta}{=} \Pr\{b^{\rm b}=k\} = \frac{\Pr\{b>k\}}{E[b]}, \quad k=0,1,\ldots,$$

$$b_k^{\rm a} \stackrel{\Delta}{=} \Pr\{b^{\rm a}=k\} = \frac{\Pr\{b\ge k\}}{E[b]}, \quad k=1,2,\ldots$$

and the joint distribution of $b^{\rm b}$ and $b^{\rm a}$ is given by

$$\Pr\{b^{\mathrm{b}} = m, b^{\mathrm{a}} = k\} = \frac{b_{m+k}}{E[b]}, \quad m \geq 0, \; k \geq 1. \tag{1}$$

The evolution equation of the queue length is given by

$$q_{t+1} = (q_t - 1)^+ + b, \quad t \geq 0, \tag{2}$$

from which we have (using standard generating function techniques [20])

$$\mathcal{Q}(z) = \frac{(1-\bar{b})(1-z^{-1})\mathcal{B}(z)}{1 - z^{-1}\mathcal{B}(z)}, \quad |z| \leq 1. \tag{3}$$

We shall consider the delay time of a cell to be 1 if the cell is transmitted in the slot immediately following its arrival slot. Using a distribution form of Little's law [18], the probability distribution of the delay of a cell, d, is given by $\Pr[d = i] = \Pr[q = i]/\Pr[q > 0]$, $i \geq 1$, from which we have

$$\mathcal{D}(z) \triangleq E[z^d] = 1 + \frac{\mathcal{Q}(z) - 1}{\bar{b}}. \tag{4}$$

In the following we assume that cells are grouped into fixed size messages, namely, every n consecutive cells form a message.

3. Message delay

Here we compute the probability generating function (PGF) of the message delay – the number of slots from the arrival slot of the first cell of the message until the last cell of that message is transmitted. Let d_n be the r.v. of the message delay for a message of size n, and define $\mathcal{D}_n(z) \triangleq E[z^{d_n}]$. Let a be the r.v. of the number of cells in the system at the departure epoch of the first cell of the message from the system. Let $d_{k,m}$, $k \geq 0$, $m \geq 1$, be the r.v. of the number of slots from the beginning of a slot in which there are k cells in the system until the next m cells leave the system (transmitted), and define $\mathcal{D}_{k,m}(z) \triangleq E[z^{d_{k,m}}]$.

Since the first cell of a message is arbitrary, we have

$$\begin{aligned} \mathcal{D}_1(z) &= \mathcal{D}(z), \\ \mathcal{D}_n(z) &= \sum_{i=0}^{\infty} \Pr[q = i] \sum_{m=0}^{\infty} b_m^{\mathrm{b}} \sum_{k=0}^{\infty} \Pr[a = k \mid q = i, \; b^{\mathrm{b}} = m] E[z^{(i-1)^+ + m + 1 + d_{k,n-1}}] \\ &= \sum_{k=0}^{\infty} f_k(z) D_{k,n-1}(z), \quad n \geq 2, \end{aligned} \tag{5}$$

where $f_k(z) \triangleq z \sum_{i=0}^{\infty} \Pr[q = i] z^{(i-1)^+} \sum_{m=0}^{\infty} b_m^{\mathrm{b}} \Pr[a = k \mid q = i, \; b^{\mathrm{b}} = m] z^m$ for $|z| \leq 1$, and $x^+ \triangleq \max(x, 0)$. In (5) we first conditioned on the number of cells in the system at the end of the slot preceding the arrival slot of the first cell of the message and on the number of cells that arrive before this cell in its arrival slot. Then we conditioned on the number of cells in the system at the departure epoch of the first cell of the message from the system which clearly departs after $(i-1)^+ + m + 1$ slots, and used the fact that the r.v. $d_{k,n-1}$ depends only on arrivals after that departure epoch.

The functions $f_k(z)$, $k \geq 0$, are computed as follows. Define the power series in the complex variable w, $F(z, w) \triangleq \sum_{k=0}^{\infty} f_k(z) w^k$, $|w| < 1$. Since $|f_k(z)| \leq 1$, $k \geq 0$, $|z| \leq 1$, then using Abel's theorem [2] it follows that the power series $F(z, w)$ for every z, $|z| \le 1$, converges absolutely and is an analytic function in the complex variable w inside the unit disk $|w| < 1$. Let $\mathcal{B}_m^{\mathrm{a}}(z)$, $m \ge 0$, be the PGF of the conditional r.v. b^{a} given that $\{b^{\mathrm{b}} = m\}$. Then $\mathcal{B}_m^{\mathrm{a}}(z) = \sum_{k=1}^{\infty} \Pr[b^{\mathrm{a}} = k | b^{\mathrm{b}} = m] z^k = (z^{-m} / \Pr[b > m]) \sum_{i=m+1}^{\infty} b_i z^i$. Note that, the probability distribution of the conditional r.v. a given that the delay of the first cell of the message is $(i-1)^+ + m + 1$ equals the $(i-1)^+ + m + 1$ fold convolution of the distribution function of the conditional r.v. $b^{\mathrm{a}} - 1$ given $\{b^{\mathrm{b}} = m\}$, and $(i-1)^+ + m$ distribution functions b_k. Therefore, $F(z, w) = z \sum_{i=0}^{\infty} \Pr[q = i][z\mathcal{B}(w)]^{(i-1)^+} \sum_{m=0}^{\infty} b_m^{\mathrm{b}} w^{-1} \mathcal{B}_m^{\mathrm{a}}(w)[z\mathcal{B}(w)]^m$. Substituting for $\mathcal{B}_m^{\mathrm{a}}(w)$ in $F(z, w)$ and using simple algebra, we have

$$F(z, w) = \frac{(1-\bar{b}) z [z\mathcal{B}(w) - 1][\mathcal{B}(w) - \mathcal{B}(z\mathcal{B}(w))]}{\bar{b}[w - z\mathcal{B}(w)][z\mathcal{B}(w) - \mathcal{B}(z\mathcal{B}(w))]}.$$

The functions $f_k(z)$ are obtained by taking the inverse transform of $F(z, w)$ in the complex variable w. If $\mathcal{B}(z)$ is a rational function of z then $F(z, w)$ is a rational function of w and the inverse transform in the variable w can be easily obtained (see the numerical example in Section 6).

To complete the computation we need to compute the PGFs $\mathcal{D}_{k,m}(z)$, $k \ge 0$, $m \ge 1$. In the following we introduce a recursive procedure for the computation of those PGFs. The recursions are obtained by conditioning on the number of cells in the system at the beginning of consecutive slots:

$$\begin{aligned} \mathcal{D}_{0,m}(z) &= \frac{z}{1 - b_0 z}\left((1-b_0)z^{m-1} + \sum_{i=1}^{m-1} b_i [\mathcal{D}_{i-1,m-1}(z) - z^{m-1}]\right), \\ \mathcal{D}_{k,m}(z) &= z\left(z^{m-1} + \sum_{i=0}^{m-k-1} b_i [\mathcal{D}_{i+k-1,m-1}(z) - z^{m-1}]\right), \quad 1 \le k \le m-1, \\ \mathcal{D}_{k,m}(z) &= z^m, \quad k \ge m. \end{aligned} \tag{6}$$

The recursions in (6) are computed for each $m = 1, 2, \ldots, n-1$ (in increasing order) and $0 \le k \le m-1$. The computation complexity is of the order of $\mathrm{O}(n^3)$.

From (5) and (6) for $k \ge m$ we have

$$\mathcal{D}_n(z) = \sum_{k=0}^{n-2} f_k(z)(D_{k,n-1}(z) - z^{n-1}) + z^{n-1}\mathcal{D}(z), \quad n \ge 2, \tag{7}$$

where in (7) we used the fact that $\sum_{k=0}^{\infty} f_k(z) = F(z, 1) = \mathcal{D}(z)$.

From (6) a set of recursive equations for any moment of the r.v.s. $\mathrm{d}_{k,m}$, $k \ge 0$, for any $m \ge 1$ can be obtained by taking the appropriate derivatives at $z = 1$. Then any moment of the message delay can be obtained from (7). The jth derivative of the function $f_k(z)$ can be obtained directly or by taking the inverse transform in the complex variable w of $(\mathrm{d}^j F(z, w) / \mathrm{d}z^j)|_{z=1}$.

3.1. Independence approximation

We note that the message delay is composed of two components (see [13]). The first is the number of slots elapsing between the arrival slot of the first cell of the message to the system until the arrival slot of the last cell of that message to the system (the message inter-arrival time). The second is the time delay of an arbitrary cell (stands for the delay of the last cell of the message) in the system and is equal to d. Let I_n

be the r.v. of the first component and denote by $\mathcal{I}_n(z)$ its PGF. Let $\hat{\mathcal{I}}_n$ be the r.v. of the number of slots from the beginning of a slot until the arrival slot of the last cell in the next n cells that arrive to the system, and denote by $\hat{\mathcal{I}}_n(z)$ its PGF. Since the first cell of a message is arbitrary, then by conditioning on the number of cells that arrive to the system after (including) the first cell within its arrival slot we have

$$\mathcal{I}_n(z) = \Pr[b^{\mathrm{a}} \geq n] + \sum_{j=1}^{n-1} b_j^{\mathrm{a}} \hat{\mathcal{I}}_{n-j}(z), \tag{8}$$

and for $\hat{\mathcal{I}}_n(z)$ we have the following recursion:

$$\hat{\mathcal{I}}_1(z) = \frac{(1-b_0)z}{1-b_0 z}, \qquad \hat{\mathcal{I}}_n(z) = \Pr[b \geq n]z + z\sum_{j=0}^{n-1} b_j \hat{\mathcal{I}}_{n-j}(z), \quad n \geq 2. \tag{9}$$

From (8) and (9) any moment of the message inter-arrival time can be simply computed.

A simple way to approximate the message delay is to assume that these two components are independent r.v.s. In Section 6 we show the relative error of the variance of the message delay under this approximation. Note that, for the average message delay, we have $E[d_n] = E[I_n] + E[d]$.

4. Maximum cell delay

We are interested in the probability distribution of the maximum delay of a cell in a message. Let $y^{\max}(n)$ be the r.v. of the maximum delay of a cell in a message of size n. Let $P_{m,n}(y)$, $m \geq 0$, $n \geq 1$, $y \geq 1$, be the conditional probability that the maximum delay of a cell in the next n cells that arrive to the system after the beginning of a slot is less or equal to y, given that m cells are present in the system at the beginning of that slot. Since the first cell of a message is arbitrary, we have

$$\begin{aligned}
&\Pr[y^{\max}(n) \leq y] \\
&= \sum_{i=0}^{y} \Pr[q=i] \sum_{k=0}^{y-(i-1)^+-1} \Bigg[\sum_{j=1}^{\min(y-(i-1)^+-k,n-1)} \Pr[b^{\mathrm{b}}=k, b^{\mathrm{a}}=j] \\
&\quad \times P_{(i-1)^++k+j-1,n-j}(y) + \Pr[b^{\mathrm{b}}=k, b^{\mathrm{a}} \geq n] 1\{(i-1)^+ + k + n \leq y\} \Bigg] \\
&= \frac{1}{\bar{b}} \sum_{i=0}^{y} \Pr[q=i] \Bigg[\sum_{k=0}^{y-(i-1)^+-1} \sum_{j=1}^{\min(y-(i-1)^+-k,n-1)} b_{k+j} P_{(i-1)^++k+j-1,n-j}(y) \\
&\quad + \sum_{k=0}^{y-(i-1)^+-n} \Pr[b \geq k+n] \Bigg],
\end{aligned} \tag{10}$$

where an empty sum vanishes. The explanation of the first equality in (10) is as follows. The probability that the r.v. $y^{\max}(n)$ does not exceed y equals the probability that the delay of each of the n cells of the message does not exceed y. First, we conditioned on the number of cells in the system at the end of the slot preceding the arrival slot of the first cell of the message $\{q=i\}$. Clearly, this number cannot exceed y in order that the delay of the first cell of the message be less or equal to y. Then we conditioned on the number of cells that arrive before and after (including) the first cell of the message within its arrival slot, $\{b^{\rm b}=k,\ b^{\rm a}=j\}$. The delay of the first cell of the message equals $(i-1)^{+}+k+1$ and must be less or equal to y. If $j<n$, then the largest delay of a cell in these j cells (which belong to the message) equals $(i-1)^{+}+k+j$ which must be less or equal to y. In order that $y^{\max}(n)$ be less or equal to y the maximum delay of the remaining $n-j$ cells (that arrive after the beginning of the next slot in which there are $(i-1)^{+}+k+j-1$ cells in the system) that belong to the message must be less or equal to y. If $j\ge n$, then the largest delay of a cell in the message equals $(i-1)^{+}+k+n$ which must be less or equal to y. The second equality in (10) follows from (1).

To complete the computation we need to compute the probabilities $P_{m,l}(y),\ 0\le m\le y,\ 1\le l\le n-1$. To that end we introduce recurrence relations. Using a recursion at the slot boundaries of consecutive slots, we have (for $0\le m\le y,\ 1\le l\le n-1$)

$$
\begin{aligned}
&P_{m,l}(y)=1, \quad m+l\le y,\\
&P_{m,l}(y)=b_0P_{(m-1)^{+},l}(y)+\sum_{i=1}^{y-m} b_iP_{i+m-1,l-i}(y), \quad m+l>y.
\end{aligned} \tag{11}
$$

The computation complexity of $P_{m,l}(y)$ for $0\le m\le y,\ 1\le l\le n-1$ is of the order of $O(n^3)$. These probabilities are then used in (10) to compute the probability distribution of the r.v. $y^{\max}(n)$ with computation complexity of the order of $O(y^2n)$. In the following section we provide upper and lower bounds on the maximum cell delay. The asymptotic behavior is investigated in Appendix B. It provides a simple formula for the distribution of the maximum cell delay for large message size n.

4.1. Bounds

Here we use the notion of associated r.v.s. (see Appendix A) to obtain an upper bound on the maximum cell delay (a similar use of associated r.v.s. can be found in [7,8,22]). Let $x_i,\ 1\le i\le n$, be the r.v. of the delay of the ith cell of a message of size n. Using an independence assumption of the cell delays, i.e., the assumption that the r.v.s. $x_i,\ 1\le i\le n$, are independent and equal in distribution to the r.v. d, the probability distribution of the r.v. of the maximum delay of a cell in a message of size n, $y^{\max}_{\rm ind}(n)$, becomes $\Pr[y^{\max}_{\rm ind}(n)\le y]=(\Pr[d\le y])^n,\ y\ge 1$. In fact, the independence assumption yields an upper bound on the message delay as is proved in the following theorem.

Theorem 1.

$$y^{\max}_{\rm ind}(n)\ge_{\rm st} y^{\max}(n),\quad n\ge 1,$$

where $\ge_{\rm st}$ stands for stochastically larger.

Proof. We shall prove that the r.v.s. $x_i,\ 1\le i\le n$, are associated r.v.s. (see Appendix A) from which Theorem 1 follows directly. Denote all cells that arrive during a slot by a batch. In any slot a batch does not arrive with probability b_0 and arrives with probability $1-b_0$, i.e., according to a Bernoulli arrival process with parameter $1-b_0$. Consider an arbitrary batch arriving to the system and denote its arrival slot by slot number 1. Denote arrival slots of consecutive batches by $i,\ i=2,3,\ldots$ Let $\hat{b}_i,\ i\ge 1$, be the r.v. of the batch size of batch number i. Let $J_i,\ i\ge 1$, be the r.v. of the ith batch inter-arrival time, i.e., the number of slots elapsing between the arrival epochs of the $(i-1)$st and the ith batches where $J_1 \triangleq 0$. Let $\hat{q}_i$, $i \geq 1$, be the r.v. of the number of cells at the beginning of slot number i. We have

$$\hat{q}_i = (\hat{q}_{i-1} + \hat{b}_{i-1} - J_i)^+, \quad i \geq 2.$$

Since $\hat{b}_i$, J_i, $i \geq 1$, are independent r.v.s. and since $\hat{q}_i$ is a nondecreasing function of $\hat{q}_{i-1}$, $\hat{b}_{i-1}$ and $-J_i$, we have by using properties (P2), (P3) and (P4) of associated r.v.s. (see Appendix A) that the r.v.s. $\hat{q}_i$, $1 \leq i \leq N$, for any $N \geq 1$ are associated r.v.s.

Consider the kth cell arriving in batch number i. Its delay equals $\hat{q}_i + k$ which is an increasing function of $\hat{q}_i$. Then, using property (P4) of associated r.v.s., the delays of all cells arriving in batches $1, \ldots, 2n$, are associated r.v.s. Since each batch contains at least one cell, a message of size n must be contained in the cells that arrive in batches $1, \ldots, 2n$, and using property (P1) of associated r.v.s., the delays of all cells belonging to this message are associated r.v.s. Since the first batch was chosen arbitrarily, this message is an arbitrary message.

Now using the property in Eq. (A.2) of Appendix A and the definition of stochastic dominance, Theorem 1 follows. □

A simple lower bound of the maximum delay of a cell in a message of size n is given by $y^{\max}(n) \geq_{\text{st}} d$. Note also that $y^{\max}(n) \geq_{\text{st}} y^{\max}(k)$ for any $1 \leq k \leq n$. In Section 6 we give a numerical example for the distribution of the r.v. $y^{\max}(n)$ and its upper and lower bounds.

4.2. *Message delay jitter*

We conclude this section by introducing an additional important measure for the design of playback algorithms (see [9]) – the maximum difference between cell delays in a message, referred to as the *message delay jitter*. Let $y^{\min}(n)$ be the r.v. of the minimum delay of a cell in a message of size n. Then the message delay jitter is defined by $y^{\max}(n) - y^{\min}(n)$, and its probability distribution can be obtained as follows. The probability $\Pr[y^{\max}(n) \leq y, y^{\min}(n) \geq x]$, $y \geq x \geq 1$, can be obtained by a similar recursion to the one introduced in this section. The probability distribution of the r.v.s. $y^{\max}(n)$, $y^{\min}(n)$ is obtained from $\Pr[y^{\max}(n) \leq y,\ y^{\min}(n) \leq x] = \Pr[y^{\max}(n) \leq y] - \Pr[y^{\max}(n) \leq y,\ y^{\min}(n) \geq x]$ where the probability distribution of the r.v. $y^{\max}(n)$ was obtained in this section. Then the probabilities $\Pr[y^{\max}(n) = y,\ y^{\min}(n) = x]$ are obtained directly. Finally, $\Pr[y^{\max}(n) - y^{\min}(n) = y] = \sum_{x=1}^{\infty} \Pr[y^{\max}(n) = y + x,\ y^{\min}(n) = x]$. Alternatively, the probability distribution of the message delay jitter can be obtained directly by a recursion similar to the one introduced in the next section.

5. Threshold constrained delay process

We are now interested in the probability distribution of the number of cells in a message whose delays exceed a pre-specified threshold. Let $P_n(y,k)$, $y, n \geq 1$, $0 \leq k \leq n$, be the probability of k cells in a message of size n whose delays exceed y. Note that for $k = 0$ this degenerates to the probability distribution of the cell maximum delay analyzed in Section 4. Let $P_{m,n}(y,k)$, $y, n \geq 1$, $0 \leq k \leq n$, $m \geq 0$, be the conditional probability of k cells, in the next n cells that arrive to the system after the beginning of a slot, whose delays exceed y, given that m cells are present in the system at the beginning of that slot. Define

$P_{m,n}(y,k) \triangleq 0,\ m \geq 0,\ k \geq 0,\ n \leq 0,\ P_{m,n}(y,k) \triangleq 0,\ m \geq 0,\ n \geq 1,\ k < 0$. Since the first cell of a message is arbitrary, we have

$$
\begin{aligned}
P_n(y,k) = & \sum_{i=0}^{\infty} \Pr[q=i] \sum_{m=0}^{\infty} \Bigg[\sum_{j=1}^{y-(i-1)^+-m} \Pr[b^{\mathrm{b}}=m, b^{\mathrm{a}}=j] P_{(i-1)^+ + m + j - 1, n-j}(y,k) \\
& + \sum_{j=(y-(i-1)^+-m)^+ + 1}^{n-1} \Pr[b^{\mathrm{b}}=m, b^{\mathrm{a}}=j] P_{(i-1)^+ + m + j - 1, n-j} \\
& \qquad \times (y, k - [j - (y - (i-1)^+ - m)^+]) \\
& + \sum_{j=n}^{\infty} \Pr[b^{\mathrm{b}}=m, b^{\mathrm{a}}=j] \big(1\{k=0\} 1\{(i-1)^+ + m + n \leq y\} \\
& + 1\{1 \leq k \leq n\} 1\{(i-1)^+ + m + n - k = y\} \big) \Bigg] \\
= & \frac{1}{\bar{b}} \sum_{i=0}^{\infty} \Pr[q=i] \sum_{m=0}^{\infty} \Bigg[\sum_{j=1}^{y-(i-1)^+-m} b_{m+j} P_{(i-1)^+ + m + j - 1, n-j}(y,k) \\
& + \sum_{j=(y-(i-1)^+-m)^+ + 1}^{n-1} b_{m+j} P_{(i-1)^+ + m + j - 1, n-j}(y, k - j + (y - (i-1)^+ - m)^+) \Bigg] \\
& + 1\{k=0\} \frac{1}{\bar{b}} \sum_{i=0}^{y+1-n} \Pr[q=i] \sum_{m=n}^{y-(i-1)^+} \Pr[b \geq m] \\
& + 1\{1 \leq k \leq n\} \frac{1}{\bar{b}} \sum_{i=0}^{y+k-n+1\{i>0\}} \Pr[q=i] \Pr[b \geq y + k - (i-1)^+]
\end{aligned}
\tag{12}
$$

The explanation of the first equality in (12) is as follows. First, we condition on the number of cells in the system at the end of the slot preceding the arrival slot of the first cell of the message, $\{q=i\}$. Then we condition on the number of cells that arrives before and after (including) the first cell of the message within its arrival slot, $\{b^{\mathrm{b}}=m,\ b^{\mathrm{a}}=j\}$. The delay of the jth cell of the message that arrives in the first slot equals $(i-1)^+ + m + j$. The first sum corresponds to the case where the delays of all cells of the message that arrive at the first slot, j, do not exceed y. This happens if $j \leq y - (i-1)^+ - m$, and in order that the delays of k cells out of the message of size n exceed y, the delays of k cells out of the next $n-j$ cells that arrive to the system after the beginning of the next slot at which there are $(i-1)^+ + m + j - 1$ cells, must exceed y. If $(y-(i-1)^+ - m)^+ + 1 \leq j \leq n-1$, then the delays of $j - (y - (i-1)^+ - m)^+$ cells of the message exceed y and the delays of $k - [j - (y - (i-1)^+ - m)^+]$ cells out of the next $n-j$ cells that arrive to the system after the beginning of the next slot at which there are $(i-1)^+ + m + j - 1$ cells, must exceed y. Note that when $(i-1)^+ + m \geq y$, the delays of all the cells of the message that arrive at the first slot, j, exceed y, and hence we used the $(\cdot)^+$ operator for the index j of the second sum. The last sum accounts for the case where all the cells of the message arrived in the first slot. Then, in order that the delays of all the cells of the message be less or equal to y, the delay of the last cell of the message $(i-1)^+ + m + n$ must not exceed y. For the case where the delays of $k > 0$ cells exceed y, we must have that the delay of the $n-k$th cell of the message equals y, so that the delays of the remaining k cells exceed y. The second equality in (12) follows from (1).

To complete the computation we need to compute the probabilities $P_{m,l}(y,k)$, $y \geq 1$, $1 \leq l \leq n-1$, $0 \leq k \leq l$, $m \geq 0$. To that end we introduce recurrence relations. Using a recursion at the slot boundaries of consecutive slots, we have

$$\begin{aligned} P_{m,l}(y,k) &= 1\{k=0\}, \quad m+l \leq y, \\ P_{m,l}(y,k) &= b_0 P_{(m-1)^+,l}(y,k) + \sum_{i=1}^{y-m} b_i P_{m+i-1,l-i}(y,k) \\ &\quad + \sum_{i=(y-m)^+ + 1}^{l-1} b_i P_{m+i-1,l-i}(y, k + (y-m)^+ - i) \\ &\quad + \Pr[b \geq l] 1\{k = l - (y-m)^+\}, \quad m+l > y. \end{aligned} \tag{13}$$

The procedure for the computation of $P_n(y,k)$ from (12) and (13) proceeds as follows. For $\bar{b} < 1$ the tail probability of the r.v. q drops exponentially (see Section 4) and the first infinite sum in (12) can be approximated by a finite sum with sufficiently large i^*. The second infinite sum in (12) can also be approximated by a finite sum with sufficiently large m^*. Then we need to compute the probabilities $P_{m,l}(y,k)$. These probabilities are computed recursively from (13) in the following order. For each l, $l = 1, 2, ..., n-1$, in increasing order, the probabilities $P_{m,l}(y,k)$ are computed for each m, $m = 0, 1, ..., i^* + m^* + n - l - 2$, in increasing order, and each k, $0 \leq k \leq l$. Note from (13) that this order is indeed a recursion. The computation complexity of this procedure is of the order of $O((i^* + m^*)n^3 + n^4)$. The probabilities $P_{m,l}(y,k)$ are then used in (12) to compute the probability distribution $P_n(y,k)$ with computation complexity of the order of $O(i^* m^* n)$.

Using the independence assumption of the cell delays (see Section 4), the number of cells in a message of size n whose delays exceed y is a r.v., binomially distributed, with parameters n, $\Pr[d > y]$. That is

$$P_n^{\text{ind}}(y,k) = \binom{n}{k} (\Pr[d > y])^k (\Pr[d \leq y])^{n-k}, \quad 0 \leq k \leq n. \tag{14}$$

In Section 6 we compare the probability distributions $P_n(y,k)$ and $P_n^{\text{ind}}(y,k)$.

5.1. Cell overload period

We conclude this section by introducing an additional important measure for real time sessions – the number of consecutive cells whose delays exceed y, referred to as the *cell overload period*. Here we consider a stream of cells with no message structure imposed on them and we study the distribution of the overload period. Let U^y be the r.v. of the length of the cell overload period, and denote by $\mathcal{U}^y(z)$ its PGF. Let U_1^y be the r.v. of the number of cells that arrive at the first slot of the overload period and whose delays exceed y, and denote by $\mathcal{U}_1^y(z)$ its PGF. Using the event equivalence $\{U_1^y = j\} \iff \{q_{t+1} = y + j, q_t \leq y\}$, $j \geq 1$, we have $\Pr[U_1^y = j] = \sum_{i=0}^{y} \Pr[q = i] b_{y+j-(i-1)^+}$, $j \geq 1$. Note that, U^y equals a busy period that starts with U_1^y and hence its PGF is $\mathcal{U}^y(z) = \mathcal{U}_1^y(z\hat{\mathcal{U}}(z))$ where $\hat{\mathcal{U}}(z) = \mathcal{B}(z\hat{\mathcal{U}}(z))$ (see, e.g., [20]).

6. Numerical examples

Assume $b_i = q^i p,\ i \geq 0$, where $q \stackrel{\Delta}{=} 1 - p$ and $0 < p < 1$. Then, $\overline{b} = q/p$ and $p > 0.5$ for stable system. The PGF of b is $\mathcal{B}(z) = p/(1-qz)$ and from (3) we have $\mathcal{Q}(z) = (p-q)(z-1)/(z(1-qz)-p)$. From (4) we have $\mathcal{D}(z) = (1-\overline{b})(z/(1-\overline{b}z))$ from which we have $\Pr[d = i] = \overline{b}^{i-1}(1-\overline{b}),\ i \geq 1$, and $\Pr[d \leq i] = 1 - \overline{b}^i,\ i \geq 1$.

6.1. Message delay

In order to compute the moments of the message delay we need to compute derivatives of the functions $f_k(z),\ 0 \leq k \leq n-2$ at $z = 1$. From the expression for $F(z, w)$ we have

$$F(z, w) = \frac{(p-q)z/(1-qz)}{1 - qw/(1-qz)}$$

from which we have (by taking the inverse transform in the variable w)

$$f_k(z) = \frac{(p-q)z[q/(1-qz)]^k}{1-qz}, \qquad k \geq 0,$$

and

$$f_k^{(j)}(1) \stackrel{\Delta}{=} \left.\frac{\mathrm{d}^j f_k(z)}{\mathrm{d}z^j}\right|_{z=1} = j(\overline{b}^{-1} - 1)\prod_{i=1}^{j-1}(k+i)\overline{b}^{k+j} + (\overline{b}^{-1} - 1)\prod_{i=1}^{j}(k+i)\overline{b}^{k+j+1}, \qquad j \geq 0,$$

where an empty product equals 1 (for each k, $f_k^{(j)}(1)$ can be computed recursively in the parameter j).

Using the independence assumption of Section 3, the variance of the message delay is obtained from (8) and (9). In Fig. 1 we plot the relative variance error of the message delay, defined as 100 * [(approximated variance)/(exact variance) −1] versus the message size n, for different values of $\overline{b}$ ($\overline{b} = 0.5,\ 0.8,\ 0.9$). For all cases observe that the approximated variance of the message delay is much larger than the exact one. Observe also that the approximation becomes worse for heavy loads in a wide range of message sizes. The same observations were made in [13] for the M/M/1 and M/G/1 systems where the differences were up to 75% for average load of 0.9.

6.2. Maximum cell delay

In this example the r.v.s. $b^{\mathrm{a}},\ b^{\mathrm{b}}$ are independent and Eq. (10) reduces to a double sum. In Fig. 2 we plot the survivor function of the maximum cell delay as computed in Section 4 for message size $n = 50$ and 0.8 load. We also plot this function under the independence assumption on the cell delays (which yields an upper bound on the maximum cell delay) and the lower bound as introduced in Section 4.

In Fig. 3 we plot the average of the maximum cell delay as a function of the message size n for 0.8 load. In the same figure we plot the natural logarithm of n, and the average of the maximum cell delay under the independence assumption. For this example the parameter α of the asymptotic analysis equals $-\mathrm{Ln}(\overline{b}) = 1/4.48142$. The quantity $\alpha E[y_{\mathrm{ind}}^{\max}(n)]/\mathrm{Ln}\, n$ approaches 1 as n becomes very large.

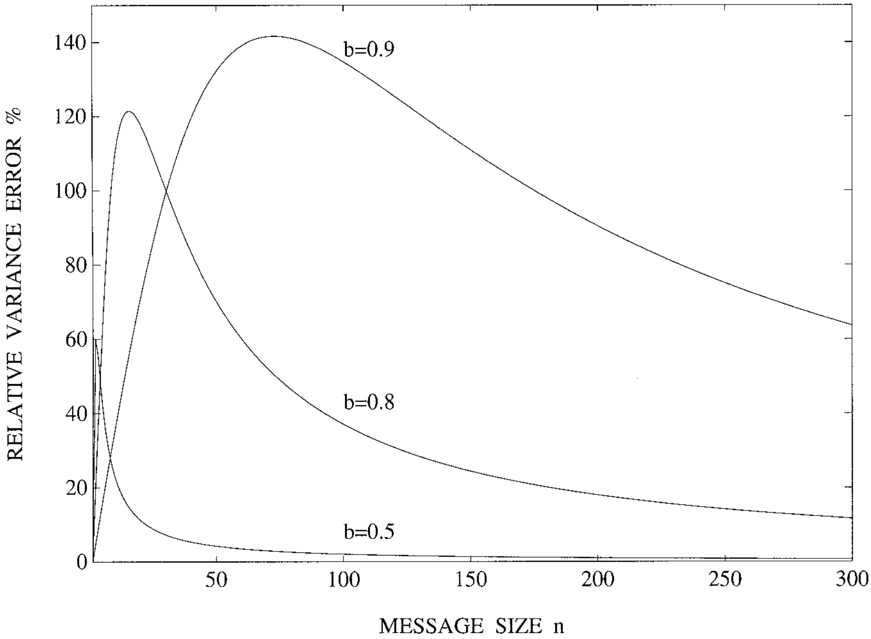

Fig. 1. The relative variance error of the message delay versus the message size n.

6.3. *Threshold constrained delay process*

The probabilities of k cells in a message of size n whose delays exceed y, $P_n(y, k)$, are given in Table 1 for a system with message size $n = 10$ cells, threshold $y = 20$ cells and for different average loads $\bar{b} = 0.7,\ 0.8,\ 0.9$. For comparison purposes we also give in the table the same probability distribution under the independence assumption $P_n^{\text{ind}}(y, k)$.

Table 1 gives a clear indication that the independence assumption may yield overly optimistic results. Furthermore, the probability distribution $P_n^{\text{ind}}(y, k)$ decreases with k while $P_n(y, k)$ is convex with k and sometimes increases with k as observed from the last rows of Table 1. This convexity is due to the positive correlation between cell delays of consecutive cells as observed in Section 4 which implies high probability in the last row of Table 1. For the time-out mechanism where a cell is lost whenever its delay exceeds a threshold y, an interesting phenomenon can be noticed from the first row of Table 1. Correlation exists not only for the cell loss process but also for the no-loss process. That is, with retransmissions at the message level, the probability of message loss under the independence assumption is higher than the exact probability of message loss, and hence we may not need to implement forward error correction scheme [24] in order to achieve a pre-specified (low) message loss probability. The same phenomenon was observed in [12] for a finite queueing system, and in [3] for moderate traffic. However, the results of [3] showed that forward error correction can reduce the message loss probability for heavy and light traffic conditions. The results of Table 1 demonstrate that for a threshold $y = 20$, even under modest utilization (0.8) there are more messages (of length $n = 10$) that contain at least 2 losses due to time-out ($\sum_{k=2}^{n} P_n(y, k) = 1.639 \times 10^{-2}$)

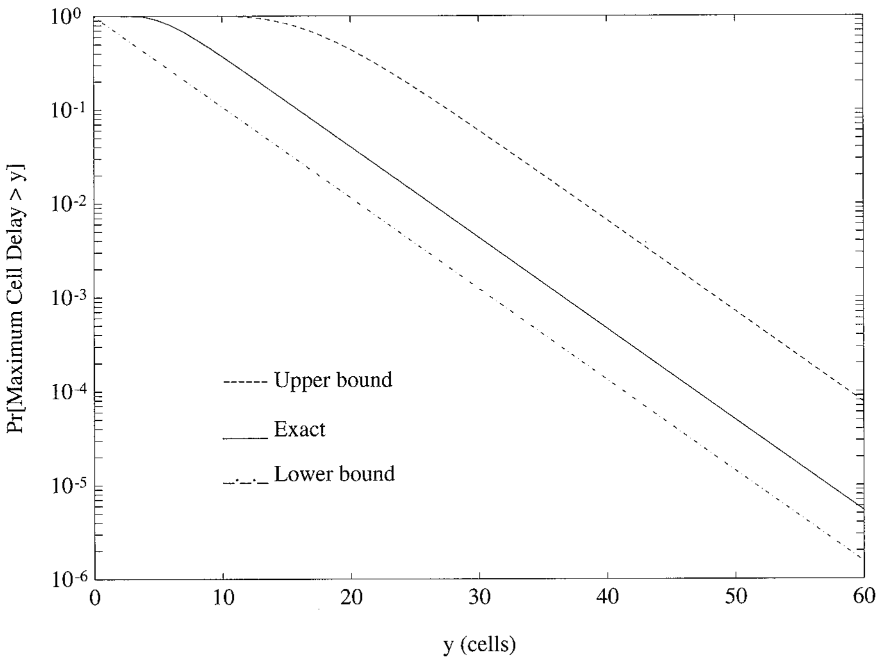

Fig. 2. Survivor function of the maximum cell delay in a message of size $n = 50$ for 0.8 load.

Table 1
Probability of k cells in a message of size $n = 10$ that their delays exceed $y = 20$

	$\bar{b} = 0.7$		$\bar{b} = 0.8$		$\bar{b} = 0.9$	
k	$P_n(y,k)$	$P_n^{\text{ind}}(y,k)$	$P_n(y,k)$	$P_n^{\text{ind}}(y,k)$	$P_n(y,k)$	$P_n^{\text{ind}}(y,k)$
0	9.983×10^{-1}	9.920×10^{-1}	9.809×10^{-1}	8.905×10^{-1}	8.435×10^{-1}	2.735×10^{-1}
1	3.690×10^{-4}	7.922×10^{-3}	2.708×10^{-3}	1.039×10^{-1}	1.089×10^{-2}	3.786×10^{-1}
2	2.470×10^{-4}	2.847×10^{-5}	1.968×10^{-3}	5.452×10^{-3}	8.511×10^{-3}	2.358×10^{-1}
3	1.859×10^{-4}	6.062×10^{-8}	1.593×10^{-3}	1.696×10^{-4}	7.343×10^{-3}	8.703×10^{-2}
4	1.480×10^{-4}	8.472×10^{-11}	1.359×10^{-3}	3.461×10^{-6}	6.654×10^{-3}	2.108×10^{-2}
5	1.221×10^{-4}	8.118×10^{-14}	1.200×10^{-3}	4.844×10^{-8}	6.229×10^{-3}	3.501×10^{-3}
6	1.035×10^{-4}	5.402×10^{-17}	1.089×10^{-3}	4.708×10^{-10}	5.995×10^{-3}	4.038×10^{-4}
7	9.005×10^{-5}	2.465×10^{-20}	1.016×10^{-3}	3.138×10^{-12}	5.942×10^{-3}	3.193×10^{-5}
8	8.090×10^{-5}	7.382×10^{-24}	9.837×10^{-4}	1.373×10^{-14}	6.141×10^{-3}	1.657×10^{-6}
9	7.748×10^{-5}	1.310×10^{-27}	1.026×10^{-3}	3.558×10^{-17}	6.901×10^{-3}	5.097×10^{-8}
10	2.760×10^{-4}	1.046×10^{-31}	6.169×10^{-3}	4.149×10^{-20}	9.192×10^{-2}	7.055×10^{-10}

than there are messages with a single cell loss ($P_n(y, 1) = 2.708 \times 10^{-3}$) where the rate of loss is only around 0.012 (an average of 0.12 lost cells per message). This can significantly impact the performance of smoothing and predictive playback algorithms.

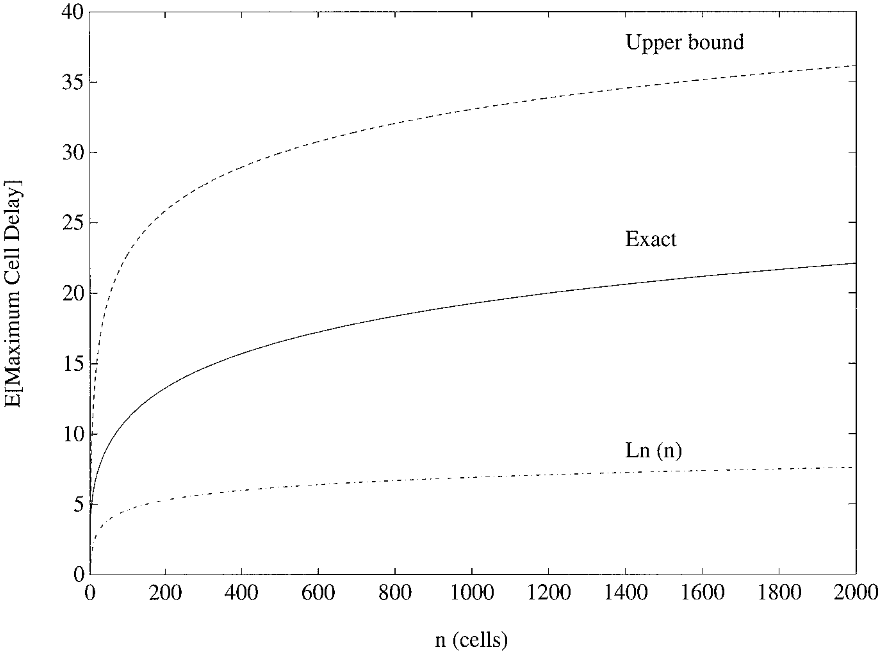

Fig. 3. Average of the maximum cell delay as a function of the message size n for 0.8 load.

7. Network model simulation

In this section we show by simulation that the general phenomena observed in Section 6 for the single node system hold for a network environment as well. Here, we concentrate on a specific session (or, virtual connection) and consider the nodes along its path from the source to the destination. In Fig. 4 we depict the eight nodes tandem queueing system used in the simulation. For each node along its path we use the same model of Section 2 and assume that slot boundaries at all nodes are synchronized. Session cells arrive at the first node according to a Poisson process with rate Λ. Background cells from other sources arrive to each node i according to Poisson process with rate λ_i. A background cell which completes service at node i along the path proceeds to the next node in the path with probability p_i or leaves the path with probability $\overline{p}_i \stackrel{\Delta}{=} 1 - p_i$. This model captures the effect of interfering traffic from other sources in the network along several hops in the path of the session. We used the Block Oriented Network Simulator (BONeS) Designer of Comdisco Systems Inc.

In Table 2 we give the mean and the variance of the cell and the message delays as obtained from the simulation. For each value of Λ we used 10 simulations with different seed generators and 10^5 sample points for each simulation. The 95% confidence interval is $1-2\%$ of the obtained values. For comparison purposes we also give the variance of the message delay and the relative variance error (RVE) under the independence assumption (see Section 3). We used a message of size 10, $p_1 = 0.5$, $\lambda_1 = 0.9 - \Lambda$, $p_i = 0.5$, $\lambda_i = 0.5(0.9 - \Lambda)$, $2 \le i \le 8$ (the overall cell arrival rate to each node equals 0.9). For all cases of Λ ($\Lambda = 0.1,\ 0.45,\ 0.8$) observe that the approximated variance of the message delay using the

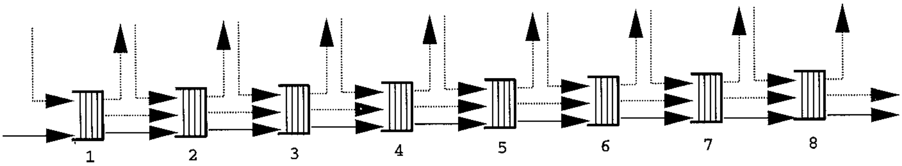

Fig. 4. An 8-node tandem queueing system.

Table 2
Cell and message delay moments for message of size 10 ($p_1 = 0.5,\ \lambda_1 = 0.9 - \Lambda,\ p_i = 0.5,\ \lambda_i = 0.5(0.9 - \Lambda),\ 2 \le i \le 8$)

Λ	Cell delay		Message delay			
	Mean	Variance	Mean	Variance	Approx. variance	RVE
0.1	29	162	119	777	1062	36
0.45	24	134	44	119	179	49
0.8	16	67	28	59	81	36

Table 3
Probability of k cells in a message of size $n = 10$ whose delays exceed $y = 20$ for message of size 10 $p_1 = 0.5,\ \lambda_1 = 0.1,\ p_i = 0.5,\ \lambda_i = 0.05,\ 2 \le i \le 8$)

	$\Lambda = 0.7$		$\Lambda = 0.77$	
k	$P_n(y,k)$	$P_n^{\text{ind}}(y,k)$	$P_n(y,k)$	$P_n^{\text{ind}}(y,k)$
0	9.365×10^{-1}	7.399×10^{-1}	7.821×10^{-1}	2.257×10^{-1}
1	1.150×10^{-2}	2.262×10^{-1}	2.260×10^{-2}	3.623×10^{-1}
2	7.100×10^{-3}	3.113×10^{-2}	1.640×10^{-2}	2.616×10^{-1}
3	6.100×10^{-3}	2.538×10^{-3}	1.390×10^{-2}	1.120×10^{-1}
4	5.300×10^{-3}	1.358×10^{-4}	1.230×10^{-2}	3.145×10^{-2}
5	3.900×10^{-3}	4.984×10^{-6}	1.140×10^{-2}	6.057×10^{-3}
6	3.800×10^{-3}	1.270×10^{-7}	1.050×10^{-2}	8.102×10^{-4}
7	3.300×10^{-3}	2.219×10^{-9}	1.030×10^{-2}	7.430×10^{-5}
8	3.400×10^{-3}	2.544×10^{-11}	1.060×10^{-2}	4.472×10^{-6}
9	3.500×10^{-3}	1.729×10^{-13}	1.140×10^{-2}	1.595×10^{-7}
10	1.560×10^{-2}	5.286×10^{-16}	9.850×10^{-2}	2.560×10^{-9}

independence assumption is significantly larger than the more realistic one (the same was observed for the single node system in Section 6). The mean and the variance of the cell and the message delay decrease as the session load Λ increases since the interference with background traffic is reduced.

In Fig. 5 we plot the average of the maximum cell delay as a function of the message size n for $\Lambda = 0.7$, $p_1 = 0.5,\ \lambda_1 = 0.1,\ p_i = 0.5,\ \lambda_i = 0.05,\ 2 \le i \le 8$ (the overall cell arrival rate to each node equals 0.8). For each point in the graph we used 10^6 sample points. From Fig. 5 we note that the average of the maximum cell delay is of the order of $O(\text{Ln}\, n)$ similarly to the results of the single node system in Section 6.

The probabilities $P_n(y,k)$ of k cells in a message of size n whose delays exceed y are given in Table 3 for a system with message size $n = 10$ cells, threshold $y = 20$ cells, background traffic parameters

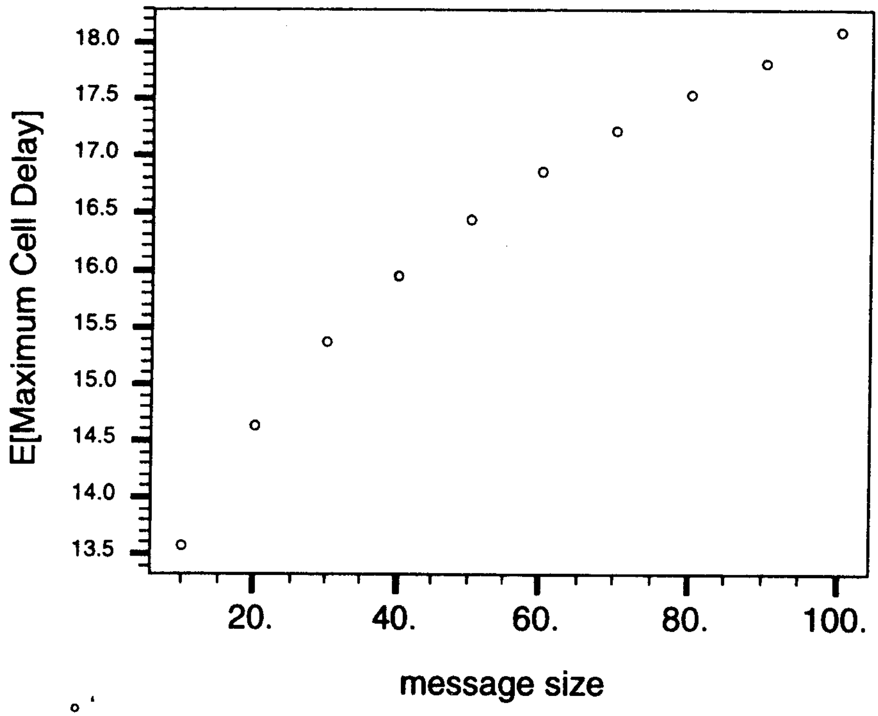

Fig. 5. Average of the maximum cell delay as a function of the message size n for $\Lambda = 0.7$, $p_1 = 0.5$, $\lambda_1 = 0.1$, $p_i = 0.5$, $\lambda_i = 0.05$, $2 \le i \le 8$.

$p_i = 0.5$, $1 \le i \le 8$, $\lambda_1 = 0.1$, $\lambda_i = 0.05$, $2 \le i \le 8$, and for $\Lambda = 0.7$, 0.77. For comparison purposes we also give in the table the same probability distribution under the independence assumption

$$P_n^{\text{ind}}(y, k) = \binom{n}{k} (\Pr[\text{cell delay } > y])^k \Pr[\text{cell delay } \le y])^{n-k}, \quad 0 \le k \le n,$$

where Pr[cell delay $> y$] $= 0.02967$, 0.1383 for $\Lambda = 0.7$, 0.77, respectively, as obtained from the simulation. Assume that a cell is lost if its delay exceeds y. The same conclusions obtained from the results of Table 1 for a single node system still hold for the tandem system. For the decoding scheme proposed in [24], a lost cell can be recovered if and only if it is the only lost cell in its message. This is done by adding one parity cell to every message of size n cells, which increases the cell arrival rate to the system by a factor of $1/n$. The average number of cells lost in a message after decoding is given by $ED = \sum_{k=2}^{n+1} kP_{n+1}(y, k)$, and the cell loss rate after decoding is given by $ED/(n + 1)$. The cell loss rates with and without this decoding scheme are computed from the simulation (with $\Lambda = 0.7$) and are equal to 0.149 and 0.0345, respectively. That is, the increase in the loss probability due to increase in the cell arrival rate, caused by adding the parity cell, supersedes the decrease in the loss probability due to the correction scheme. The fact that forward error correction schemes becomes less efficient due to the bursty nature of the cell loss process was first observed in [12,24] for a single node system with finite number of buffers.

Appendix A

A.1. Associated r.v.s.

The notion of association, a type of positive dependence among r.v.s., was introduced in [15].

Definition A.1. We say that r.v.s. $T_1, \dots, T_n$ are associated if

$$\mathrm{Cov}[f(T), g(T)] \geq 0 \tag{A.1}$$

for all nondecreasing functions f and g for which $Ef(T)$, $Eg(T)$, $Ef(T)g(T)$ exist, where $T \triangleq (T_1, \dots, T_n)$.

A key property of association that makes it valuable in a variety of applications is the following: If $X_1, \dots, X_n$ are associated, then

$$\Pr\left[\bigcap_{i=1}^{n}\{X_i \leq (>)\, r_i\}\right] \geq \prod_{i=1}^{n} \Pr[X_i \leq (>)\, r_i] \tag{A.2}$$

for all reals $r_1, \dots, r_n$.

Association has the following properties proved in [15]:

(P1) Any subset of associated r.v.s. are associated.
(P2) If two sets of associated r.v.s. are independent of one another, then their union is a set of associated r.v.s.
(P3) The set consisting of a single r.v. is associated.
(P4) Nondecreasing functions of associated r.v.s. are associated.

A.2. Asymptotic distribution of maximum of r.v.s.

The following theorem was proved in [21, Theorem 1.5.1].

Theorem A.2. *Let $\{\xi_i\}$ be an i.i.d. sequence. Denote by $F(x) \triangleq \Pr[\xi_1 \leq x]$, and $M_n \triangleq \max\{\xi_1, \dots, \xi_n\}$. Let $0 \leq \tau \leq \infty$ and suppose that $\{u_n\}$ is a sequence of real numbers such that*

$$n(1 - F(u_n)) \to \tau \quad \text{as } n \to \infty. \tag{A.3}$$

Then

$$\Pr[M_n \leq u_n] \to \mathrm{e}^{-\tau} \quad \text{as } n \to \infty. \tag{A.4}$$

Conversely, if (A.4) *holds for some τ, $0 \leq \tau \leq \infty$, then so does* (A.3).

Appendix B

The behavior of the maximum delay of a cell in a message as the message size n increases is important for the study of the behavior of playback algorithms. In the following we investigate the asymptotic behavior of the upper bound of the message delay. The asymptotic behavior of the r.v. $y_{\rm ind}^{\max}(n)$ is obtained using Theorem A.2 described in Appendix A.

It was shown in [1] that under quite general conditions (semi-Markov queues where the underlying state space is infinite and the utilization is less than 1), the tail probability $\Pr[d > n]$ of the delay time is of the form $c\,e^{-\alpha n}$ where $c, \alpha > 0$ ($-\alpha$ is obtainable as the negative real root of a certain functional equation which lies closest to the origin). Under the independence assumption, x_i, $1 \le i \le n$, are independent r.v.s. equal in distribution to the r.v. d. Using Theorem A.2 with x_i, $\Pr[d \le x]$ and $y_{\rm ind}^{\max}(n)$ replacing for ξ_i, $F(x)$ and M_n, respectively, and $u_n = (\alpha^{-1} - \epsilon){\rm Ln}(n)$ with $\epsilon > 0$ arbitrarily small, we have $n\ \Pr[d > u_n] \to_{n\to\infty} \infty$ and from Theorem A.2 (with $\tau = \infty$) we have $\Pr[y_{\rm ind}^{\max}(n)/{\rm Ln}(n) > \alpha^{-1} - \epsilon] \to_{n\to\infty} 1$. Similarly, for $u_n = (\alpha^{-1} + \epsilon){\rm Ln}(n)$ we have $\Pr[y_{\rm ind}^{\max}(n)/{\rm Ln}(n) \le \alpha^{-1} + \epsilon] \to_{n\to\infty} 1$. Combining these two limits we have $\Pr[|y_{\rm ind}^{\max}(n)/{\rm Ln}(n) - \alpha^{-1}| > \epsilon] \to_{n\to\infty} 0$, which implies that $y_{\rm ind}^{\max}(n)/{\rm Ln}(n)$ converges to α^{-1} in probability. In Section 6 we give numerical example for the mean of the maximum cell delay and its upper bound as a function of the message size n, and show that it approaches $\alpha^{-1}{\rm Ln}\,n$ as n becomes large.

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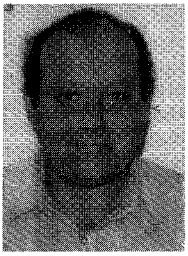

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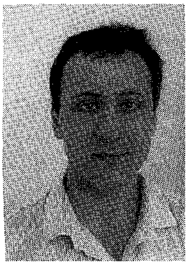

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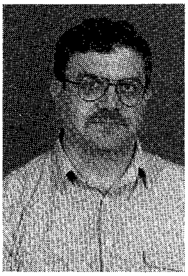

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