Defying Nyquist in Analog to Digital Conversion

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In collaboration with my students at the Technion

Digital Revolution

"The change from analog mechanical and electronic technology to digital technology that has taken place since c. 1980 and continues to the present day."

- Cell phone subscribers: 4 billion (67% of world population)
- Digital cameras: 77% of American households now own at least one
- Internet users: 1.8 billion (26.6% of world population)
- PC users: 1 billion (16.8% of world population)



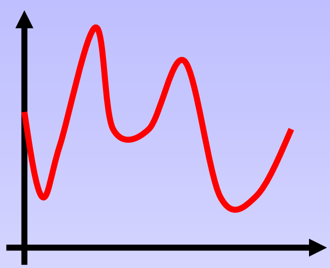
Sampling: "Analog Girl in a Digital World…" Judy Gorman 99

Analog world

Digital world

$x(t)$ → Sampling A2D → $c[n]$

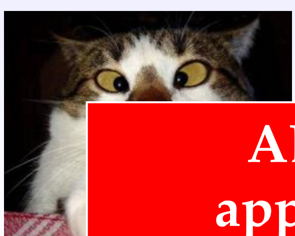
$\tilde{x}(t)$ ← Reconstruction D2A (Interpolation) ←



- Music
- Radar
- Image…

- Very high sampling rates: hardware excessive solutions
- High DSP rates

- Signal processing
- Image denoising
- Analysis…




ADCs, the front end of every digital application, remain a major bottleneck

Today's Paradigm

- Analog designers and circuit experts design samplers at Nyquist rate or higher
- DSP/machine learning experts process the data
- Typical first step: Throw away (or combine in a "smart" way) much of the data …
- Logic: Exploit structure prevalent in most applications to reduce DSP processing rates

Can we use the structure to reduce sampling rate + first DSP rate (data transfer, bus …) as well?

Key Idea

Exploit structure to improve data processing performance:

- Reduce storage/reduce sampling rates
- Reduce processing rates
- Reduce power consumption
- Increase resolution
- Improve denoising/deblurring capabilities
- Improved classification/source separation

Goal:

- Survey the main principles involved in exploiting "sparse" structure
- Provide a variety of different applications and benefits

Talk Outline

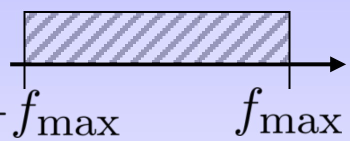
- Motivation
- Classes of structured analog signals
- Xampling: Compression + sampling
- Sub-Nyquist solutions
  - Multiband communication: Cognitive radio
  - Time delay estimation: Ultrasound, radar, multipath medium identification
- Applications to digital processing

Shannon-Nyquist Sampling

Signal Model

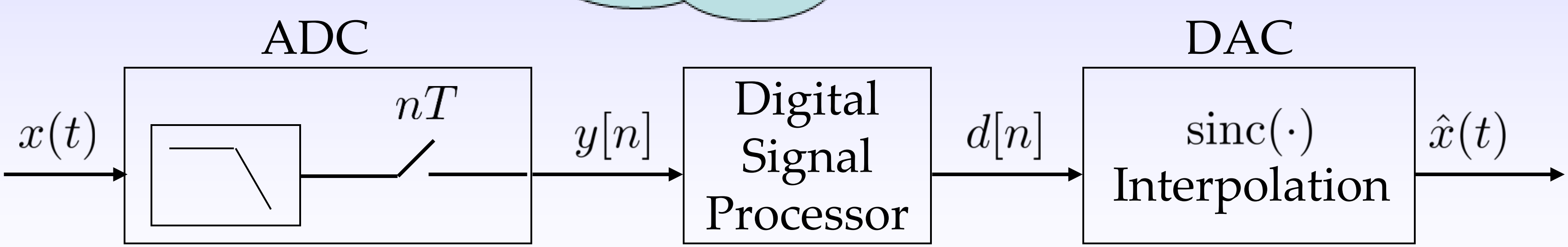
Minimal Rate

Bandlimited signals



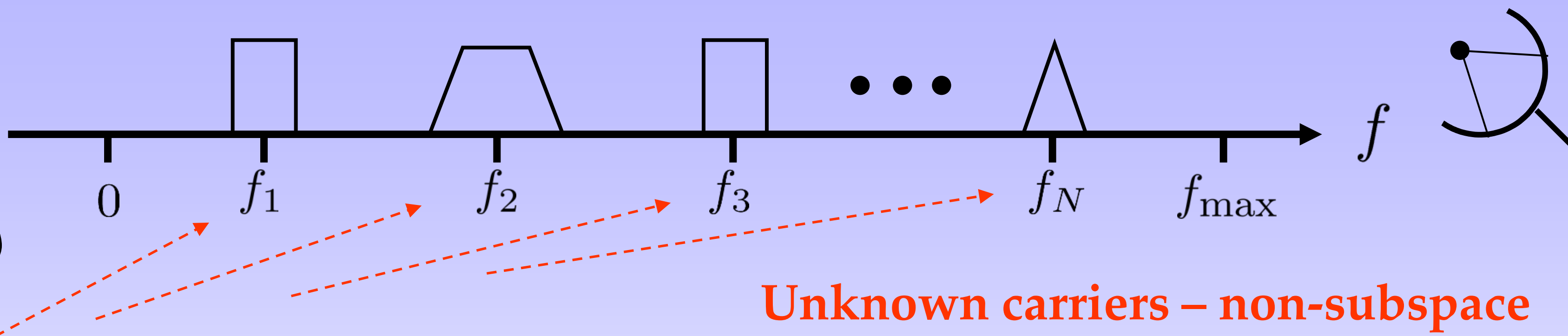
Shannon-Nyquist:
$f_{\text{sample}} \geq 2f_{\max}$

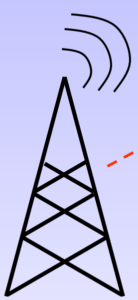
Analog+Digital Implementation



Structured Analog Models

Multiband communication:





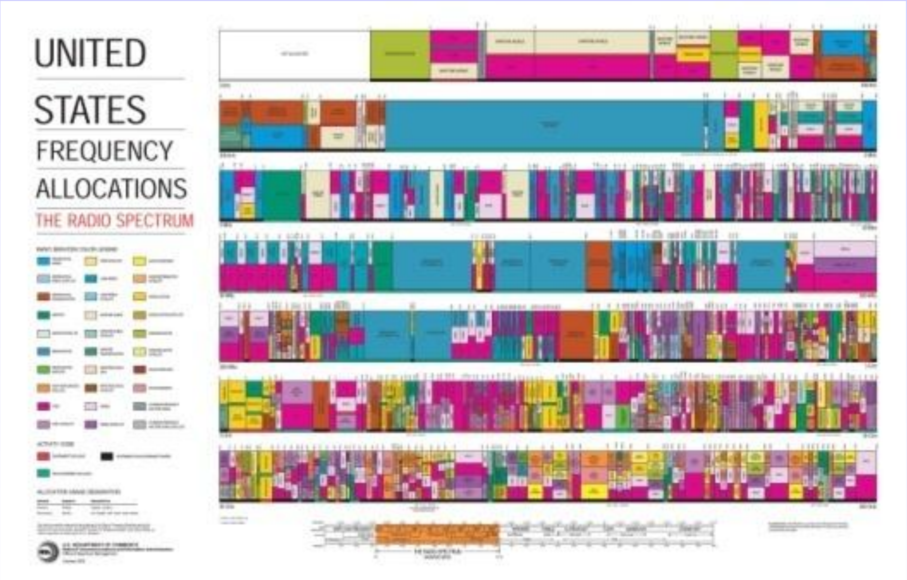
- Can be viewed as $f_{\max}$–bandlimited (subspace)
- But sampling at rate $\geq 2f_{\max}$ is a waste of resources
- For wideband applications Nyquist sampling may be infeasible

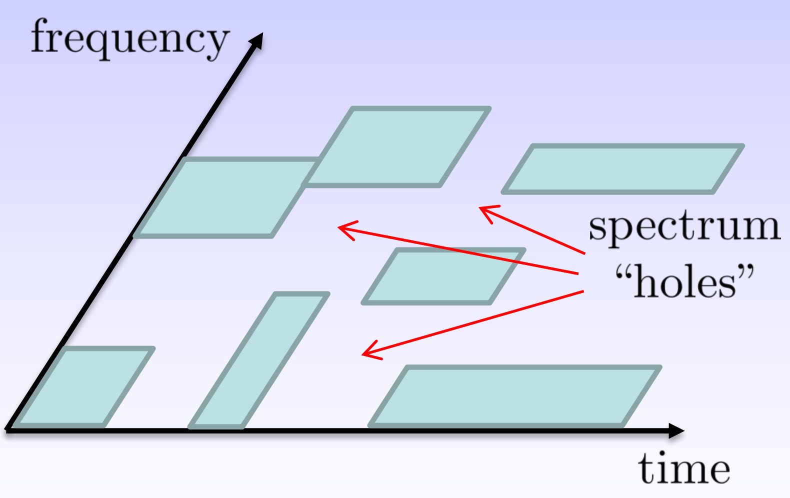
Question:
How do we treat structured (non-subspace) models efficiently?

Cognitive Radio

- Cognitive radio mobiles utilize unused spectrum ``holes''
- Spectral map is unknown a-priori, leading to a multiband model

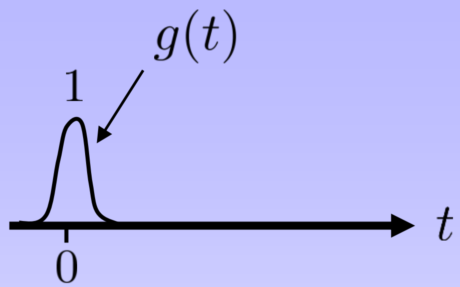
Federal Communications Commission (FCC) frequency allocation

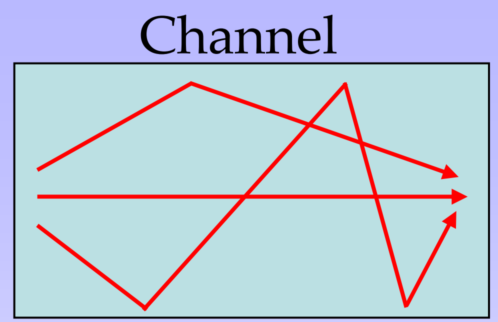


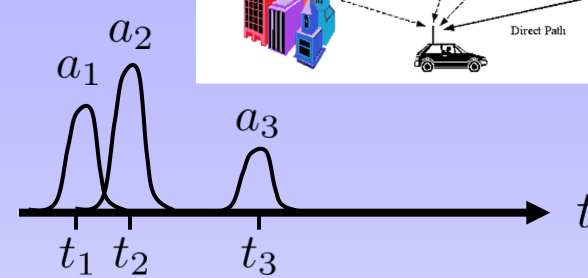


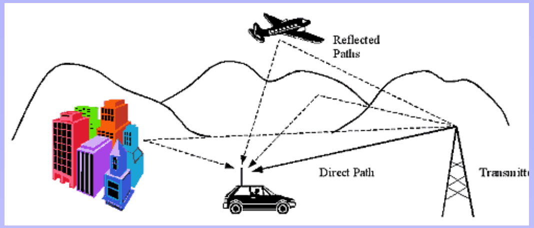
Structured Analog Models

Medium identification:









Similar problem arises in radar, UWB communications, timing recovery problems …

Unknown delays – non-subspace

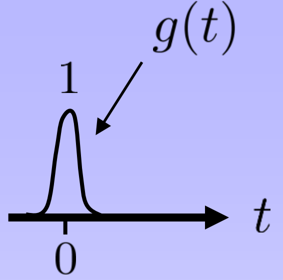
- Digital match filter or super-resolution ideas (MUSIC etc.) *(Brukstein, Kailath, Jouradin, Saarnisaari …)*
- But requires sampling at the Nyquist rate of $g(t)$
- The pulse shape is known – No need to waste sampling resources !

Question (same):
How do we treat structured (non-subspace) models efficiently?

Ultrasound

- High digital processing rates
- Large power consumption

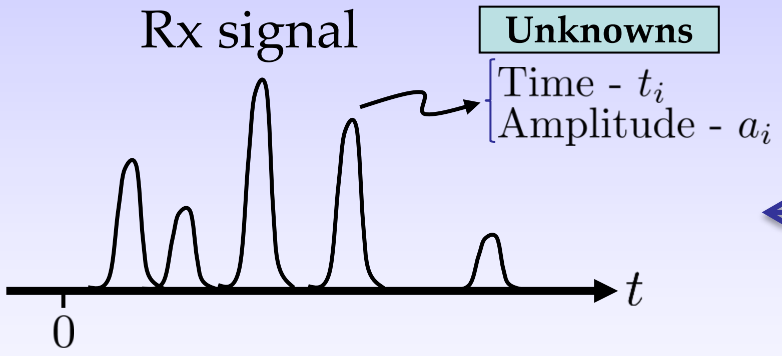
Tx pulse

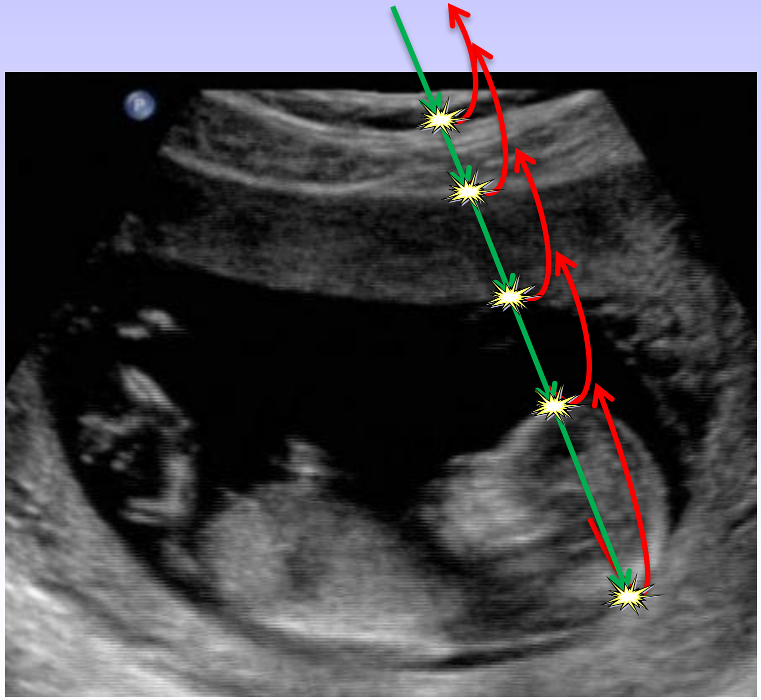


Ultrasonic probe

(Collaboration with General Electric Israel)

Rx signal

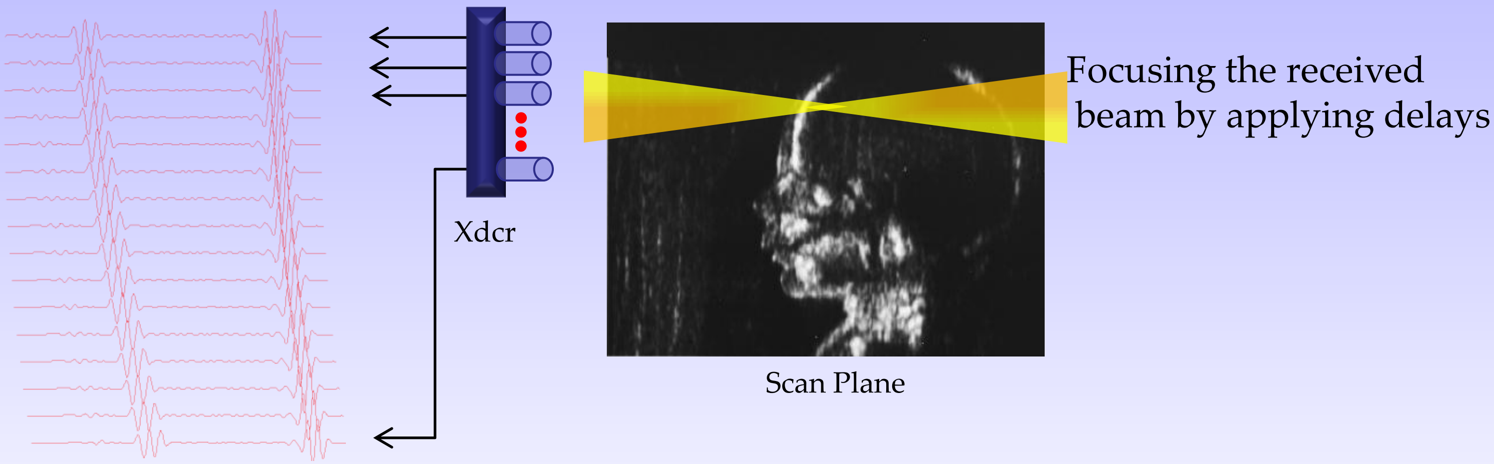




- Echoes result from scattering in the tissue
- The image is formed by identifying the scatterers

Processing Rates

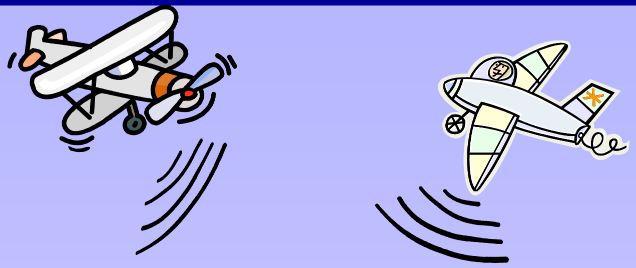
- To increase SNR the reflections are viewed by an antenna array
- SNR is improved through beamforming by introducing appropriate time shifts to the received signals



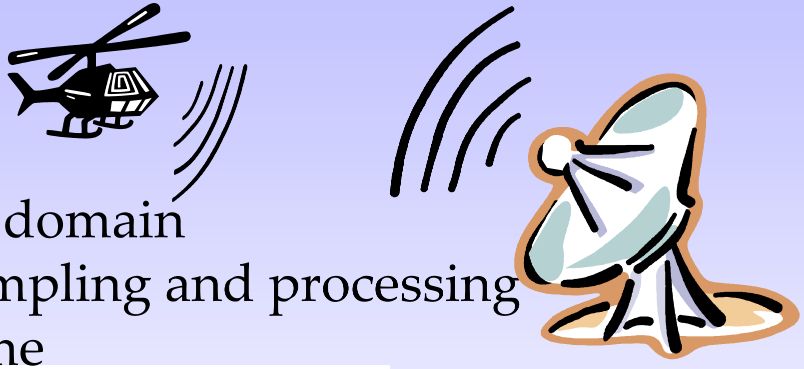
- Requires high sampling rates and large data processing rates
- One image trace requires 128 samplers @ 20M, beamforming to 150 points, a total of 6.3x10^6 sums/frame

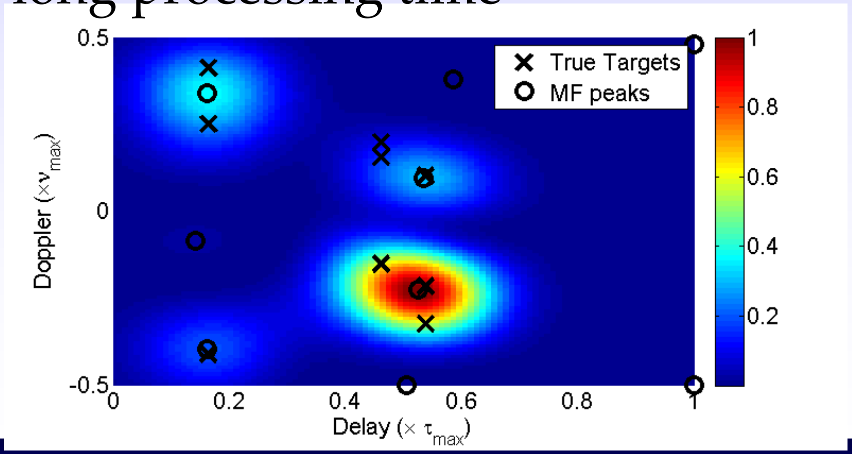
Resolution (1): Radar

- Principle:
  - A known pulse is transmitted
  - Reflections from targets are received
  - Target's ranges and velocities are identified



- Challenges:
  - Targets can lie on an arbitrary grid
  - Process of digitizing
    → loss of resolution in range-velocity domain
  - Wideband radar requires high rate sampling and processing which also results in long processing time



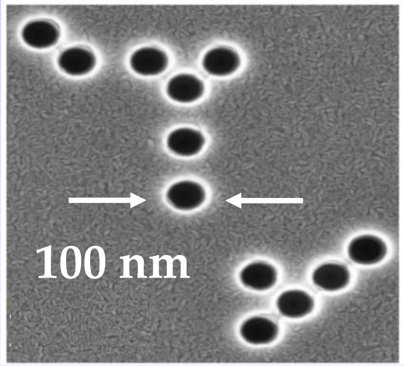


Resolution (2): Subwavelength Imaging

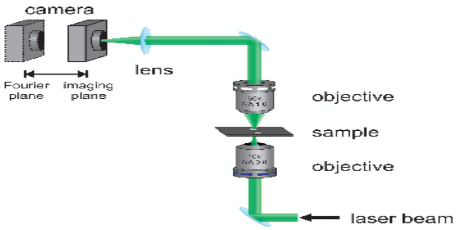
(Collaboration with the groups of Segev and Cohen)

Diffraction limit: Even a perfect optical imaging system has a resolution limit determined by the wavelength λ

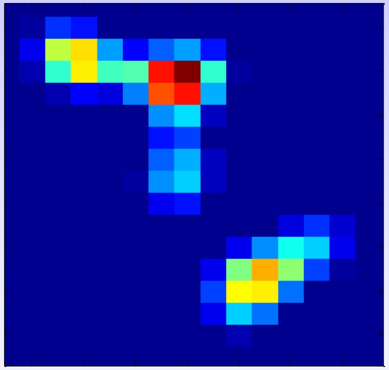
- The smallest observable detail is larger than ~ $\lambda/2$
- This results in image smearing



Nano-holes as seen in electronic microscope



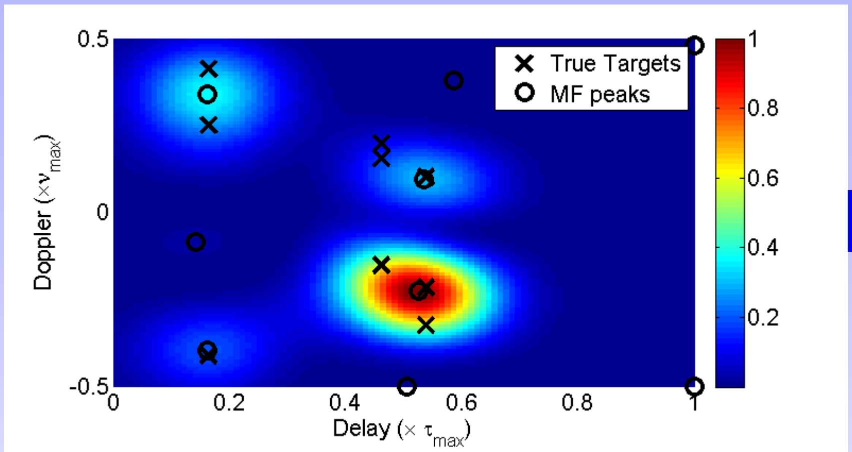
Sketch of an optical microscope: the physics of EM waves acts as an ideal low-pass filter

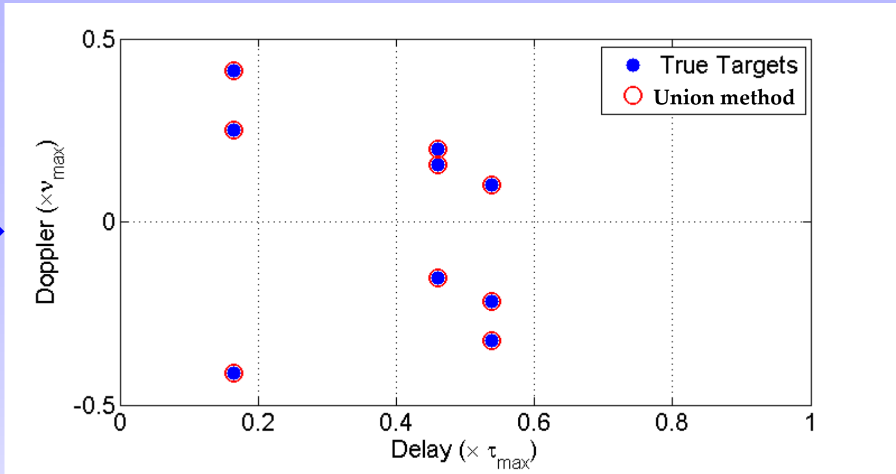


Blurred image seen in optical microscope

Imaging via "Sparse" Modeling

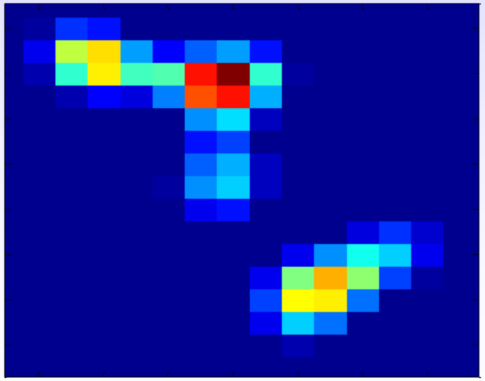
- Radar:

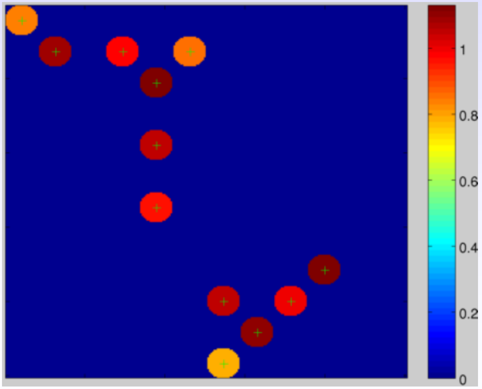




Bajwa *et al.*, '11

- Subwavelength Coherent Diffractive Imaging:





Recovery of sub-wavelength images from highly truncated Fourier power spectrum

Szameit *et al.*, Nature Photonics, '12

Proposed Framework

- Instead of a single subspace modeling use **union of subspaces** framework

- Adopt a new design methodology – **Xampling**
  - Compression+Sampling = Xampling
  - X prefix for compression, e.g. DivX

- Results in simple hardware and low computational cost on the DSP

Union + Xampling = Practical Low Rate Sampling

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- Xampling: Compression + sampling
- Sub-Nyquist solutions
  - Multiband communication: Cognitive radio
  - Time delay estimation: Ultrasound, radar, multipath medium identification
- Applications to digital processing

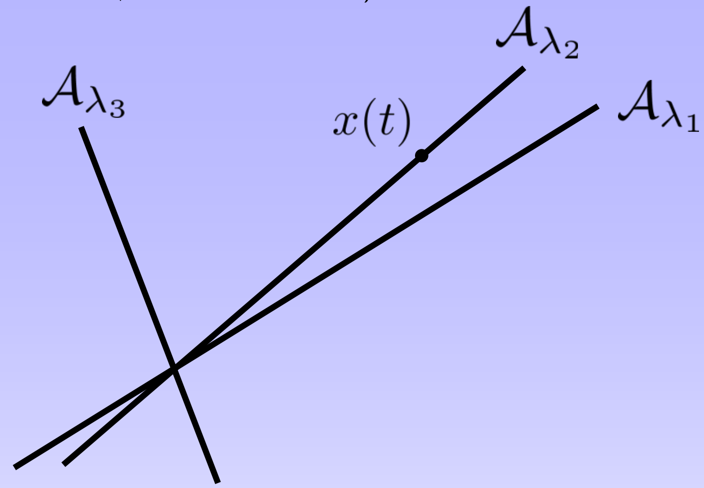
Union of Subspaces

(Lu and Do 08, Eldar and Mishali 09)

- Model: $\mathcal{U} = \bigcup_{\lambda \in \Lambda} \mathcal{A}_\lambda$

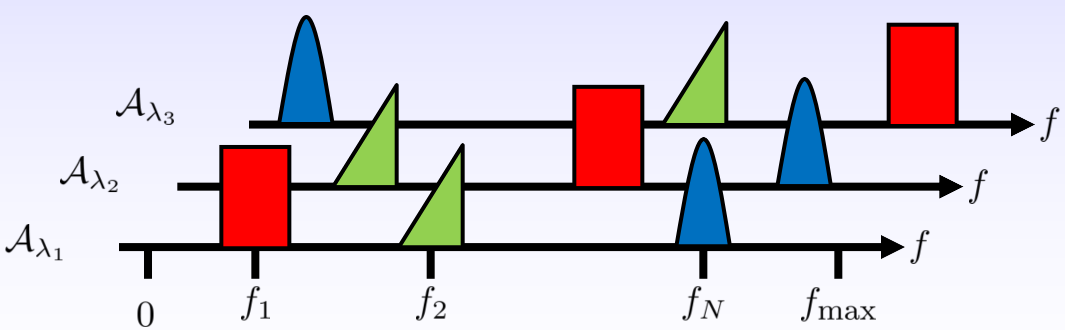
  $x(t) \in \mathcal{A}_{\lambda^*} \rightarrow \lambda^*$ is unknown a-priori

  Each $\mathcal{A}_\lambda$ has low dimension



- Examples:

Multiband communication

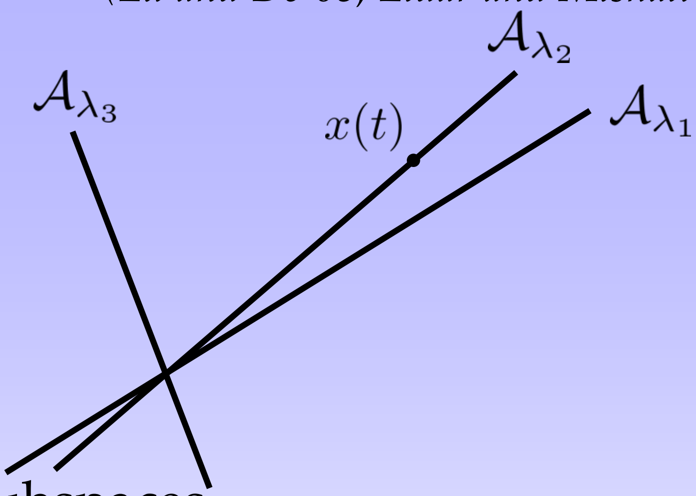


Union over possible band positions $f_i \in [0, f_{\max}]$

Union of Subspaces

(Lu and Do 08, Eldar and Mishali 09)

- Model: $\mathcal{U} = \bigcup_{\lambda \in \Lambda} \mathcal{A}_\lambda$

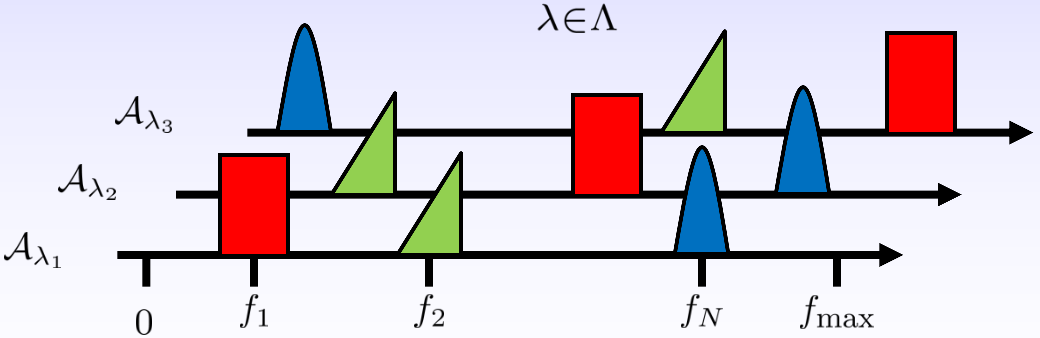


$x(t) \in \mathcal{A}_{\lambda^*}$ $\rightarrow$ λ^* is unknown a-priori

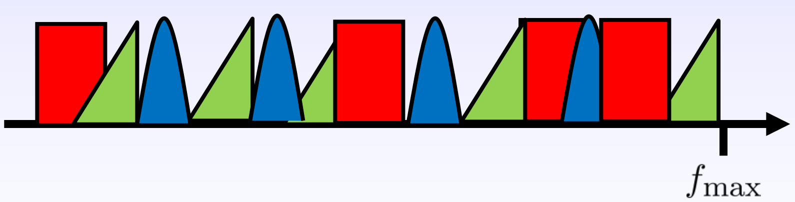
Each $\mathcal{A}_\lambda$ has low dimension

- Standard approach: Look at sum of all subspaces

$$\mathcal{U} = \bigcup_{\lambda \in \Lambda} \mathcal{A}_\lambda$$



$$\mathcal{U} = \oplus_{\lambda \in \Lambda} \mathcal{A}_\lambda$$



Signal bandlimited to $f_{\max}$

$\longrightarrow$ High rate

Union of Subspaces

(Lu and Do 08, Eldar and Mishali 09)

- Model: $\mathcal{U} = \bigcup_{\lambda \in \Lambda} \mathcal{A}_\lambda$

  $x(t) \in \mathcal{A}_{\lambda^*}$ $\rightarrow \lambda^*$ is unknown a-priori

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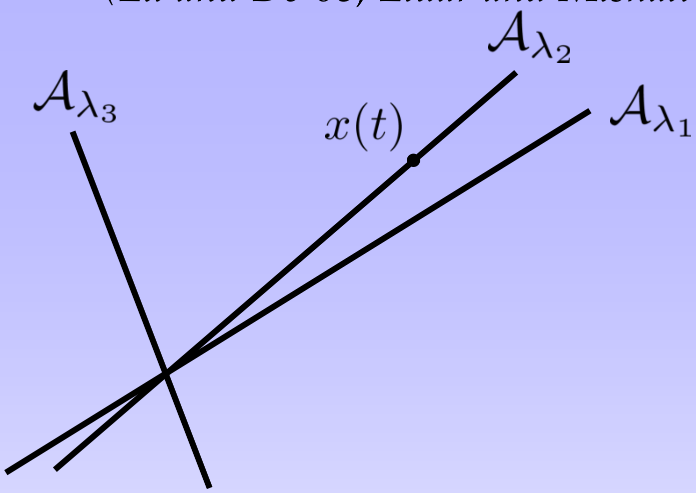
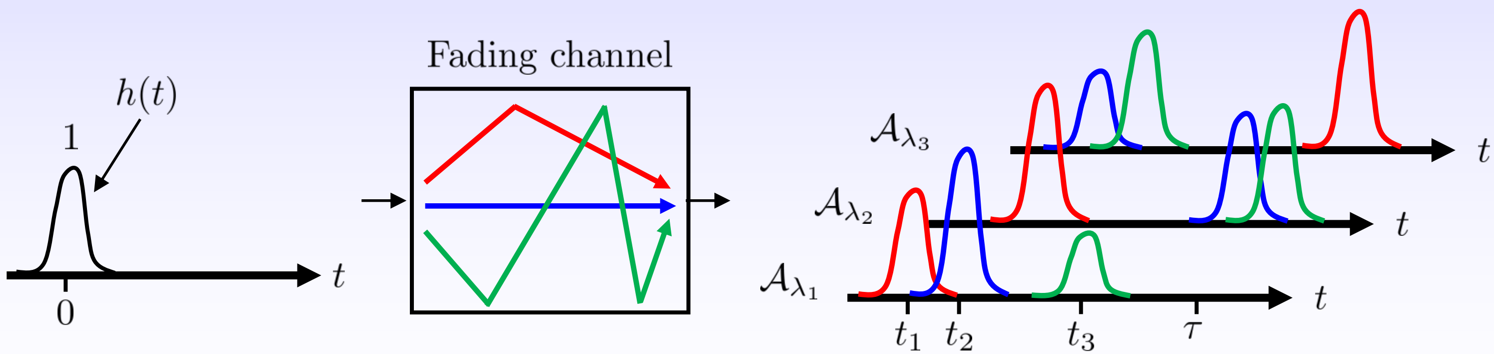


- Examples:

Time-delay estimation



Union over possible path delays $t_i \in [0, \tau]$

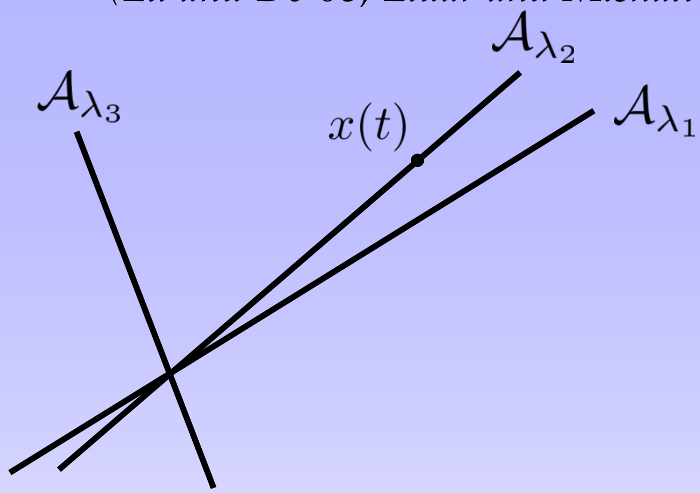
Union of Subspaces

(Lu and Do 08, Eldar and Mishali 09)

- Model: $\mathcal{U} = \bigcup_{\lambda \in \Lambda} \mathcal{A}_\lambda$

  $x(t) \in \mathcal{A}_{\lambda^*} \rightarrow \lambda^*$ is unknown a-priori

  Each $\mathcal{A}_\lambda$ has low dimension



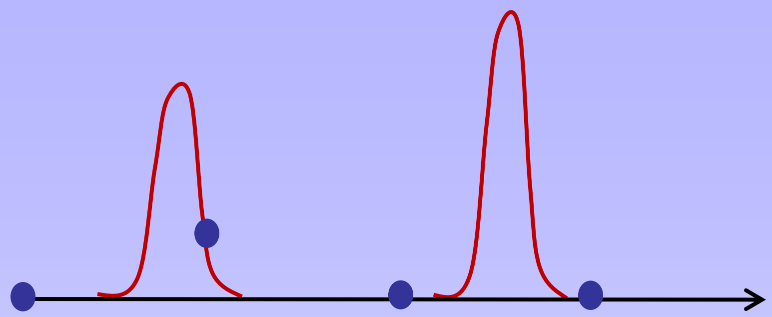
- Allows to keep low dimension in the problem model
- Low dimension translates to low sampling rate

Talk Outline

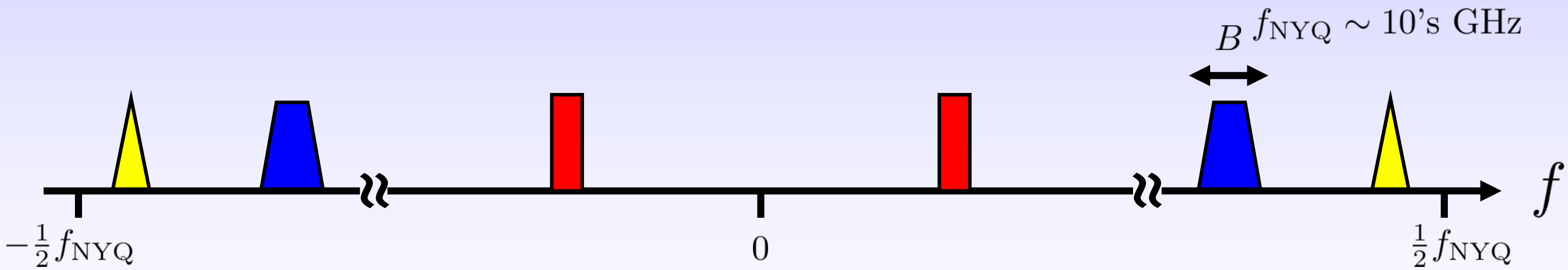
- Motivation
- Classes of structured analog signals
- Xampling: Compression + sampling
- Sub-Nyquist solutions
  - Multiband communication: Cognitive radio
  - Time delay estimation: Ultrasound, radar, multipath medium identification
- Applications to digital processing

Difficulty

- Naïve attempt: direct sampling at low rate
- Most samples do not contain information!!

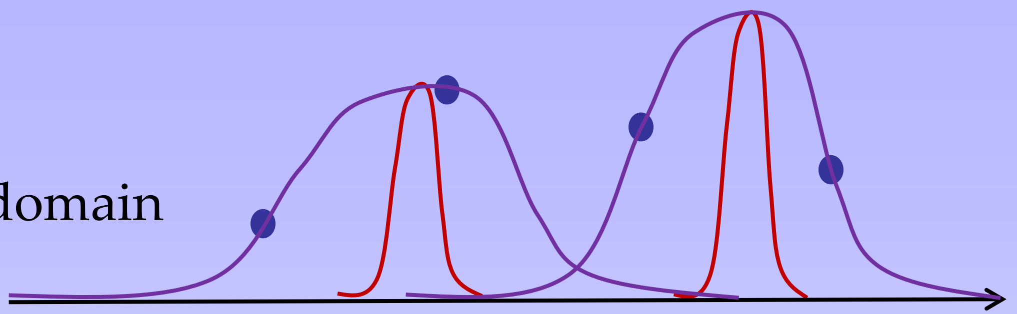


- Most bands do not have energy – which band should be sampled?

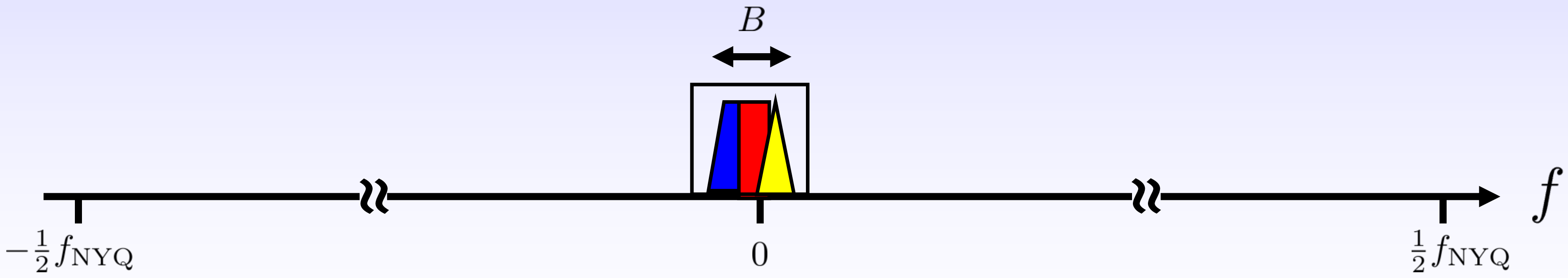


Intuitive Solution: Pre-Processing

- Smear pulse before sampling
- Each samples contains energy
- Resolve ambiguity in the digital domain



- Alias all energy to baseband
- Can sample at low rate
- Resolve ambiguity in the digital domain



Xampling: Main Idea

- Create several streams of data
- Each stream is sampled at a low rate
  (overall rate much smaller than the Nyquist rate)
- Each stream contains a combination from different subspaces

Hardware design ideas

- Identify subspaces involved
- Recover using standard sampling results

DSP algorithms

Subspace Identification

For linear methods:

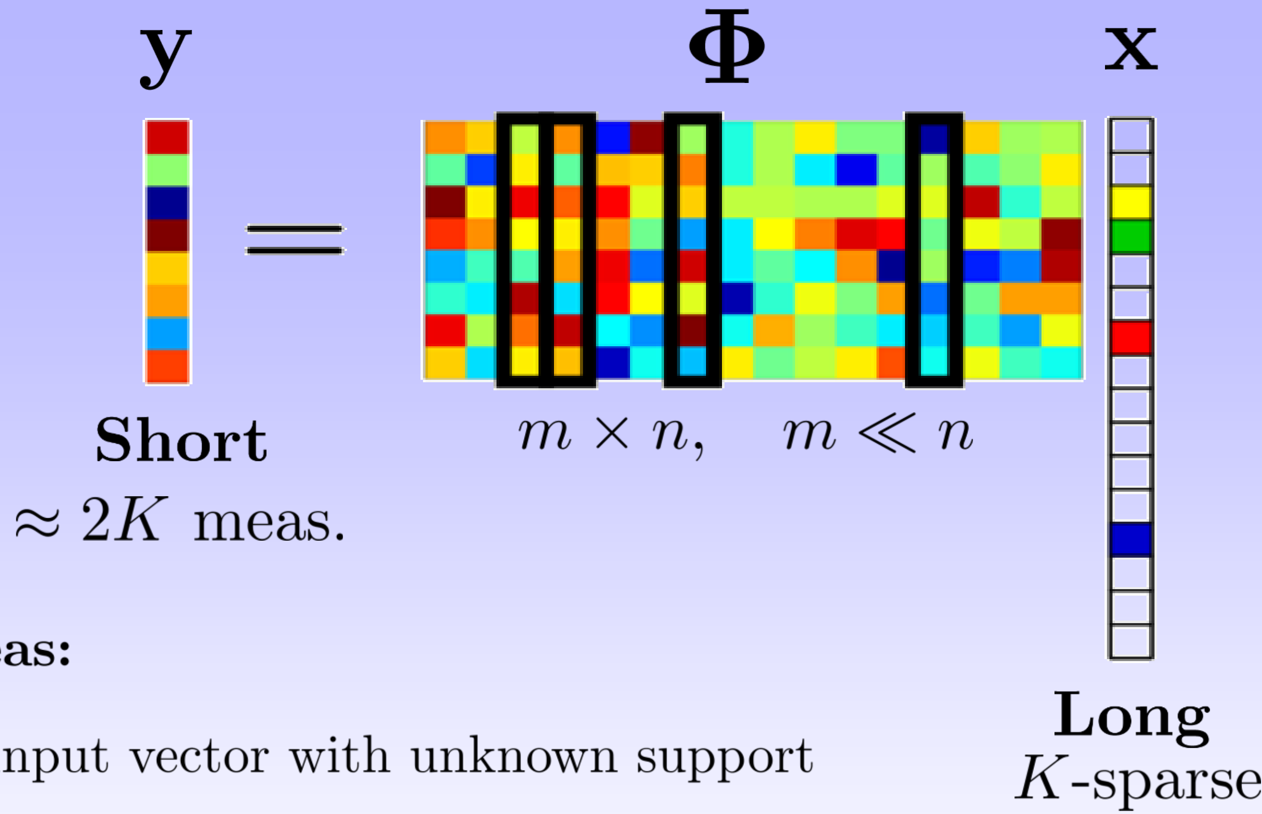
- Subspace techniques developed in the context of array processing (such as MUSIC, ESPRIT etc.)
- Compressed sensing

(Deborah and Noam's talks this afternoon)

For nonlinear sampling:

- Specialized iterative algorithms (Tomer's talk this afternoon)

Compressed Sensing



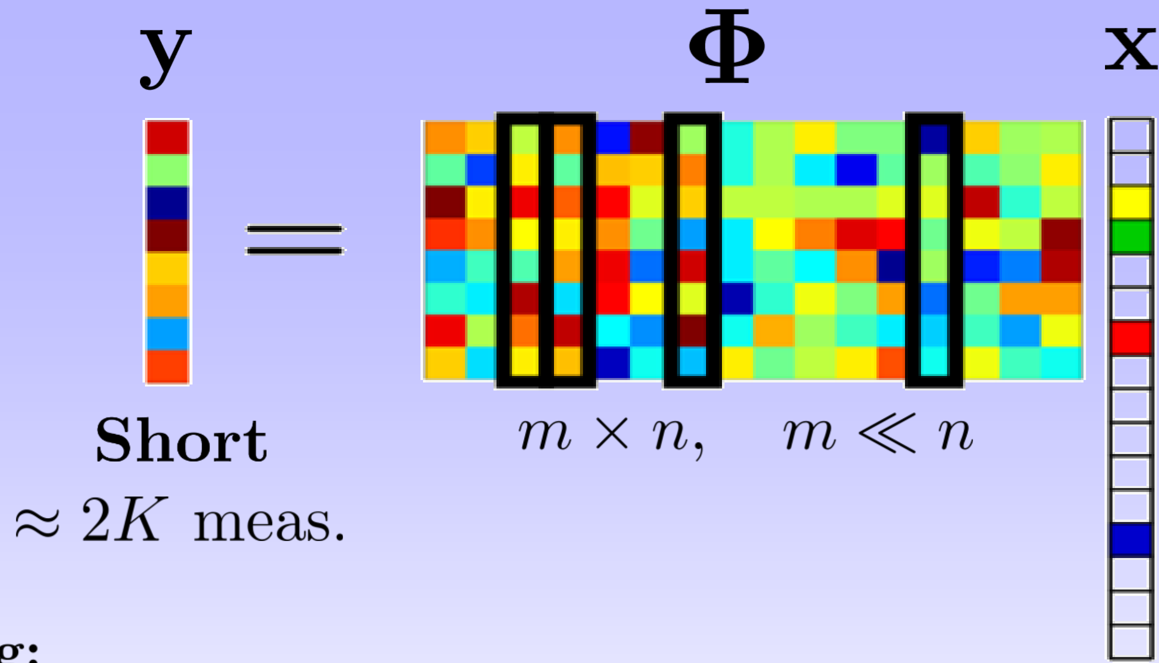
Main ideas:

- Sparse input vector with unknown support
- Sensing by sufficiently incoherent matrix (semi-random)
- Polynomial-time recovery algorithms

(*Candès, Romberg, Tao 2006*)

(*Donoho 2006*)

Compressed Sensing



Xampling:

- Sparsity of $\boldsymbol{x}$ represents that only a few subspaces participate
- The matrix $\boldsymbol{\Phi}$ represents the aliasing of the hardware
- Support detection is equivalent to subspace detection

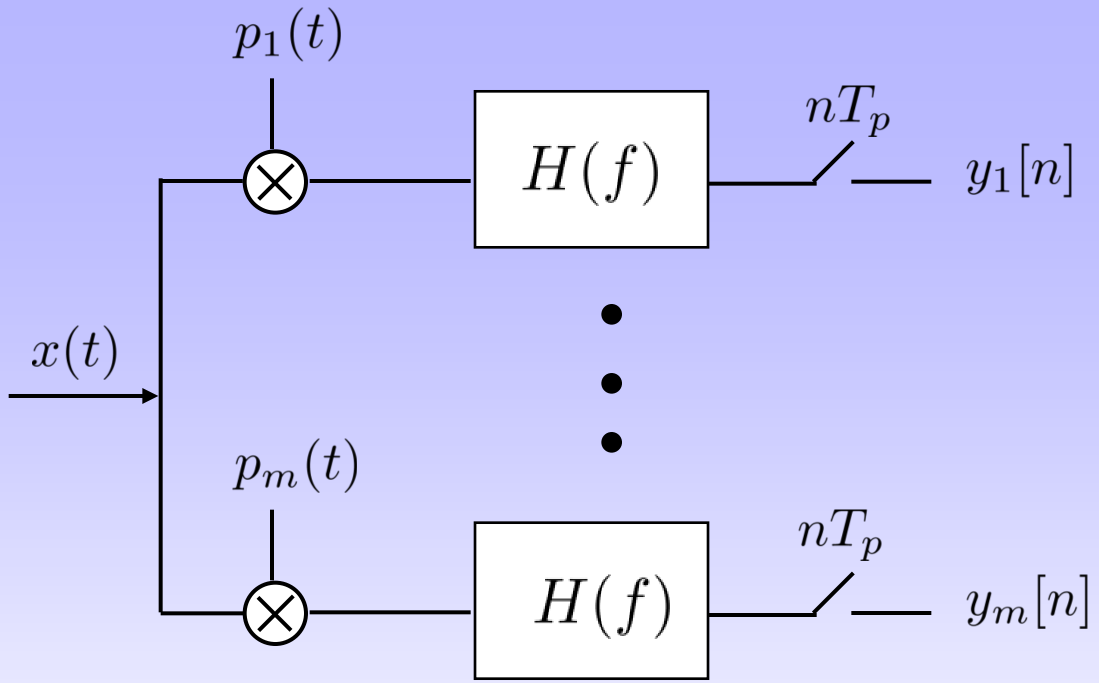
Compressed Sensing and Hardware

- Explosion of work on compressed sensing in many digital applications
- Many papers describing models for CS of analog signals
- None of these models have made it into hardware
- CS is a digital theory – treats vectors not analog inputs

	Standard CS	Analog CS
Input	vector x	analog signal $x(t)$
Sparsity	few nonzero values	?
Measurement	random matrix	real hardware
Recovery	convex optimization greedy methods	need to recover analog input

We use CS only after sampling and only to detect the subspace
Enables real hardware and low processing rates

Xampling Hardware



- $p_i(t)$ - periodic functions
- $p_i(t) = \sum a_{in} e^{-j\frac{2\pi}{T_p}nt}$ sums of exponentials
- The filter *H(f)* allows for additional freedom in shaping the tones
- The channels can be collapsed to a single channel

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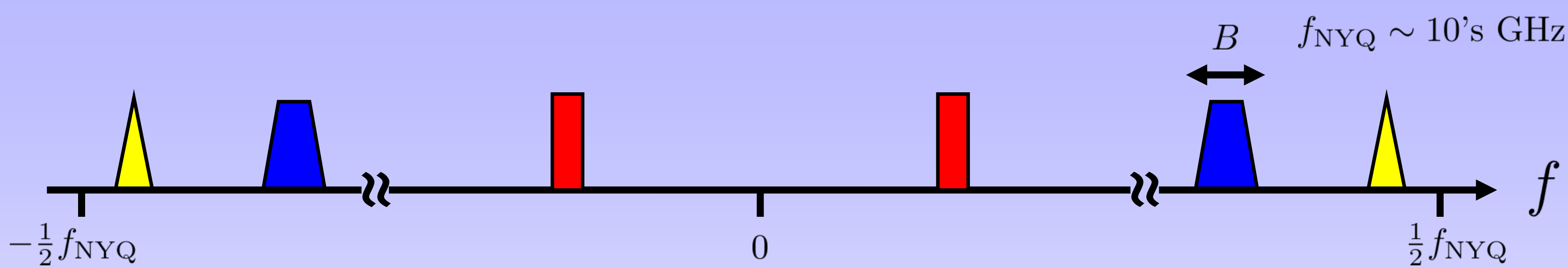
Signal Model

(Mishali and Eldar, 2007-2009)



1. Each band has an uncountable number of non-zero elements
2. Band locations lie on the continuum
3. Band locations are unknown in advance

$\mathcal{M} = \{\, x(t) \mid$ no more than N bands, max width B, bandlimited to $[-\frac{1}{2}f_{\mathrm{NYQ}}, +\frac{1}{2}f_{\mathrm{NYQ}})\}$

Rate Requirement

Theorem (Single multiband subspace)

Let R be a sampling set for $\mathcal{B}_{\mathcal{F}} = \{x(t) \in L^2(\mathbb{R}) \mid \operatorname{supp} X(f) \subseteq \mathcal{F}\}$. Then,

$$\underbrace{D^-(R)}_{\text{Average sampling rate}} \geq \lambda = |\mathcal{F}|$$

(Landau 1967)

Theorem (Union of multiband subspaces)

Let R be a sampling set for $\mathcal{N}_\lambda = \bigcup_{|\mathcal{F}| \leq \lambda} \mathcal{B}_{\mathcal{F}}$.

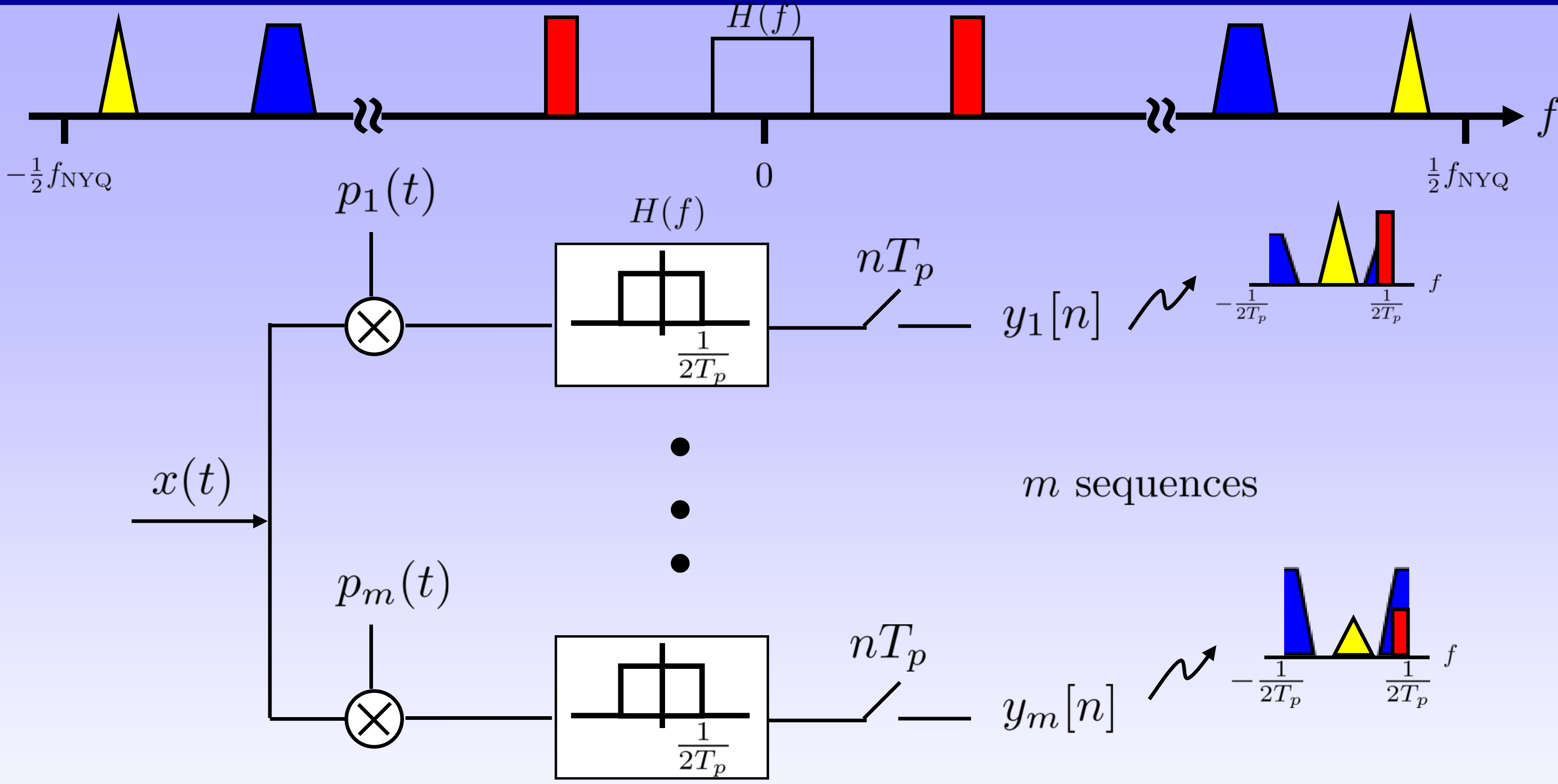
Then,

$$D^-(R) \geq \min\{2\lambda, f_{\mathrm{NYQ}}\}$$

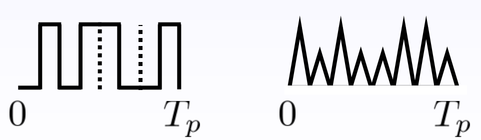
(Mishali and Eldar 2007)

1. The minimal rate is doubled.
2. For $x(t) \in \mathcal{M}$, the rate requirement is $2NB$ samples/sec (on average).

The Modulated Wideband Converter

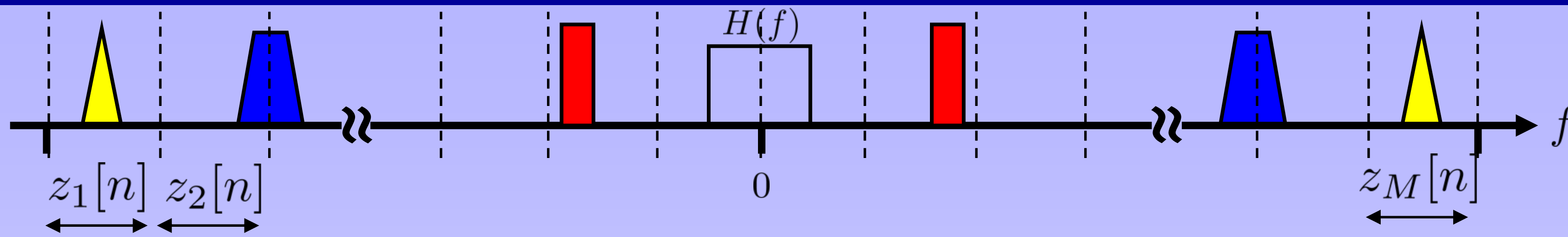


T_p–periodic $p_i(t)$ gives the desired aliasing effect



and many more...

Recovery From Xamples

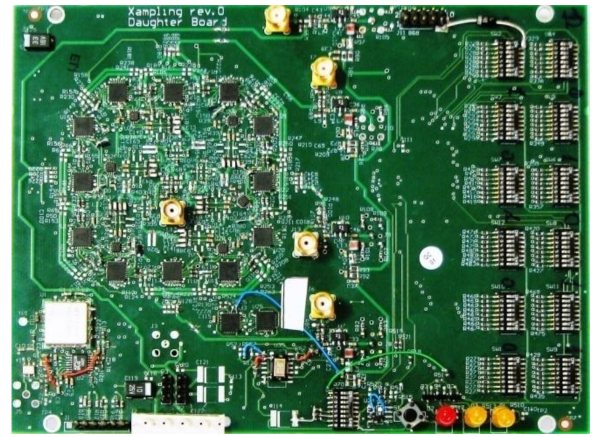


$$\begin{pmatrix} y_1[n] \\ \vdots \\ y_m[n] \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{il} = c_{il} \\ \\ m \times M \end{pmatrix} \begin{pmatrix} z_1[n] \\ z_2[n] \\ \vdots \\ \\ z_M[n] \end{pmatrix}$$

- Using ideas of compressed sensing
- Modifications to allow for real time computations and noise robustness
- Cleverly combine data across samples to improve support detection
- Details in Deborah's talk this afternoon

A 2.4 GHz Prototype

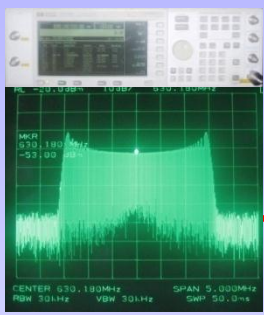
(Mishali, Eldar, Dounaevsky, and Shoshan, 2010)



- 2.3 GHz Nyquist-rate, 120 MHz occupancy
- 280 MHz sampling rate
- Wideband receiver mode:
  - 49 dB dynamic range
  - SNDR > 30 dB over all input range
- ADC mode:
  - 1.2 volt peak-to-peak full-scale
  - 42 dB SNDR = 6.7 ENOB
- Off-the-shelf devices, ~5k$, standard PCB production

Sub-Nyquist Demonstration

Carrier frequencies are chosen to create overlayed aliasing at baeband

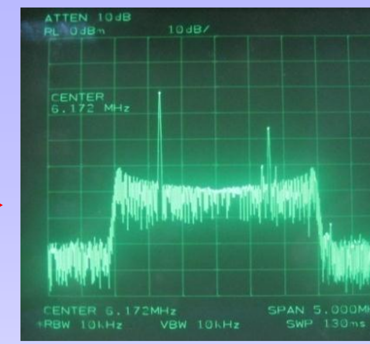


FM @ 631.2 MHz

AM @ 807.8 MHz

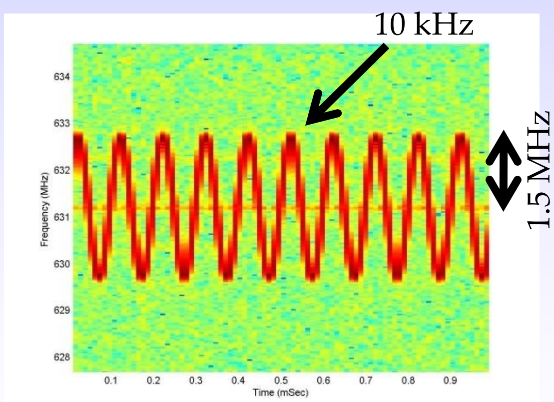
Sine @ 981.9 MHz

MWC prototype

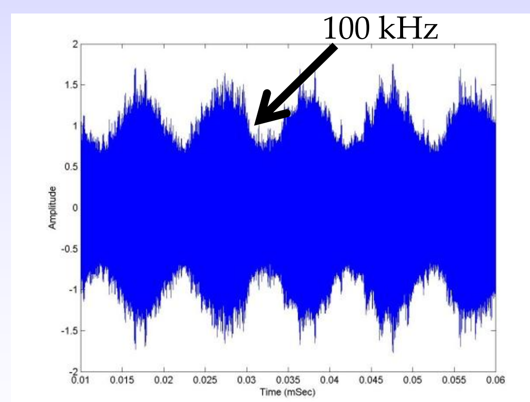


aliasing around 6.171 MHz

Reconstruction
(CTF)



FM @ 631.2 MHz

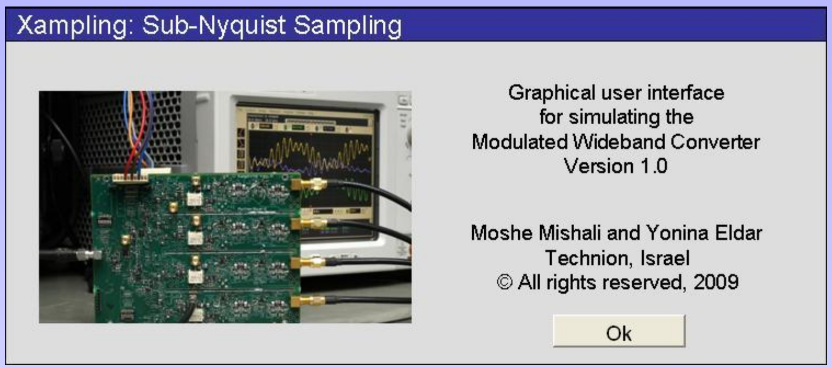


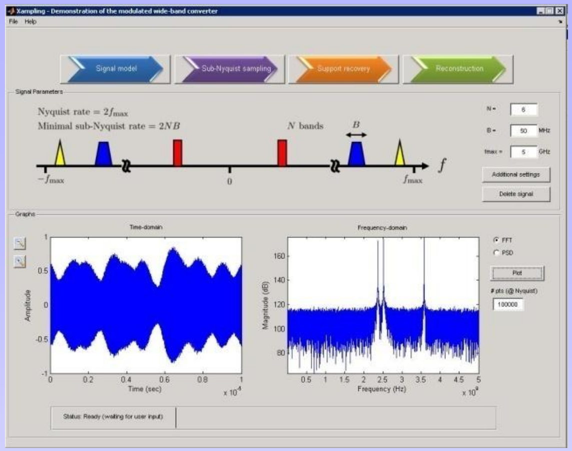
AM @ 807.8 MHz

Mishali et al., 10

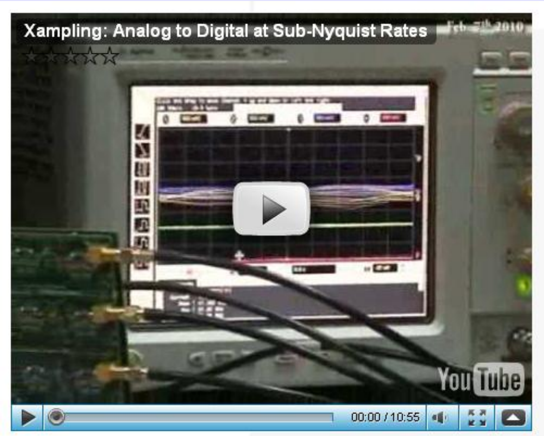
Online Demonstrations

- GUI package of the MWC



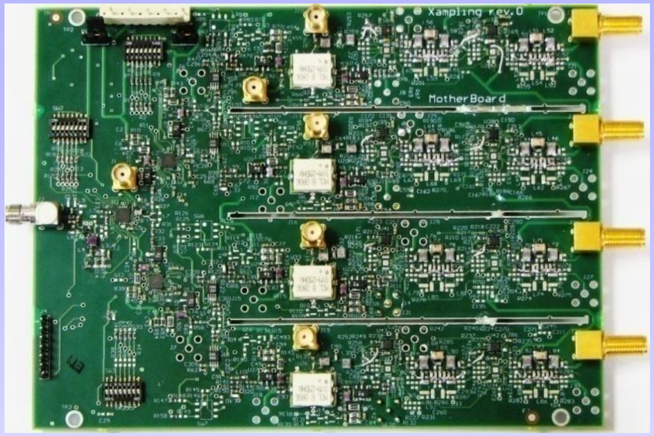


- Video recording of sub-Nyquist sampling + carrier recovery in lab



Demos – Supported By NI

Demo this afternoon by Rolf and Idan



2010 ICASSP Dallas

IEEE Signal Processing Society

2010 IEEE International Conference on Acoustics, Speech and Signal Processing

March 14 - 19, 2010 • Sheraton Dallas Hotel • Dallas, Texas, U.S.A.

ISCAS 2011

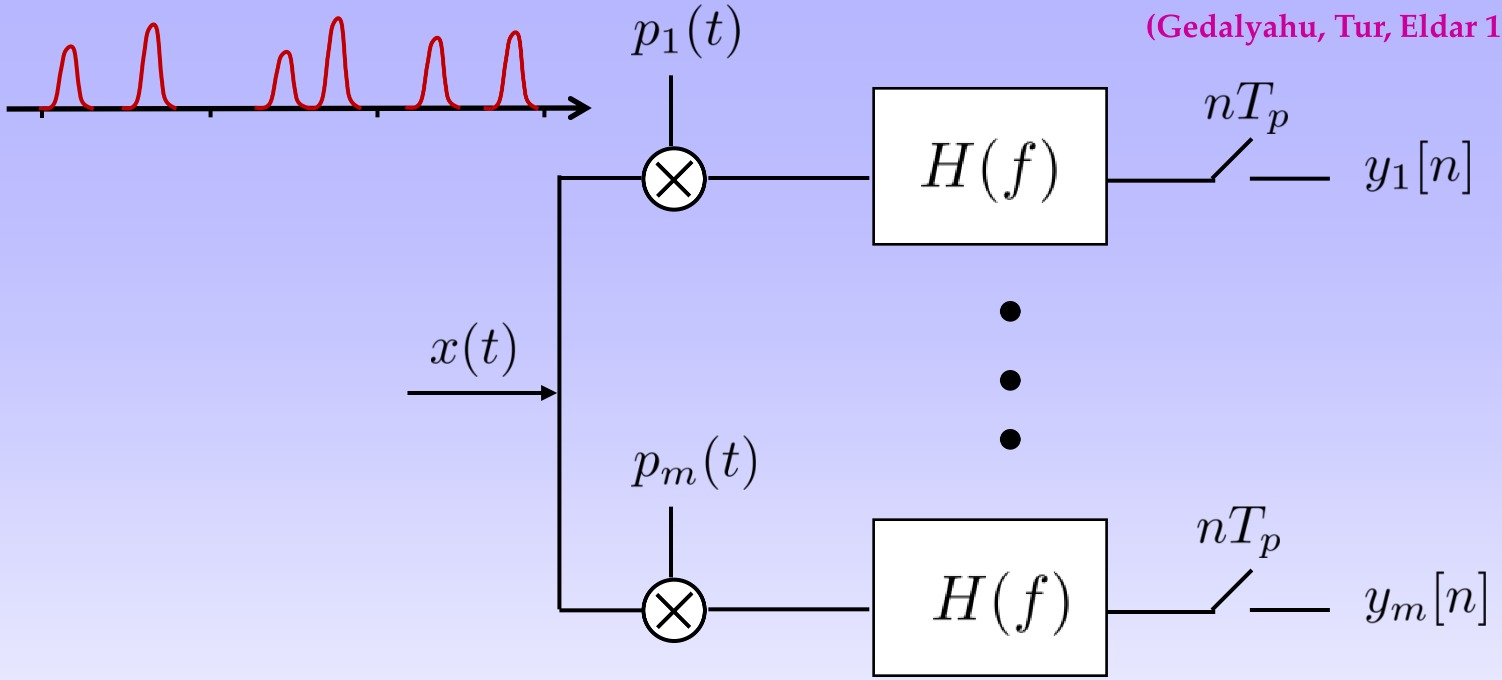
Talk Outline

- Brief overview of standard sampling
- Classes of structured analog signals
- Xampling: Compression + sampling
- Sub-Nyquist solutions
  - Multiband communication
  - Time delay estimation:

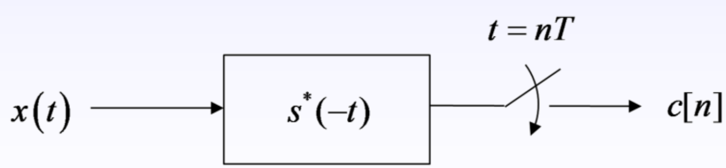
    Ultrasound, radar, multipath medium identification
- Applications to digital processing

Streams of Pulses

(Gedalyahu, Tur, Eldar 10, Tur, Freidman, Eldar 10)



- $H(f)$ is replaced by an integrator
- Can equivalently be implemented as a single channel with $T = T_p/m$

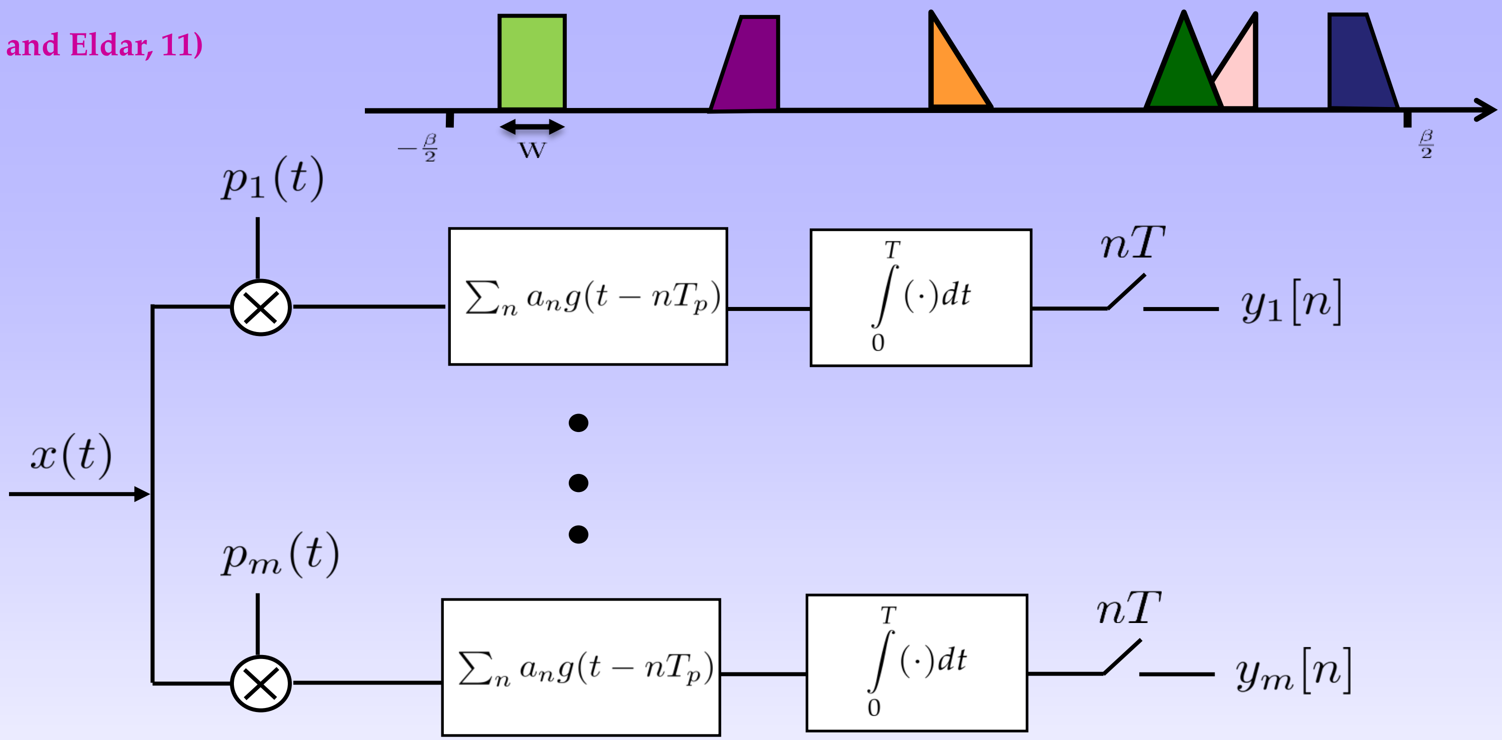


$$s(t) = \sum_n b_n e^{j\frac{2\pi}{T_p}nt} \mathrm{rect}(t/T_p)$$

- Application to radar, ultrasound and general localization problems such as GPS

Unknown Pulses

(Matusiak and Eldar, 11)



- Output corresponds to aliased version of Gabor coefficients
- Recovery by solving 2-step CS problem $Y = AZB^T$

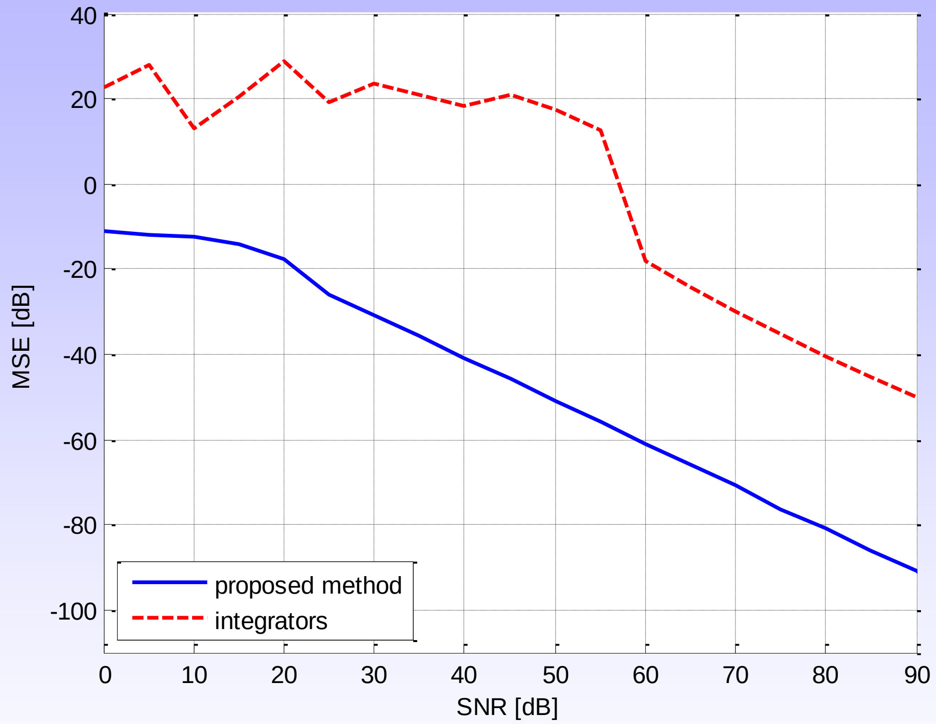
Row-sparse Gabor Coeff.

1. Solve $Y = AC$ with $C = ZB^T \Rightarrow$ Since Z is row-sparse C is row-sparse
2. Solve CS problem $C^T = BZ$ where Z is row sparse

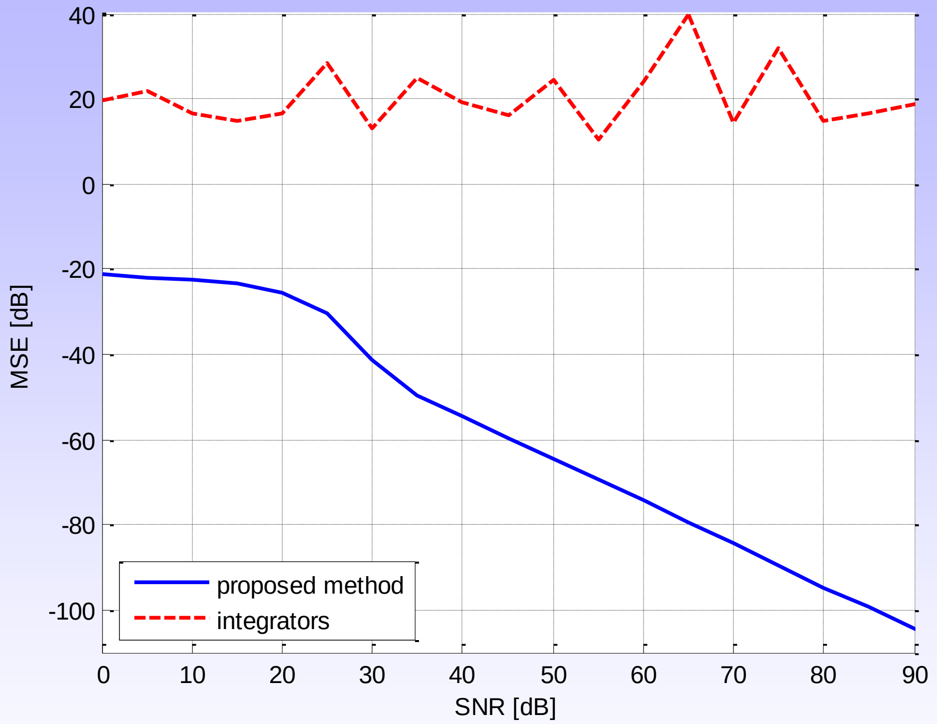
Noise Robustness

- MSE of the delays estimation, versus integrators approach *(Kusuma & Goyal)*

L=2 pulses, 5 samples



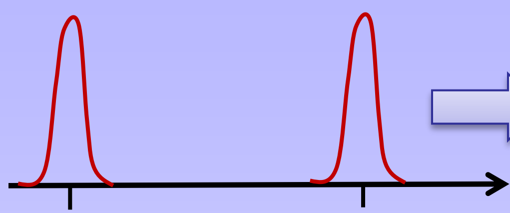
L=10 pulses, 21 samples



The proposed scheme is stable even for high rates of innovation!

Application: Multipath Medium Identification

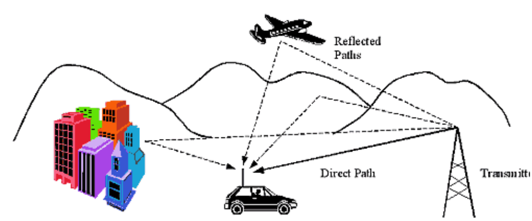
(Gedalyahu and Eldar 09-10)

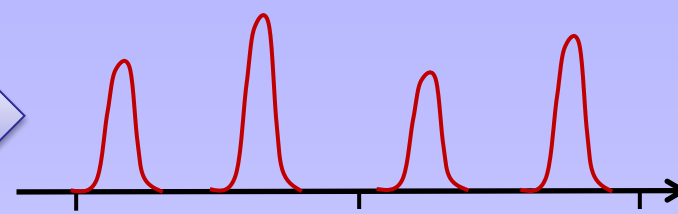


$$x_T(t) = \sum_{n \in \mathbb{Z}} h(t - nT)$$

LTV channel

L propagation paths





$$x_R(t) = \sum_{l \in \mathbb{Z}} a_l h(t - t_l)$$

L pulses per period T

- Medium identification (collaboration with National Instruments):
  - Recovery of the time delays
  - Recovery of time-variant gain coefficients

The proposed method can recover the channel parameters from sub-Nyquist samples

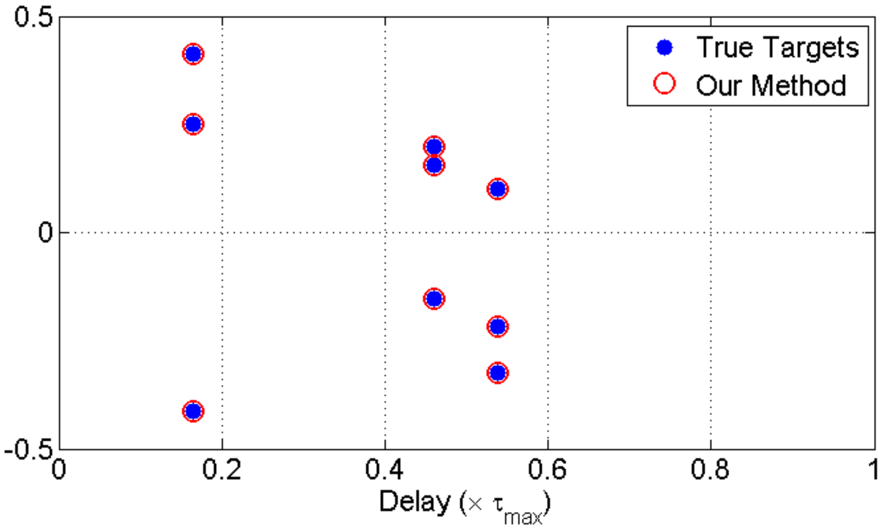
Application: Wireless Communications

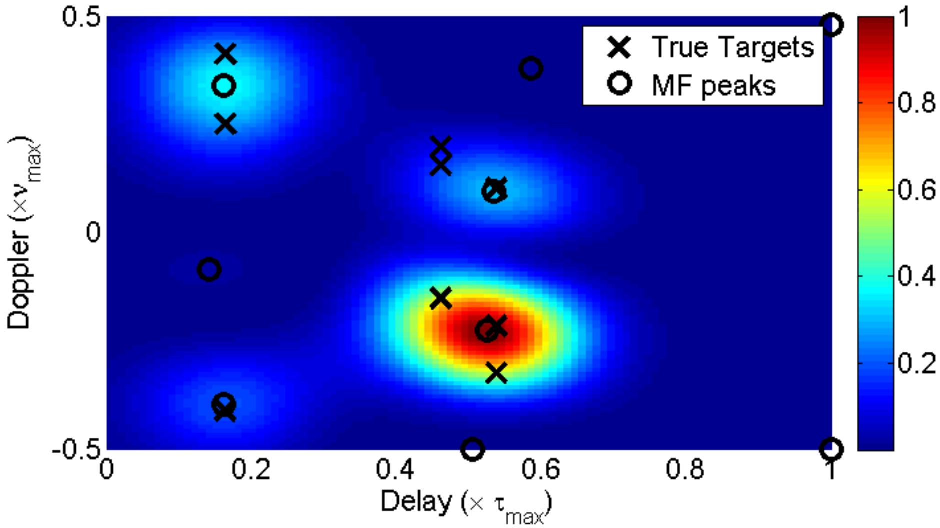
- New paradigm for wireless communications: Joint effort with Prof. Andrea Goldsmith from Stanford (Transformative Science Grant)
  - Main bottleneck today in wireless are ADCs
  - Multiuser detection, which enables many users to share joint resources, is not implemented because of high rates – channels are interference limited
  - SDR and Cognitive radio are limited by ADCs
  - Capacity tools are limited to Nyquist-rate channels
- New multiuser receiver that substantially reduces hardware requirements
- Capacity expressions for sampling-rate limited channels
- Applications to LTE standards (with Prof. Murmann and Ericsson)

Application: Radar

(Bajwa, Gedalyahu and Eldar, 10)

- Each target is defined by:
  - Range – delay
  - Velocity – doppler
- Targets can be identified with **infinite resolution** as long as the time-bandwidth product satisfies $\mathcal{TW} \geq 2\pi(K+1)^2$
- Previous results required infinite time-bandwidth product

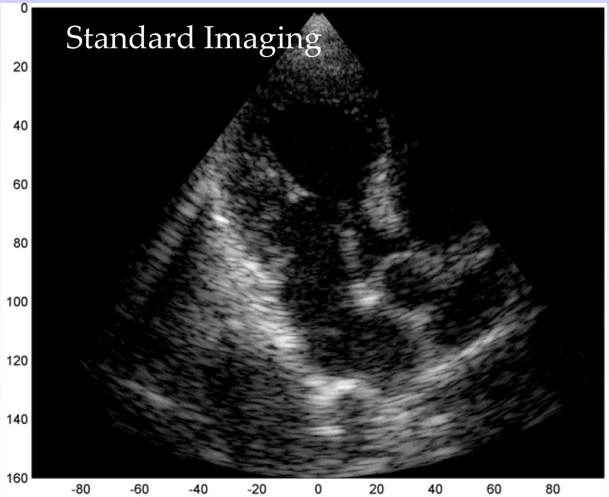




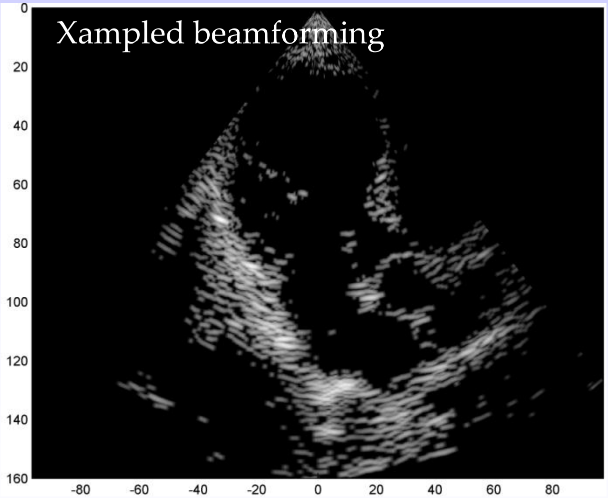
Xampling in Ultrasound Imaging

Wagner, Eldar, and Friedman, ’11

- A scheme which enables reconstruction of a two dimensional image, from samples obtained at a rate 10-15 times below Nyquist
- The resulting image depicts strong perturbations in the tissue
- Obtained by beamforming in the compressed domain
- More details in Noam’s talk



1662 real-valued samples, per sensor per image line

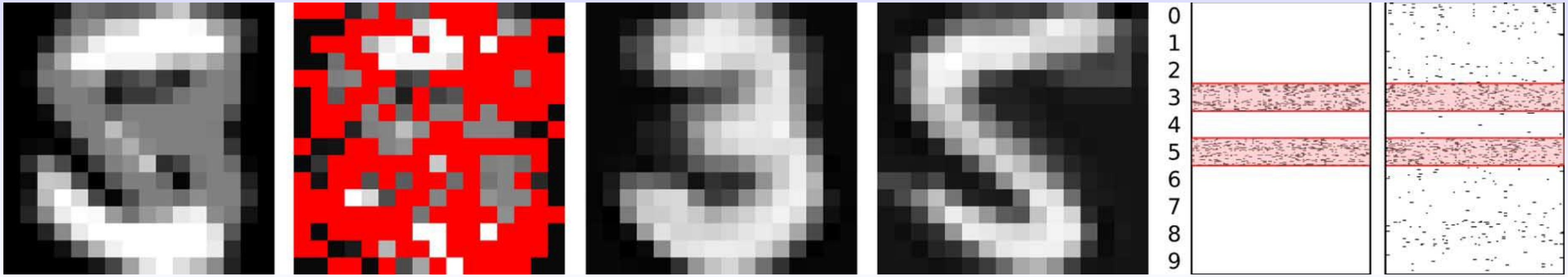


200 real-valued samples, per sensor per image line (assume L=25 reflectors per line)

Structure in Digital Problems

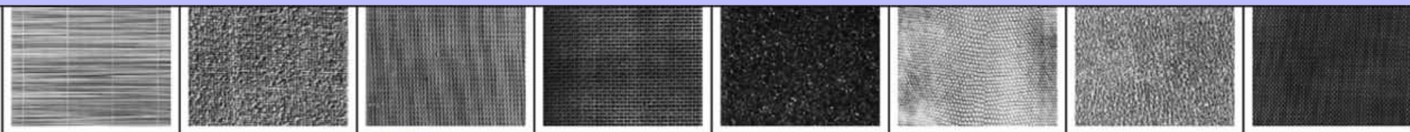
- Union of subspaces structure can be exploited in many digital models
- Subspace models lead to block sparse recovery
- Block sparsity: algorithms and recovery guarantees in noisy environments (Eldar and Mishali 09, Eldar et. al. 10, Ben-Haim and Eldar 10)
- Hierarchical models with structure on the subspace level and within the subspaces (Sprechmann, Ramirez, Sapiro and Eldar, 10)

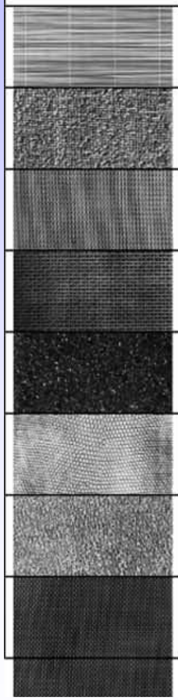
Noisy merged Missing pixels Separated



Source Separation Cont.

Texture Separation:





1			110	214	**18**	074	63	78	19	47	85	174	107	447	7	43
			117	**69**	069	**18**	126	**38**	47	**18**	132	**51**	102	**42**	27	**3**
2	2.80	0.42			107	76	141	129	91	83	191	234	240	219	68	105
	1.36	**0.00**			182	**68**	209	**102**	100	**78**	257	**141**	245	**178**	95	**19**
3	0.33	0.25	3.65	**0.00**			52	**42**	35	62	105	112	162	141	21	93
	2.06	**0.00**	2.67	0.02			158	43	83	**29**	214	**62**	200	**107**	102	**10**
4	0.96	0.01	3.69	0.07	1.74	**0.00**			**49**	72	123	145	182	148	26	89
	1.97	**0.00**	2.30	**0.00**	2.42	**0.00**			81	55	224	**98**	214	**107**	85	**10**
5	1.02	1.00	3.55	1.00	1.42	1.00	2.25	1.00			85	76	120	87	15	63
	2.25	**0.09**	2.52	**0.94**	3.39	**0.16**	2.85	**0.35**			120	**59**	107	**71**	41	**9**
6	2.26	0.32	4.12	**0.53**	3.48	0.44	3.49	0.32	3.16	1.00			229	240	56	95
	2.50	**0.00**	3.23	0.82	3.54	**0.20**	3.11	**0.01**	4.07	**0.40**			245	**162**	117	**27**
7	4.37	1.39	4.47	**0.08**	4.09	0.13	4.23	0.12	4.20	1.00	4.42	0.42			100	112
	2.51	**0.02**	2.39	0.22	2.42	**0.02**	2.76	**0.02**	2.24	**0.20**	2.96	**0.11**			102	**51**
8	0.09	0.98	3.77	1.00	0.31	1.00	1.83	1.00	1.13	1.00	3.14	0.97	4.30	1.00		
	0.53	**0.00**	1.75	**0.01**	2.04	**0.00**	1.82	**0.00**	2.18	**0.00**	3.04	**0.24**	1.90	**0.18**		

Subspace Learning

- Prior work has focused primarily on learning a single subspace (Vidal et. al., Ma et. al., Elhamifar …)
- We developed methods for multiple subspace learning from training data (Rosenblum, Zelnik-Manor and Eldar, 10)
- Subspace learning from reduced-dimensional measured data: Blind compressed sensing (Gleichman and Eldar 10)
- Current work: Extending these ideas to more practical scenarios (Carin, Silva, Chen, Sapiro)

50% Missing Pixels

Interpolation by learning the basis from the corrupted image

Conclusions

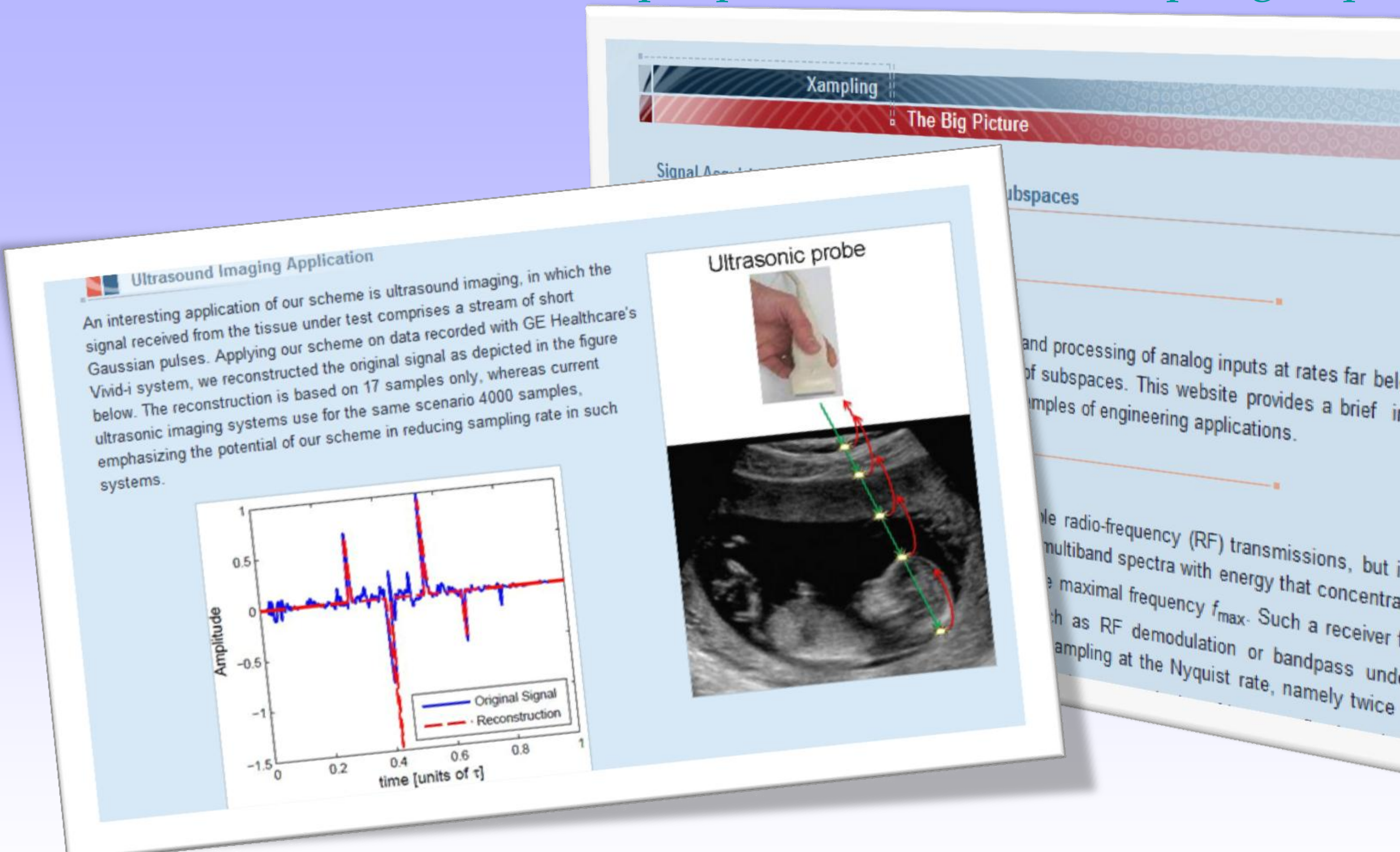
- Compressed sampling and processing of many signals
- Wideband sub-Nyquist samplers in hardware
- Union of subspaces: broad and flexible model
- Practical and efficient hardware
- Many applications and many research opportunities: extensions to other analog and digital problems, robustness, hardware …

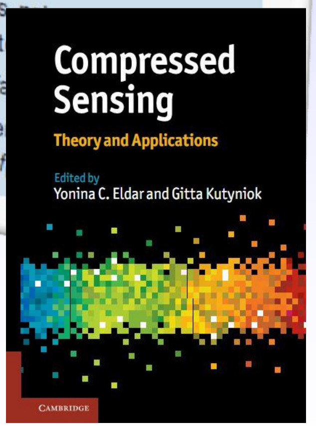
Exploiting structure can lead to a new sampling paradigm which combines analog + digital

More details in:
M. Duarte and Y. C. Eldar, "Structured Compressed Sensing: From Theory to Applications," Review for TSP.
M. Mishali and Y. C. Eldar, "Xampling: Compressed Sensing for Analog Signals", book chapter available at http://webee.technion.ac.il/Sites/People/YoninaEldar/books.html

Xampling Website

webee.technion.ac.il/people/YoninaEldar/xampling_top.html





Y. C. Eldar and G. Kutyniok, "Compressed Sensing: Theory and Applications", Cambridge University Press, to appear in 2012

Thank you