

Carrier Sense Access in an Environment of Two Interfering Channels

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Interference problems in radio networks are investigated. A general model is developed for the case of two interfering channels. The model is used to analyze the performance of a two-station packet radio network and a CSMA network with hidden terminals. Performance evaluations for both slotted and unslotted systems are presented.

Keywords: Packet radio, Interference, Carrier sense, Busy tone multiple access, Hidden terminal, Performance evaluation



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1. Introduction

Interference is the phenomenon where the activity of one system disturbs the activity of another as a result of contention for a shared resource. In radio computer networks, this resource is the radio channel on which transmission takes place. Interference in such networks is characterized by the ability or inability of one user to hear or be heard by another. This paper investigates some aspects of the interference problem in packet radio networks.

Interference issues arise in a variety of situations, the most common being packet collision in random access schemes. A large variety of access protocols have been designed to handle the interference problem and to resolve collisions once they occur [1,2]. However, several more subtle interferences still exist that are due to the operation of several networks (or several parts of the same network) on the same radio channel or to an imperfect environment for the operation of random access protocols. For example, consider two separate networks using the same communication channel. Here, the interference is manifested by the mutual disturbance to the operation of each network. Some or all of the activity in one of these networks may disrupt the operation of the random access protocol in the other network. In another situation, groups of users belonging to the same system, who should coordinate their transmission over a shared channel are unable to do so because range or line of sight limitations prevent some from sensing the activity of the others.

In both examples, interference arises because the sources cannot communicate directly and because channel activity is sensed at the source rather than at the destination of the packets.

In both examples, interference arises because the sources cannot communicate directly and because channel activity is sensed at the source rather than at the destination of the packets. The problem is very common in random access protocols which use a "listen before transmission"

policy. Here there is no guarantee that an interfering source for the receiver can be sensed at the transmitter. The performance of systems in the presence of such interferences is analyzed in this paper.

To model these phenomena, users are divided into interference groups each containing users that have identical characteristics as to whom they hear or are heard by. The model developed is suitable for solving various interference problems, two of which—the two-station packet radio network, and the hidden terminal problem—are analyzed in this paper.

Section II analyzes a busy-tone multiple-access scheme in a two-station packet radio network. In such configurations, so common in cellular systems [3], nodes communicate through stations that serve as packet forwarders. We offer the analysis for both slotted and unslotted systems and for two different forwarding schemes.

Section III uses the same model to analyze the hidden-terminal problem in CSMA networks. The problem arises when not all users hear each other and, as a consequence, a user wishing to transmit may deem the channel free when it is not [4]. The analysis we offer assumes neither symmetry nor independence.

2. Two-station, Busy-Tone, Multiple-Access (BTMA) System

Consider a multiple-station radio network consisting of nodes, stations, and three different (and independent) channels—node to station, station to node, and station to station [5]. Communication among nodes is implemented by having the source node transmit its message to a station that, if

necessary, forwards the message to a destination station (on the station-to-station channel), which finally transmits the message to the destination node. As indicated in [5], the problematic link in this chain is the node-to-station channel since it is shared by a large number of users whose communication devices must be kept simple.

We confine ourselves to a two-station configuration and focus on the node-to-station channel. Thus, the configuration consists of two stations listening to a common collision-type radio channel using a busy-tone access—that is, each station transmits a busy tone whenever it senses a carrier on the common channel. The busy one is transmitted using the station-to-node communication system and does not interfere with transmission from the nodes or the busy tone of another station.

Nodes obey a nonpersistent busy-tone carrier sense access discipline [6]. Before transmission, the node listens to the busy-tone channel and transmits only if no busy tone is sensed. Should a busy tone be sensed, the node reschedules transmission to some random time in the future. Once transmission starts, the node transmits the entire message. Each node listens to the busy tone of a single station. Hence, the nodes are divided into two major groups depending on the station they listen to. Each node is heard by the station to which it listens; however, some nodes are also heard by the other station, which is the cause of interference. Thus, within each group the nodes are further subdivided into two subgroups depending on whether or not they are heard by another station.

Since the busy tone is generated only by the stations it is necessary to track channel activities only at the stations. We refer to the activity tracked by Station 1 as the “first channel” (or CH1) and

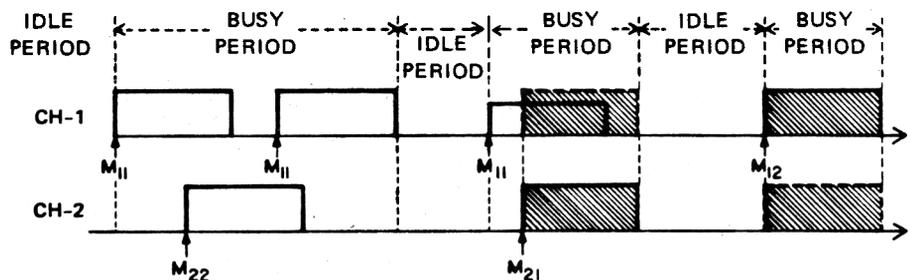


Fig. 1. Two-Channel (BTMA) Busy and Idle Periods

that tracked by Station 2 as the second channel (CH2).

For analysis purposes, we assume that each subgroup generates equally long messages of unit length according to an independent Poisson process resulting in four different such processes, P_{11} , P_{12} , P_{21} , P_{22} , with the respective parameters g_{11} , g_{12} , g_{21} , g_{22} , where g_{ij} ($i, j = 1, 2$) is the arrival rate (in messages per unit time) at the nodes listening to (and heard by) Station i and heard also by station j . For convenience, we denote $g_i \triangleq g_{i1} + g_{i2}$ and $g \triangleq g_1 + g_2 = g_{11} + g_{12} + g_{21} + g_{22}$. We also define interference indices $I_1 \triangleq g_{12}/g_1$ and $I_2 \triangleq g_{21}/g_2$.

In the following we analyze separately two types of channels-unslotted and slotted.

2.1. Unslotted BTMA

We consider a continuous-time system in which nodes may initiate transmission at any time. We assume the propagation delay to be negligible, i.e., nodes hear the busy tone as soon as transmission starts (this is similar to the zero propagation delay for CSMA channels). Having zero propagation delay excludes the possibility of collision among nodes of the same group; this assumption therefore allows us to isolate the effect of interference among groups.

Observing both channels over time, we identify a succession of busy and idle periods forming an alternating renewal process. An idle period is a period in which no transmission takes place in either channel. A busy period is the time between two consecutive idle periods.

Figure 1 depicts several busy periods. The first busy period starts with the transmission of message M_{11} from process P_{11} . Some time later, message M_{22} from process P_{22} arrives and is transmitted (since no busy tone is heard on CH2). Another M_{11} message following the first is transmitted without interference. Note that such a busy period can potentially last arbitrarily long and that all three messages are successfully transmitted.

The second busy period starts with an M_{11} message followed by an M_{21} message. The busy period terminates since the M_{21} message causes a busy tone to be generated by both stations. Only M_{21} is transmitted successfully. The third busy period consists of a single (successful) transmis-

sion of an M_{12} message.

The end of transmission of a message from P_{11} or P_{21} always terminates the busy period.

2.1.1. Throughput Analysis

Because the process is a renewal process the throughput is calculated by dividing the average time of successfully transmitted messages by the average length of the busy and idle periods. Because two overlapping messages may both be successful, the throughput can exceed 1 (but cannot exceed 2).

We define several types of busy periods depending on the first message in the period. Let \tilde{B}_{ij} be the busy period in which the first message belongs to process P_{ij} and let B_{ij} be its average length. In the same manner, let U_{ij} be the average total time of successfully transmitted messages in busy period \tilde{B}_{ij} . We further denote by \bar{I} the average length of an idle period.

With the above definitions, the throughput is given by

$$S = \frac{g_{11}U_{11} + g_{22}U_{22} + g_{12}U_{12} + g_{21}U_{21}}{g_{11}B_{11} + g_{22}B_{22} + g_{12}B_{12} + g_{21}B_{21} + g\bar{I}} \quad (1)$$

The individual components are evaluated next.

The length of the idle period is determined by the lack of arrival of any message. Since all processes are Poisson, their sum is a Poisson process with parameter g and thus

$$\bar{I} = \frac{1}{g} \quad (2)$$

Busy periods of the type B_{12} and B_{21} each contain a single message of length $T = 1$ and thus

$$B_{21} = B_{12} = 1. \quad (3)$$

For these periods we also have

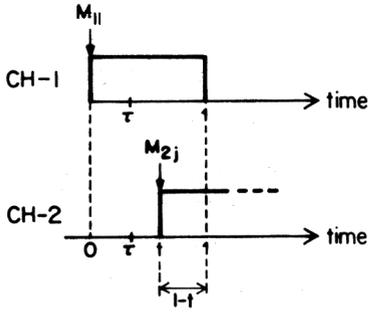
$$U_{21} = U_{12} = 1. \quad (4)$$

The harder case is B_{11} , which we evaluate next. Define $B_{11}(\tau)$ as the average length of a busy period starting at time $t = 0$ with a message from P_{11} given that no message transmission started in CH2 until time τ . From this definition, clearly

$$\begin{aligned} B_{11} &= B_{11}(0) \\ B_{11}(\tau) &= \tau \geq 0 \end{aligned} \quad (5)$$

Similar definitions and relations hold for B_{22} .

Consider now the situation shown in Figure 2 describing a busy period starting with a message


 Fig. 2. Channel Activity in \tilde{B}_{11} Period.

from P_{11} and no message transmitted on CH2 until $t > \tau$. At time t , an arrival occurs on CH2. If this is a message from P_{12} , the length of the busy period is $1 + t$; otherwise, this is a message from P_{22} resulting in an average length of $t + B_{22}(1 - t)$ since clearly no message transmission can be started in the first channel during the initial $1 - t$ period of transmission on CH2.

The last possibility is that no arrival occurs ($t > 1$), in which case the period is of length 1. Averaging all three cases according to their probability of occurrence, we get

$$\begin{aligned}
 B_{11}(\tau) &= e^{-g_2(1-\tau)} + g_{21} \int_{\tau}^1 e^{-g_2(t-\tau)}(1+t) dt \\
 &\quad + g_{22} \int_{\tau}^1 e^{-g_2(t-\tau)}(t + B_{22}(1-t)) dt \\
 &= \tau + \frac{g_{21} + 1}{g_2} (1 - e^{-g_2(1-\tau)}) \\
 &\quad + g_{22} \int_{\tau}^1 e^{-g_2(t-\tau)} B_{22}(1-t) dt. \quad (6)
 \end{aligned}$$

Similar equations hold for $B_{22}(\tau)$ (with the indices 1 and 2 exchanged).

With the same approach we define $U_{ij}(\tau)$ and compute their values along the same lines:

$$\begin{aligned}
 U_{11} &= U_{11}(0), \\
 U_{11}(\tau) &= 1, \quad \tau \geq 1, \quad (7)
 \end{aligned}$$

$$\begin{aligned}
 U_{11}(\tau) &= 1 \cdot e^{-g_2(1-\tau)} + g_{21} \int_{\tau}^1 e^{-g_2(t-\tau)} dt \\
 &\quad + g_{22} \int_{\tau}^1 e^{-g_2(t-\tau)}(1 + U_{22}(1-t)) dt \\
 &= 1 + g_{22} \int_{\tau}^1 e^{-g_2(t-\tau)} U_{22}(1-t) dt. \quad (8)
 \end{aligned}$$

Equation (8) stems from the same arguments that led to equation (6), except that every successful message adds $T = 1$ to U_{11} .

This completes the derivation of all the components of Equation (1).

Although Equations (6) and (8) seem simple, we were unable to find a closed-form solution (Kingman [7] considers this an open problem). We therefore chose numerical solutions. Figure 3 depicts a sample computation of throughput versus total offered load for several interference indices in a symmetric system (i.e., $g_1 = g_2$ and $I_1 = I_2 = I$).

We note in these graphs an asymptotic behavior similar to that existing in zero-delay, nonpersistent CSMA. This behavior is due to the almost certain failure of transmission from P_{11} and P_{22} and the almost certain success of transmissions from P_{12} and P_{21} . To calculate the value of the asymptote, we keep intact all relations among the g_{ij} while causing $g \rightarrow \infty$. This clearly causes the average idle periods to approach 0. As before, $U_{12} = U_{21} = 1$, $B_{12} = B_{21} = 1$. When the load increases the channels work in a synchronized manner, i.e., transmission starts simultaneously in both channels, causing $B_{11} = B_{22} = 1$. The value of U_{11} is either 1 if M_{21} is concurrently transmitted on CH2 or 2 if M_{22} is transmitted there. On the

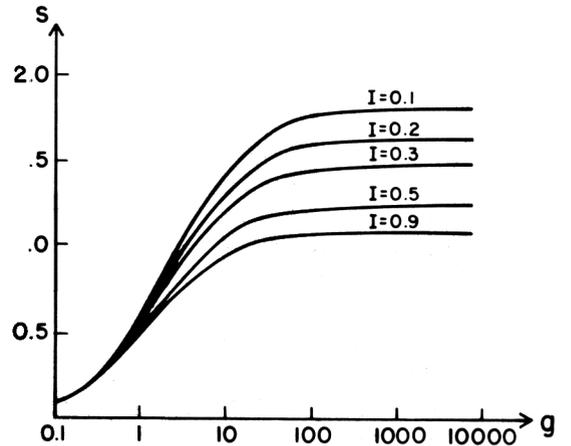


Fig. 3. Throughput vs Offered Load For Unslotted BTMA Channels.

average, therefore

$$U_{11} = 1 \cdot \frac{g_{21}}{g_2} + 2 \cdot \frac{g_{22}}{g_2} = 1 + \frac{g_{22}}{g_2} \quad (9)$$

and similarly

$$U_{22} = 1 + \frac{g_{11}}{g_1}. \quad (10)$$

Substituting these into Equation (1) yields

$$S = \frac{g_{11} \left(1 + \frac{g_{22}}{g_2}\right) + g_{22} \left(1 + \frac{g_{11}}{g_1}\right) + g_{12} + g_{21}}{g} \\ = 1 + (1 - I_1)(1 - I_2). \quad (11)$$

These results are verified by the graphs of Figure 3.

2.2. Slotted BTMA

The analysis of the previous section excluded, because of the zero propagation delay, any interference between messages of the same process. The slotted version we present here accounts for such interference. We choose a slot long enough to reflect the round-trip propagation delay in the system and so that all activities can be assumed to be detected at all places within a single slot time.

The activities of the nodes and stations are identical to those of the unslotted system except for the following two changes:

1. Message transmission starts on the slot boundary only. An arrival during a slot entails waiting for the end of that slot before any subsequent transmission.
2. The busy tone is generated by the station on the slot boundary and is heard immediately by all nodes. The busy tone is turned off one slot time after transmission stops.

This model therefore accounts for collisions within the same group (if arrivals occur during the same slot) as well as propagation delays (since the slot size accounts for the round-trip delays) because the node's decision on its activity in a given slot is based on the activity of the channel (busy tone) in the previous slot.

For the slotted, system we analyze two different forwarding schemes: fixed and random [5]. In the fixed forwarding scheme (FFS), every node has an assigned station serving it and therefore a message from P_{ij} is considered successful if it is correctly received at Station i . In the random forwarding

scheme (RFS), such an assignment does not exist and therefore a successful message is one that is properly received by any station.

2.2.1. Throughput Calculation

The calculation here follows the same line as that of the unslotted system. The RFS and FFS have the same average busy and idle periods and differ only in terms of what is considered a useful transmission.

The throughput is calculated from

$$S = \frac{U}{B + \bar{I}} \quad (12)$$

the components of which we now derive.

The arrival processes P_{ij} are again independent Poisson processes with parameters g_{ij} (measured in messages per slot). The length of the idle period is geometrically distributed with parameter e^{-g} leading to the average

$$\bar{I} = \frac{1}{1 - e^{-g}} \quad (13)$$

Note that in the last slot of the idle period, arrivals take place; this slot is referred to as the *arrival slot*.

To evaluate the length of the busy period, we break it down to \tilde{B}_1 and \tilde{B}_2 depending on the first message to arrive in the arrival slot. Thus, for example, \tilde{B}_1 is the busy period in which a message from P_1 is first to arrive in the arrival slot. B_1 is the average length (in slots) of \tilde{B}_1 . Thus we have

$$B = \frac{g_1 B_1 + g_2 B_2}{g} \quad (14)$$

B_1 is evaluated by breaking \tilde{B}_1 down to two subcases – \tilde{B}_{11} and \tilde{B}_{12} – depending on whether or not messages from P_{12} arrive in the arrival slot. Thus \tilde{B}_{11} is the \tilde{B}_1 busy period whose arrival slot does not contain messages from P_{12} . B_1 is therefore given by

$$B_1 = \frac{(1 - e^{-g_{11}})e^{-g_{12}}}{1 - e^{-g_1}} B_{11} + \frac{1 - e^{-g_{12}}}{1 - e^{-g_1}} B_{12} \quad (15)$$

Clearly

$$B_{12} = N + 1 \quad (16)$$

where N is the length of the transmitted message.

To compute B_{11} , we first define $B_{11}(i)$ as the average length of a \tilde{B}_{11} period in which no arrival

occurs in CH2 until the i -th slot; thus,

$$B_{11} = B_{11}(0)$$

$$B_{11}(i) = N + 1 \quad i \geq N + 2 \quad (17)$$

and $B_{11}(i)$ is given by

$$B_{11}(i) = e^{-g_2(N+2-i)}(N+1)$$

$$+ \sum_{k=i}^{N+1} e^{-g_2(k-i)}(1 - e^{-g_{21}})(k + N + 1)$$

$$+ \sum_{k=i}^{N+1} e^{-g_2(k-i)}e^{-g_{21}}(1 - e^{-g_{22}})$$

$$\times [k + B_{22}(N + 2 - k)]. \quad (18)$$

In Equation (18) the first term accounts for no arrivals whatsoever in CH2; the second term accounts for the case in which messages from P_{22} arrive. Similar results hold for the second channel (B_{21} and B_{22}).

Using the same arguments that led to Equations (14) and (15), we have

$$U = \frac{g_1 U_1 + g_2 U_2}{g} \quad (19)$$

$$U_1 = \frac{(1 - e^{-g_{11}})e^{-g_{12}}}{1 - e^{-g_1}} U_{11} + \frac{1 - e^{-g_{12}}}{1 - e^{-g_1}} U_{12} \quad (20)$$

with similar equations for U_{12} , U_{21} , and U_1 . The values of U_{ij} are derived separately for the RFS and FFS schemes.

2.2.1.1. Computation for FFS. Recall that U_{12} is the average total time of successfully transmitted messages within \tilde{B}_{12} . The only successful message, therefore, is only the first message from P_{12} , provided it is not disturbed by a message from P_{11} , P_{21} , or P_{12} processes. Thus

$$U_{12} = \frac{g_{12}e^{-g_1}e^{-g_{21}}}{1 - e^{-g_{12}}} N. \quad (21)$$

The value of U_{11} is evaluated by computing $U_{11}(i)$, which is given by

$$U_{11}(i) = \frac{g_{11}e^{-g_{11}}}{1 - e^{-g_{11}}}$$

$$\times \left[e^{-g_2(N+2-i)} \right.$$

$$\left. \sum_{k=i}^{N+1} e^{-g_2(k-i)}e^{-g_{21}}(1 - e^{-g_{22}}) \right.$$

$$\left. + e^{-g_2(N+1-i)}(1 - e^{-g_{21}}) \right]$$

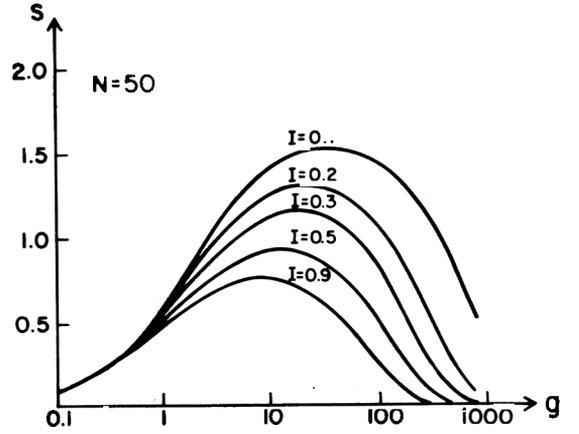


Fig. 4. Throughput vs Offered Load for Slotted BTMA Channels Fixed Forwarding Scheme (FFS). Message length $N = 50$.

$$+ \sum_{k=i}^{N+1} e^{-g_2(k-i)}g_{21}e^{-g_2N}$$

$$+ \sum_{k=i}^{N+1} e^{-g_2(k-i)}e^{-g_{21}}(1 - e^{-g_{22}})$$

$$\times U_{22}(N + 2 - k). \quad (22)$$

In Equation (22) the bracketed term accounts for success of messages from the P_{11} process: that is, a single message arrives in the arrival slot and (a) either nothing arrives on CH2, or (b) only messages from P_{22} arrive but not before the i -th slot, or (c) messages from P_{21} arrive in the $(N + 1)$ -st slot. The second-term accounts for the success of a message from P_{21} : it arrives alone on CH2

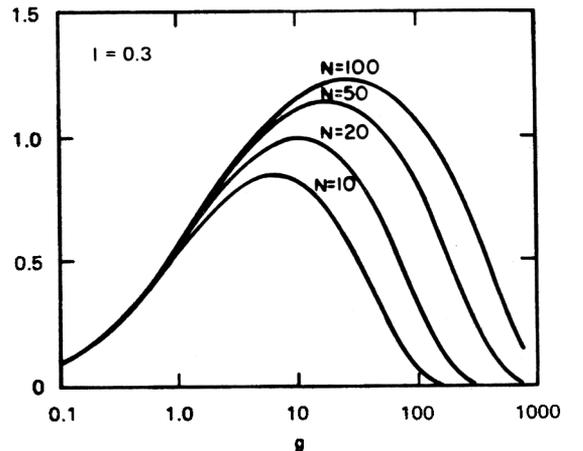


Fig. 5. Throughput vs Offered Load for Slotted BTMA Channels Fixed Forwarding Scheme (FFS). Interference Index 0.3.

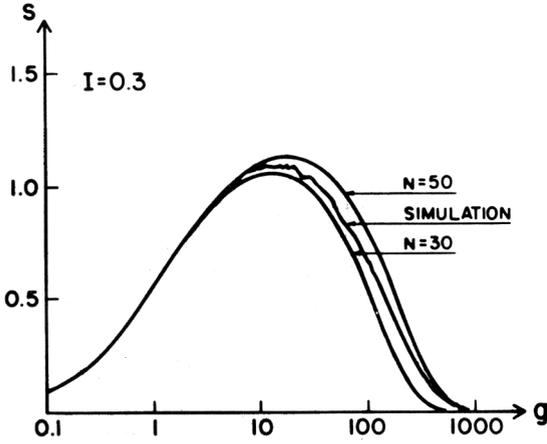


Fig. 6. Throughput Comparison Between Simulation and Computation Interference Index = 0.3 Computation: FFS Slotted System, $N = 30$ and $N = 50$, Simulation: $a = 0.01$.

sometime after the $(i-1)$ st slot. The third term accounts for the success of arrivals from P_{22} .

This completes the derivation of all components needed to compute the throughput. Figure 4 depicts the throughput versus offered load for a symmetric system with $N = 50$ and various interference indices. Note that the maximal throughput is achieved at an offered load which depends only lightly on the interference index. Figure 5 shows the influence of message length on throughput. Clearly, the larger N is the greater will be the throughput achieved. These graphs are similar to those presented in [8]. Figure 6 shows the results of a simulation of an unslotted system with a propagation delay of 0.01 (measured in message length units). Simulation results are compared with computed results for slotted systems with $N = 30$ and $N = 50$. Except for medium offered-loads a slotted system with $N = 50$ should behave like a slotted system with propagation delay of 0.01 since arrival typically occur at the middle of a slot. The graph demonstrates the accuracy of the computation.

2.2.1.2. Computation for RFS. The values of U_{ij} in RFS differ only slightly from those of the FFS. Here we have

$$U_{12} = g_{21} \frac{e^{-(g_{21} + g_{12})}}{1 - e^{-g_{12}}} [e^{-g_{11}} + e^{-g_{22}} - e^{-(g_{11} + g_{22})}] N \quad (23)$$

since a message is successful either when it is successful in its own channel (no other message from P_{11} , P_{12} or P_{21} is transmitted) or it is successful in the other channel (i.e., no concurrent transmission of messages from P_{21} , P_{12} , and P_{22}).

$U_{11}(i)$ is derived similarly to the derivation of Equation (22) except that another term must be added to account for the case where a message from P_{21} is successful in the first channel and fails in the second channel which can happen only if a single message from P_{21} arrives during the $(N+1)$ st slot with at least one message from P_{22} . Thus,

$$\begin{aligned} U_{11}(i) &= \frac{g_{11}e^{-g_{11}}}{1 - e^{-g_{11}}} \\ &\times \left[e^{-g_2(N+2-i)} \right. \\ &\quad + \sum_{k=i}^{N+1} e^{-g_2(k-i)} e^{-g_{21}} (1 - e^{-g_{22}}) \\ &\quad + e^{-g_2(N+1-i)} (1 - e^{-g_{21}}) N \\ &\quad + \sum_{k=i}^{N+1} e^{-g_2(k-i)} g_{21} e^{-g_2 N} \\ &\quad + e^{-g_2(N+1-i)} g_{21} e^{-g_{21}} (1 - e^{-g_{22}}) N \\ &\quad \left. + \sum_{k=i}^{N+1} e^{-g_2(k-i)} e^{-g_{21}} (1 - e^{-g_{22}}) \right] \\ &\quad \times U_{22}(N+2-k). \end{aligned} \quad (24)$$

The throughput versus offered load for various

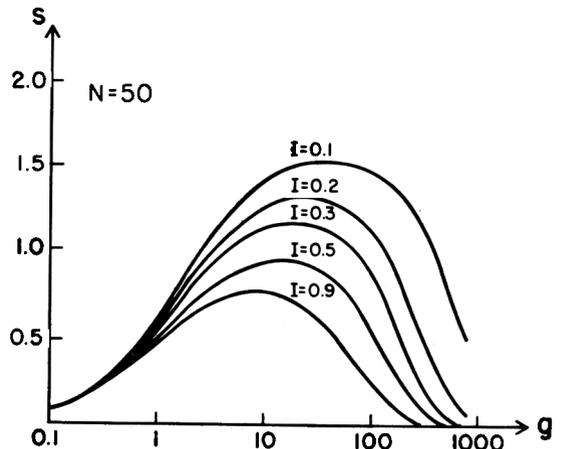


Fig. 7. Throughput vs Offered Load for Slotted BTMA Channels Random Forwarding Scheme (RFS). Message length $N = 50$.

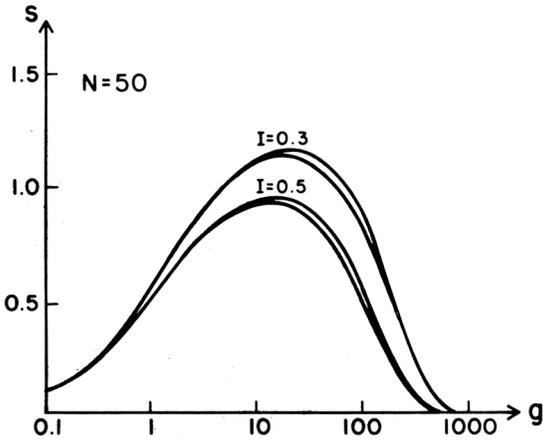


Fig. 8. Throughput Comparison Between Forwarding Schemes Message length $N = 50$.

interference indices is shown in Figure 7. Figure 8 compares the performance of the FFS and RFS schemes. It is seen that RFS performs better (as intuitively expected) but the difference is slight and the choice of a forwarding scheme must be based on issues other than throughput (e.g., routing).

3. CSMA With Hidden Terminals

The model presented above can be used to analyze CSMA systems with hidden terminals. The hidden terminal problem in the CSMA context arises when a user cannot sense the carrier generated by another and thus their messages potentially collide. This definition is asymmetric since one terminal can be hidden from the other and not vice versa. Tobagi and Kleinrock [4] presented an analysis for a symmetric independent case with any number of terminal groups, and Takagi and Kleinrock [9] approximated the distribution of interdeparture times under heavy load in similar circumstances. Here we present the complete solution for the case of two groups.

Consider a set of nodes using nonpersistent CSMA to access a channel. All nodes transmit to a common destination which hears them all. The nodes, however, do not necessarily hear one another. We assume they are divided into two groups, P_1 and P_2 , depending upon who they hear—each node in P_1 hears every other node in P_1 , and similarly for P_2 . Within P_1 there are

nodes that are also heard by nodes of P_2 , giving rise to a finer division: P_{ij} ($i, j = 1, 2$) are nodes of P_i that are also heard by nodes in P_j . Thus nodes from P_{11} are heard only by nodes of P_1 and are therefore completely hidden from nodes of P_2 .

Consider now the activity sensed at the destination. We denote by CH1 the activity at the destination due to transmissions from P_1 , and CH2 the activity due to P_2 . In terms of busy and idle periods these activities are identical to those described by the model of Section II. As a consequence, the same equations—notably (1) through (6) and (12) through (18)—hold. The difference is only in the definition of successful messages; in this case a message is considered successful if it is received at the station undisturbed, i.e., if no other message is transmitted concurrently on the other channel.

In the following we compute the throughput for both slotted and unslotted systems.

1. Unslotted System

We define U_{ij} as we did for the BTMA system, as the portion of success during \tilde{B}_{ij} . For any packet to be successful, it must be the only one to be transmitted in either channel. Thus,

$$U_{12} = U_{21} = 1,$$

$$U_{11} = e^{-g};$$

$$U_{22} = e^{-g_1}$$

The throughput is now computed using Equa-

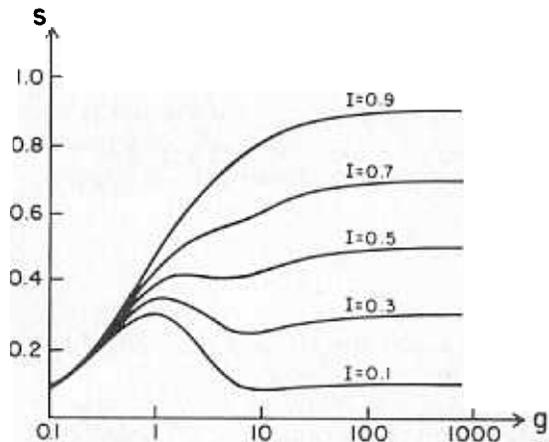


Fig. 9. Throughput of Interfering Terminals Unslotted Channel. Message length $N = 50$.

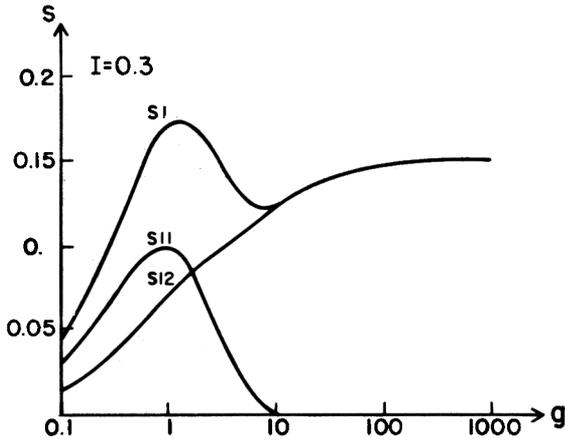


Fig. 10. Throughput Components of Interfering Terminals Unslotted Channel. Message length $N = 50$. Interference Index = 0.3.

tion (1). Figure 9 shows the throughput vs. offered load for various interference indices in a symmetric network.

Denote by S_{ij} the throughput due to messages from process P_{ij} . Figure 10 shows S_{11} and S_{12} for a specific interference index. The figure clearly demonstrates that throughput of the hidden terminals (generating messages according to P_{11}) decrease sharply with load while those terminals not hidden generate most of the total throughput. Clearly, therefore, at high loads the hidden terminals cause only damage since their contribution to total throughput is negligible while the number of collisions in which they are involved does not decrease.

3.2. Slotted System

To derive the equation for a slotted system, we note that the length of the busy and idle period are expressed identically to those presented by Equations (13) through (18). We now derive the values of U_{ij} .

For a message to be successful it must be transmitted alone; thus,

$$U_{12} = \frac{g_{12}e^{-g}}{1 - e^{-g_2}} N.$$

To derive U_{11} , we define a function $U_{11}(i)$ in a slightly different manner than before. $U_{11}(i)$ is the portion of successful messages in a B_{11} period given that another transmission on CH2 ended at

the $(i - 1)$ st slot. Because of this definition, special attention must be given to the cases $i = 0$ and $i = 1$. Thus,

$$\begin{aligned} U_{11}(i) = & \frac{g_{11}e^{-g_{11}}}{1 - e^{-g_{11}}} e^{-g_2(N+2-i)} (\delta_i + \delta_{i-1}) N \\ & + g_{21} e^{-g_2(N+2-i)} N \\ & + \sum_{k=i}^{N+1} e^{-g_2(k-i)} e^{-g_{21}} (1 - e^{-g_{22}}) \\ & \times U_{22}(N+2-k) \end{aligned} \quad (25)$$

where δ_i equals 1 if $i = 0$ and 0 otherwise.

In Equation (25) the first term accounts for success of a message from P_{11} , which requires that it arrives alone and that transmission on CH2 ceases prior to the first slot. The second term accounts for a successful message from P_{21} which can arrive only at the $(N + 1)$ st slot. The last term accounts for success due to arrival of P_{22} .

Putting all this together and using Equations (12) through (20) yields the total throughput. Figure 11 shows the throughput vs offered load for various interference indices in a symmetric network.

Conclusion

In this paper we developed a model for analyzing interference in packet radio networks. The model was applied to the analysis of a two-station

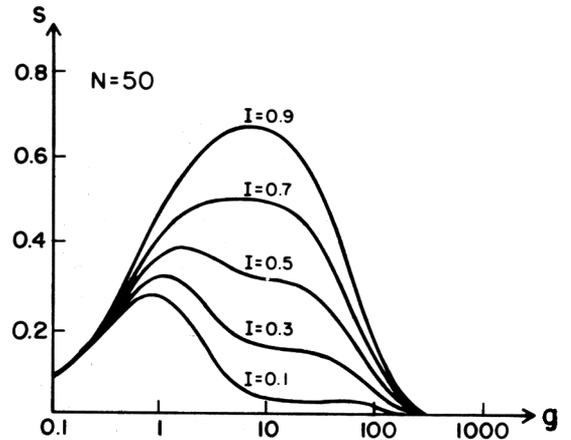


Fig. 11. Throughput of Interfering Terminals Slotted Channel. Message length $N = 50$.

BTMA system, and to the throughput analysis of CSMA systems in the presence of hidden terminals.

The analysis of the two-station BTMA system revealed the difference between the fixed and random forwarding schemes. As might be expected, random forwarding is uniformly better than fixed forwarding since there is always a positive probability that a packet will be received at the non-designated station. Yet, it is demonstrated that the gains in throughput are relatively small so that other considerations should dominate the selection of a forwarding scheme.

The analysis of hidden terminals confirmed the intuitive expectation that in high loads, hidden terminals cause only harm. Their contribution to the total throughput is negligible yet their transmissions still cause collisions to the detriment of the directly accessible users.

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