Sub-Nyquist Sampling of Wideband Signals

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Sub-Nyquist Sampling (Xampling) – Smart Sampling Seminar

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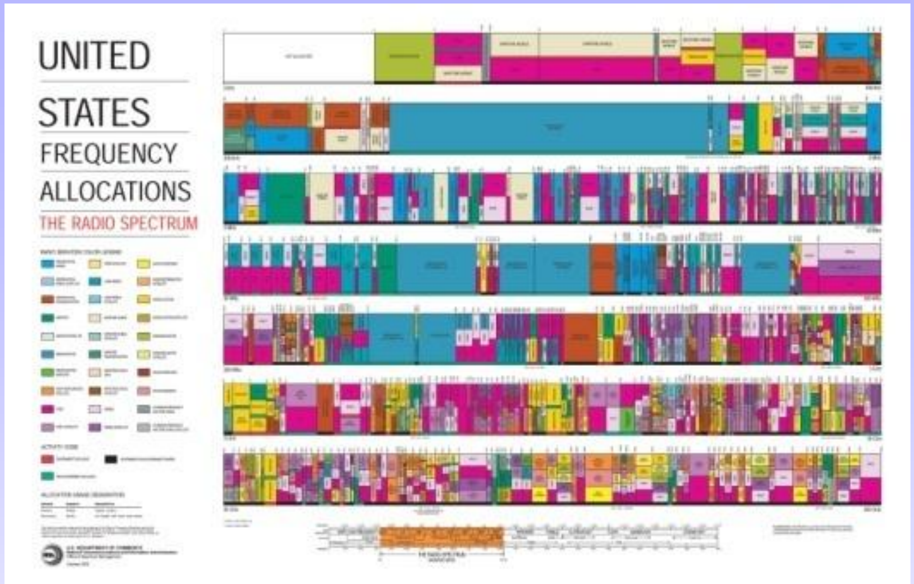
Outline

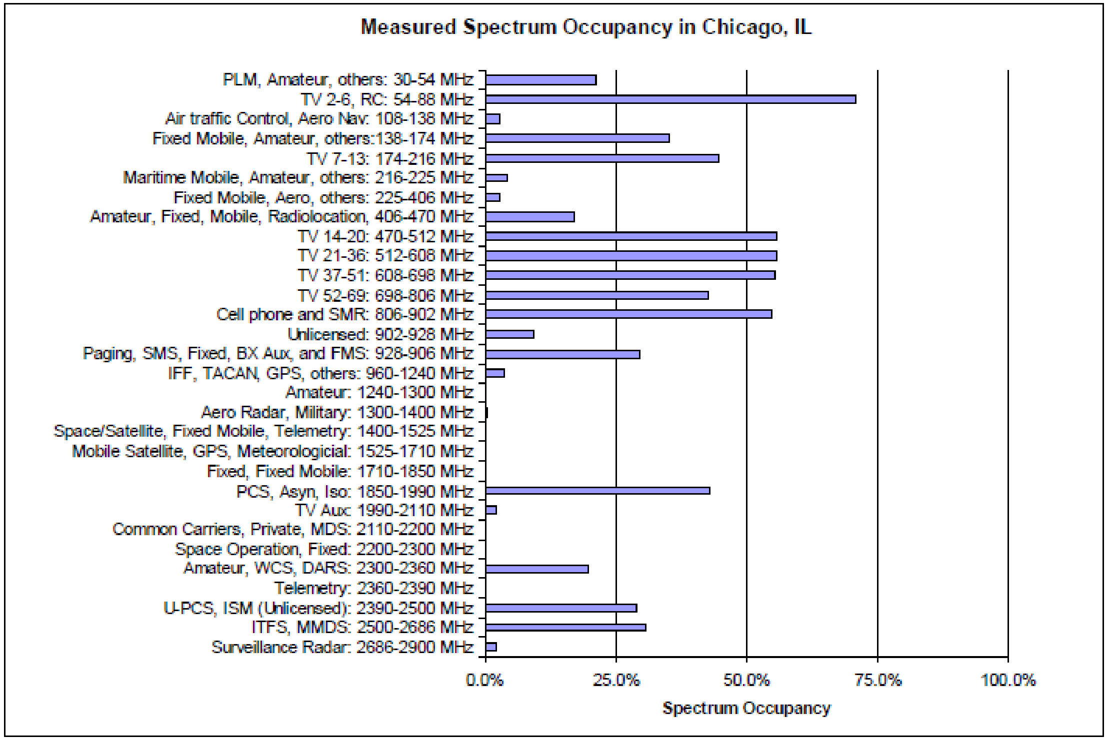
- Motivation
- Algorithms
  - Sampling: MWC and Multicoset
  - Recovery
- Challenges and Trade-Offs
- Treatment of Noise

Spectrum Saturation



- Licensed frequency bands to Primary Users (PUs):
  TV, radio stations, mobile carriers, air traffic control…)
- Spectrum is too crowded
- Cannot allocate frequency bands to new users!

Spectrum Sparsity



Shared Spectrum Company (SSC) – 16-18 Nov 2005

- Spectrum is underutilized
- In a given place, at a given time, only a small number of PUs transmit concurrently

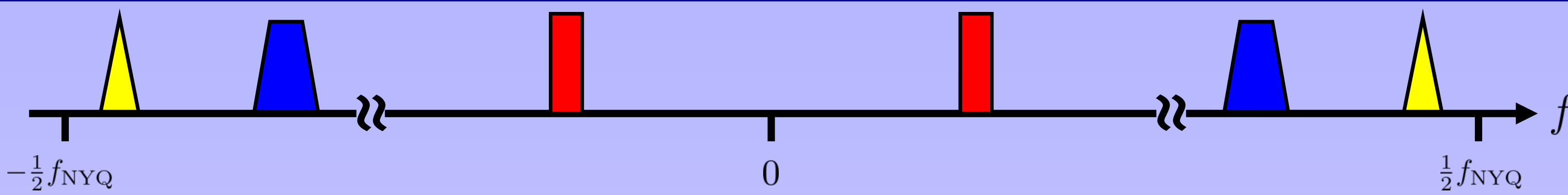
Can we exploit temporarily available spectrum holes for opportunistic transmissions?

Cognitive Radios

- Principle:
  - Perform spectrum sensing to search for available spectrum holes
  - Monitor spectrum during transmission to detect any change in PUs' activity
- Requirements:
  - Wideband spectrum sensing
  - Real-time
  - Reliability
  - Minimal hardware and software resources (mobile)

Nyquist sampling is not an option!
How do we efficiently perform detection on a wideband signal?

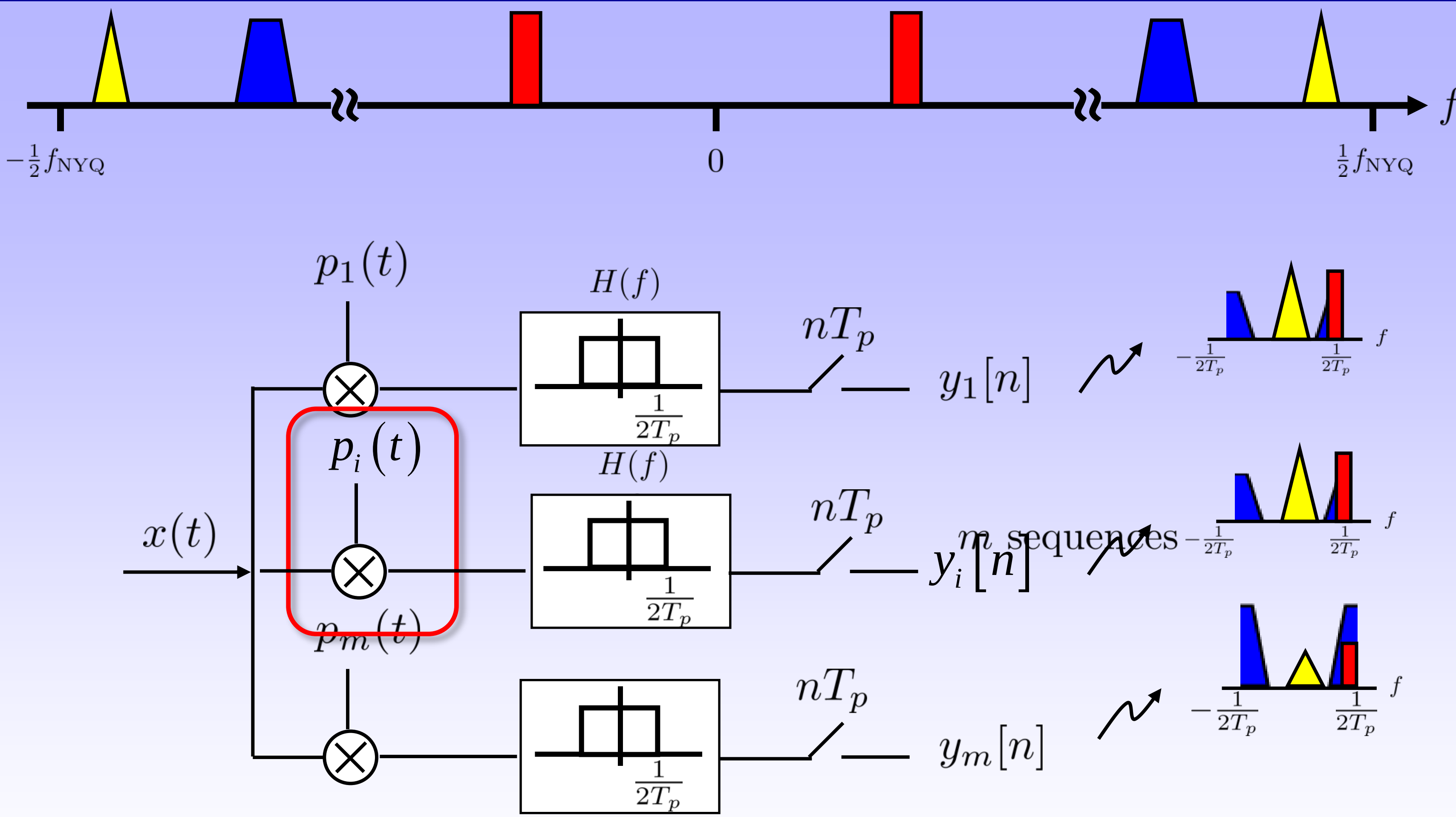
Model



- Multiband model:
  - N – max number of transmissions
  - B – max bandwidth of each transmission
- Goal: blind detection
- Minimal achievable rate: $2NB \ll f_{NYQ}$

The Modulated Wideband Converter (MWC)

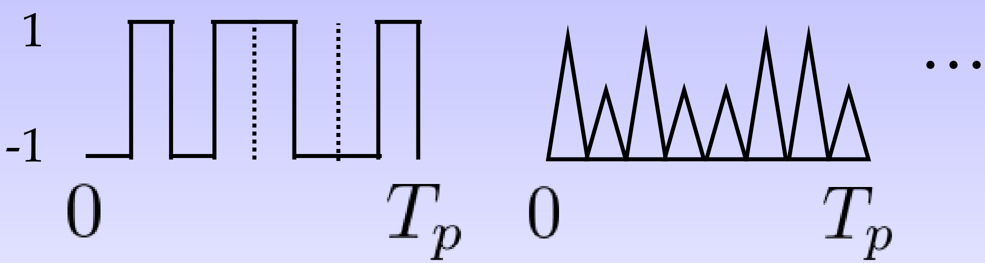
Mishali & Eldar '10

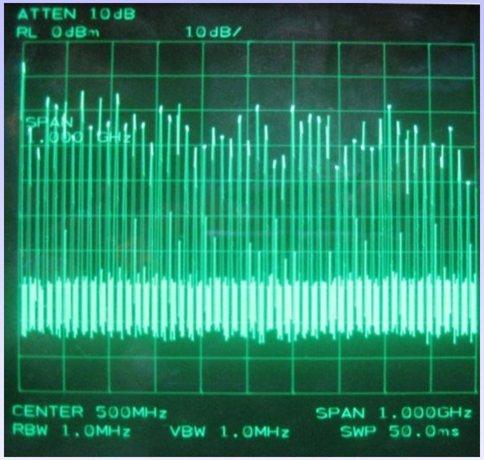


MWC – Mixing & Aliasing

- Mixing function $p_i(t)$ periodic with period T_p

- Examples:

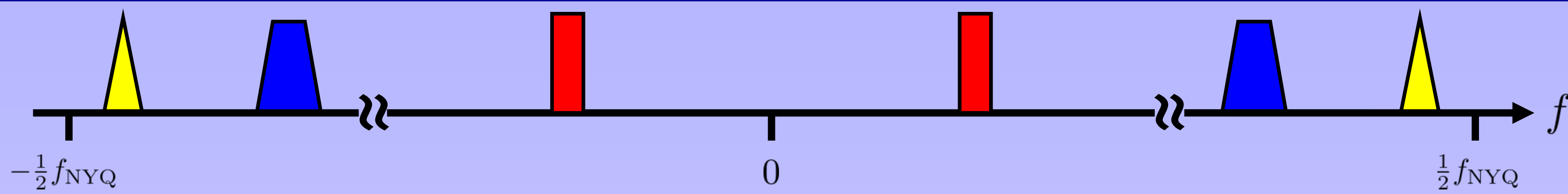


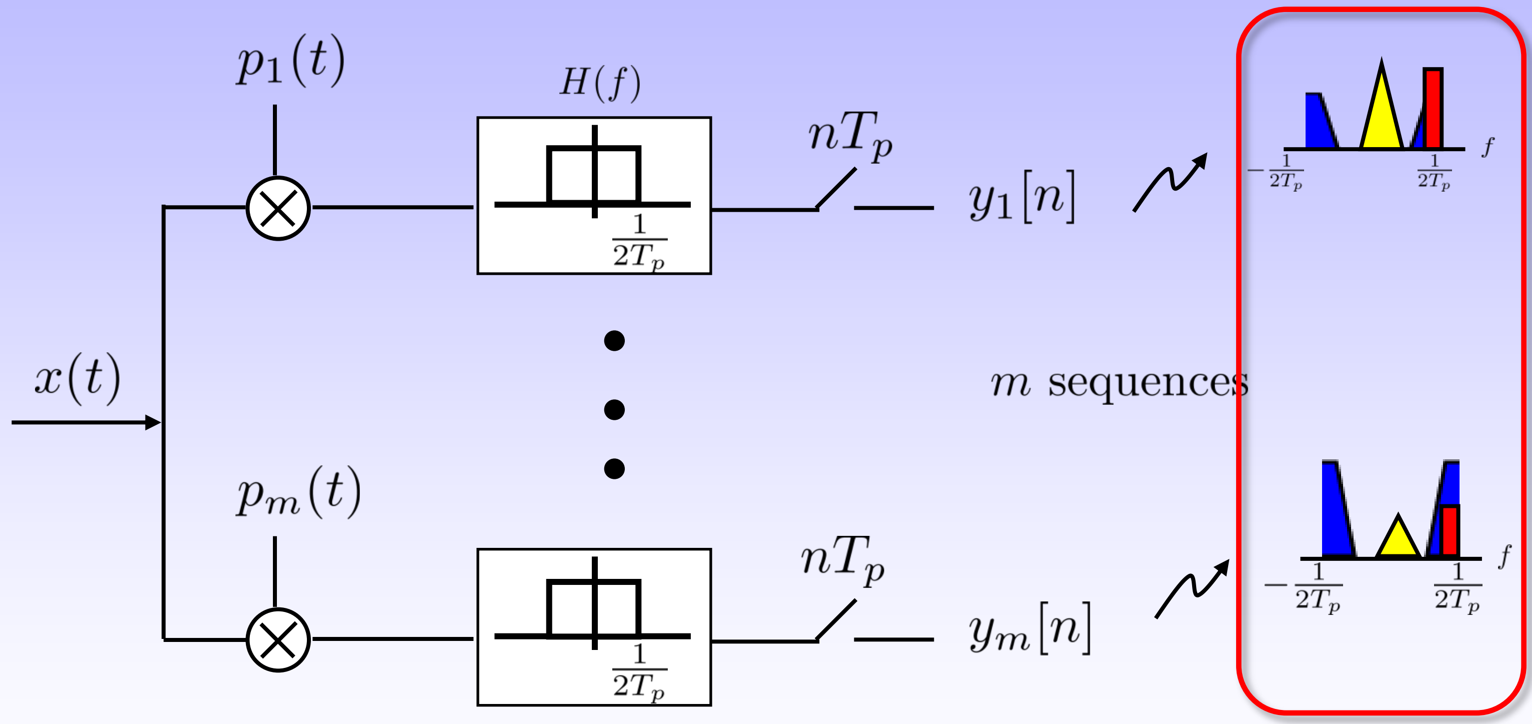


Frequency domain

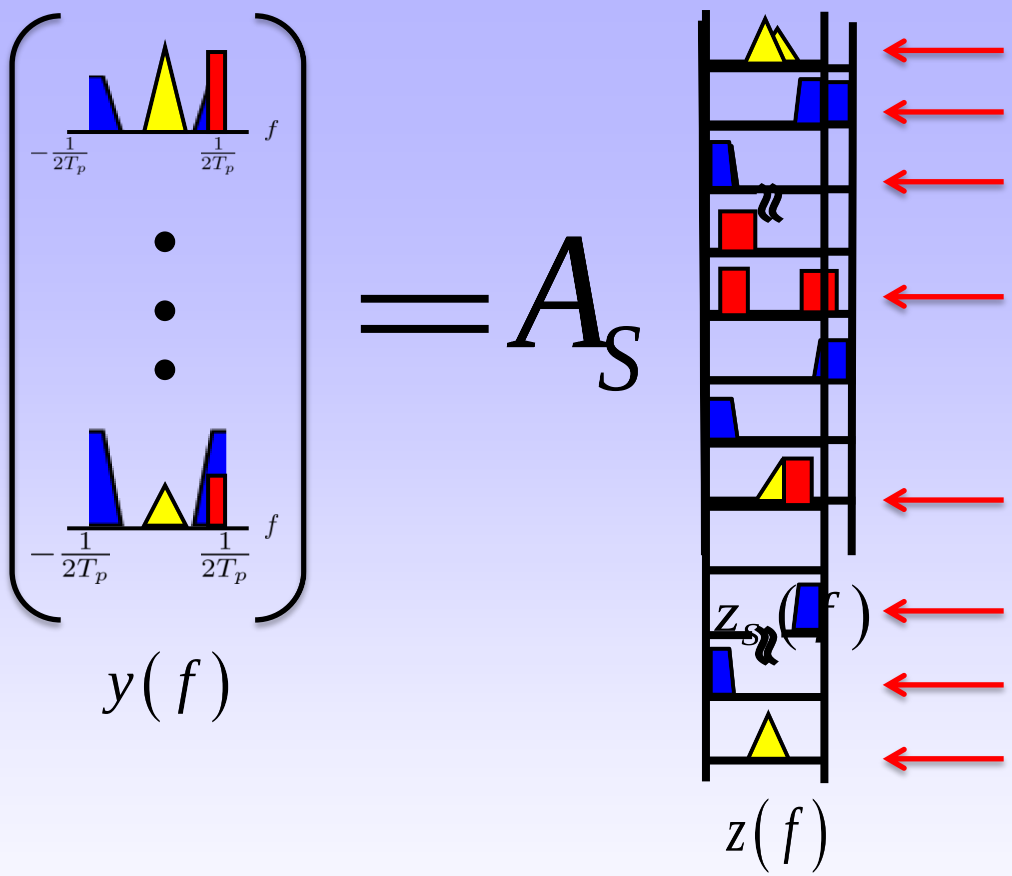
- Practical considerations:
  - Can’t design nice sign patterns at high frequency
  - Only periodicity and frequency smoothness matter

MWC – Aliasing





MWC – Recovery



- Support S recovery
- Signal reconstruction: $z_S(f) = A_S^{\dagger} y(f)$

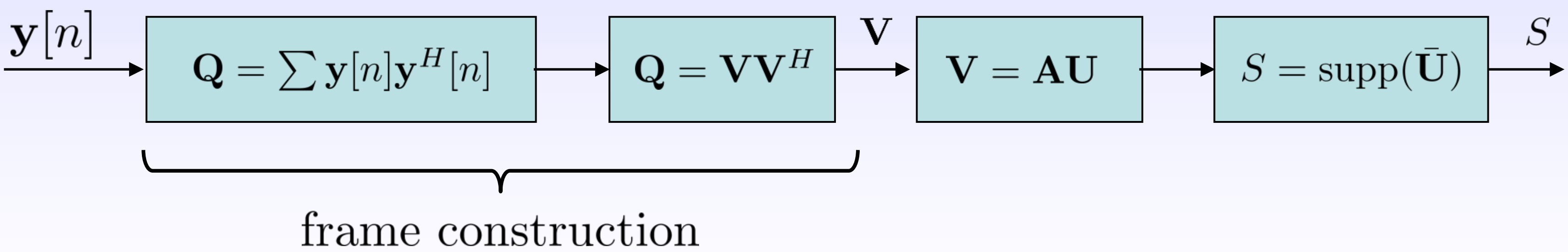
MWC – Support Recovery (CTF)

Problem: infinite number of linear systems (f is continuous)

- Solve in the time domain for each n:

$$\mathbf{y}(f) = \mathbf{A}\mathbf{z}(f) \quad \longleftrightarrow \quad \mathbf{y}[n] = \mathbf{A}\mathbf{z}[n]$$

  - Time consuming
  - Not robust to noise
- CTF (Continuous To Finite):



Infinite problem (IMV) → One finite-dimensional problem

MWC – Single Channel

m channels at rate f_s → 1 channel at rate mf_s

- A system with $f_s = qf_p$ provides q equations for each physical channel
- Trade-off:
  - Fewer channels: big hardware savings
  - Increased rate in each channel

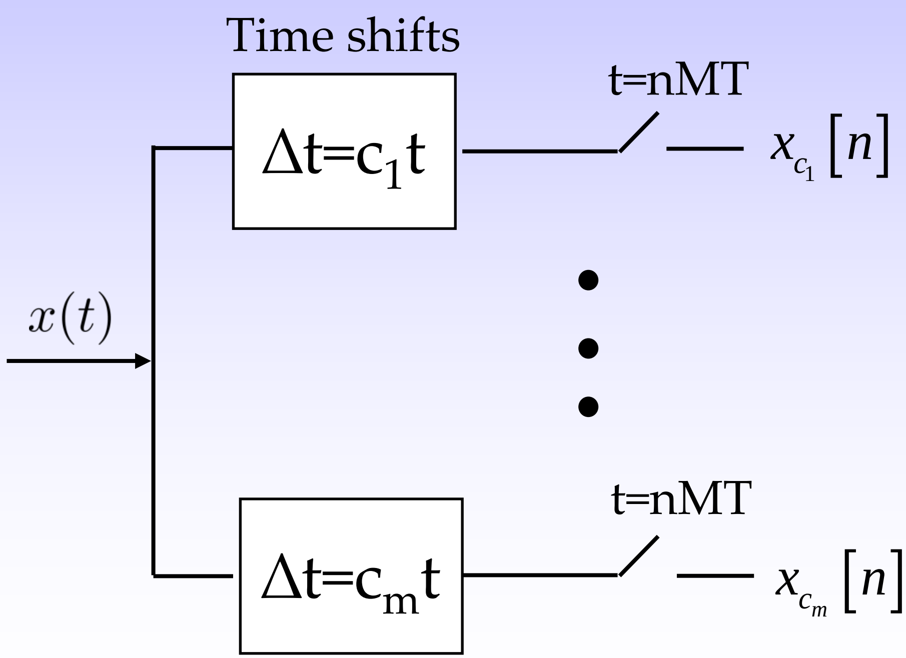
Alternative: Multicoset Sampling

Mishali & Eldar '09

- Selection of certain samples from the Nyquist grid at rate $f_s = \frac{1}{MT}$:

$$x_{c_i}[n] = x(nMT + c_i T), \qquad 0 \le c_i \le M-1$$

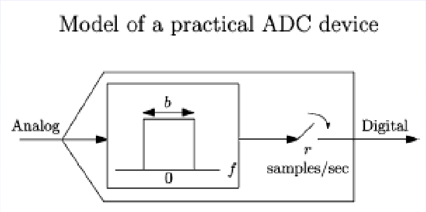
$$1 \le i \le m$$



Multicoset vs. MWC

- Same…
  - Minimal sampling rate
  - Relation between samples and original signal
  - Reconstruction scheme

- … But Different
  - ✘ Difficult to maintain accurate time shifts
  - ✘ Practical ADCs distort the samples

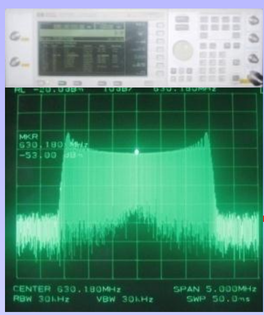


  - ✔ Easier to implement – less hardware
  - ✔ Solve digital bottleneck in case of low bandwidth

Sub-Nyquist Demonstration

Mishali & Eldar, '10

Carrier frequencies are chosen to create overlayed aliasing at baseband



FM @ 631.2 MHz

+

AM @ 807.8 MHz

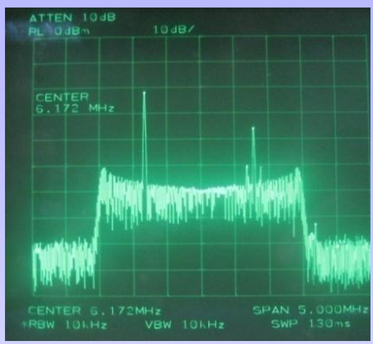
+

Sine @ 981.9 MHz

→

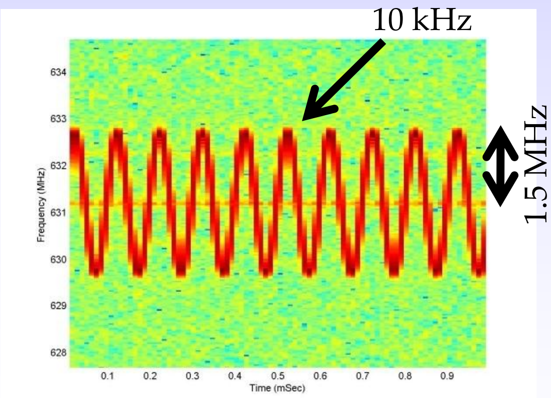
MWC prototype

→

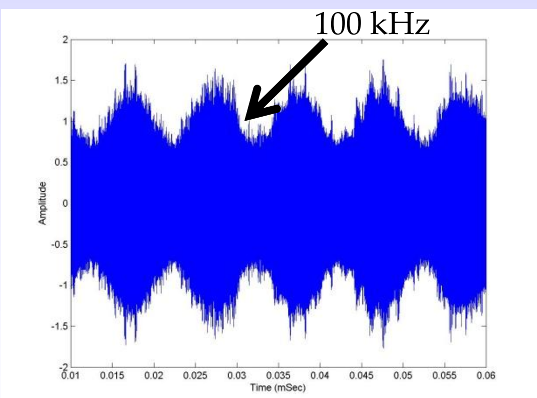


aliasing around 6.171 MHz

Reclassification: Reconstruction (CTF) →



FM @ 631.2 MHz



AM @ 807.8 MHz

But…

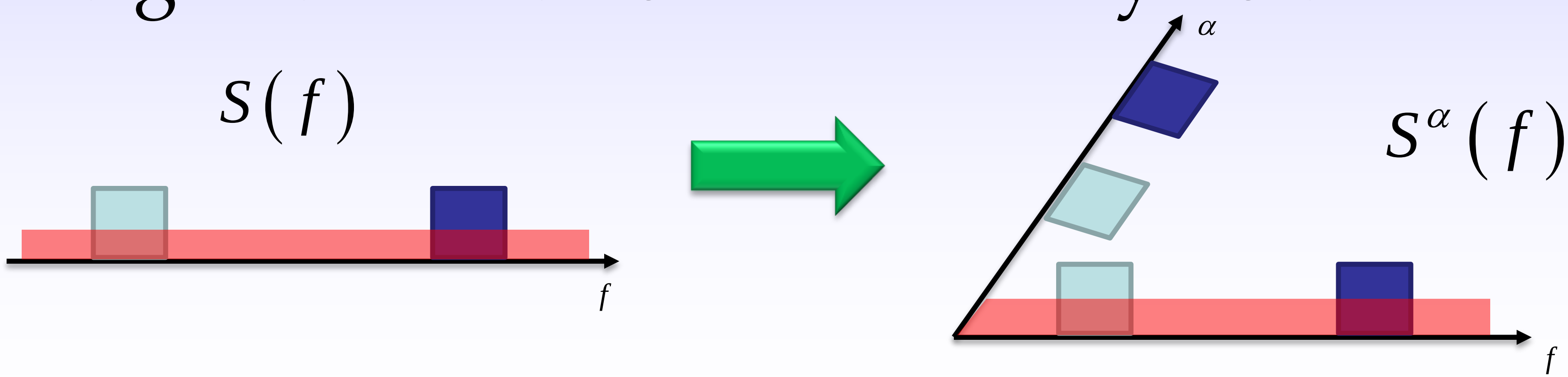
Joint work with Cores, UCLA

Problem: High sensitivity to noise

- Energy detection fails in low SNR regimes

Solution: New detection scheme

- Using a property of communication signals that is not exhibited by noise

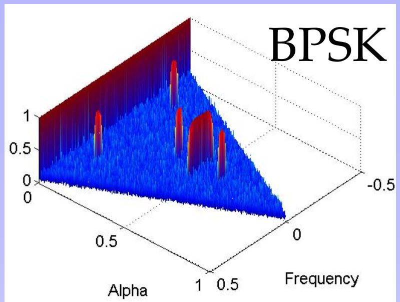


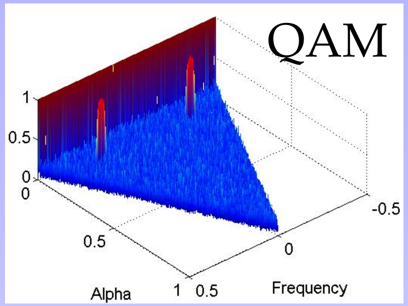
Cyclostationarity

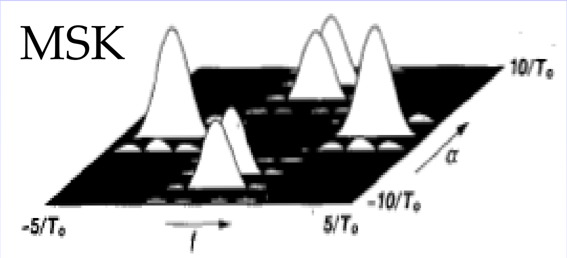
- Definition:
  - Process whose statistical characteristics vary periodically with time
- Example:
  - Communication signals
- Characterization:
  - Spectral correlation function (SCF)
  - Exhibits spectral peaks at certain frequency locations called cycle frequencies

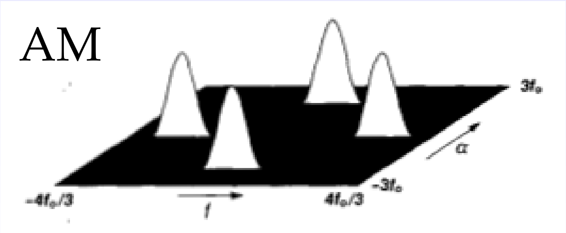
SCF – Examples

(Gardner)





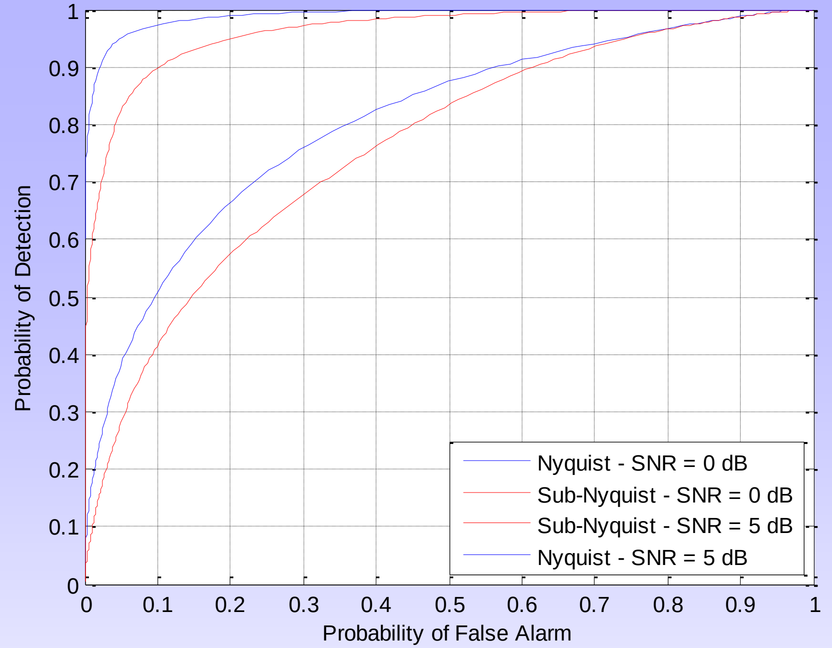




Modulation	Peaks at (α,f)
BPSK	$\left(\frac{1}{T}, f_c\right), (2f_c, 0), \left(2f_c \pm \frac{1}{T}, 0\right)$
MSK	$\left(\frac{1}{T}, f_c\right), \left(2f_c \pm \frac{1}{2T}, 0\right)$
QAM	$\left(\frac{1}{T}, f_c\right)$
AM	$(2f_c, 0)$

Results

Cohen, Rebeiz et. Al, '11



	Nyquist	Sub-Nyquist
Sampling rate	$f_{nyq} = 10GHz$	$m \cdot f_s = 30 \cdot 12MHz = 360MHz$

We can perform recovery from MWC samples in low SNR regimes using cyclostationary detection

Conclusions

- Cognitive radios: solve the spectrum congestion issue
- Crucial task: wideband analog spectrum sensing
- Sensing mechanism: low-rate, quick, efficient and reliable
- Robustness to noise: exploit communication signals cyclostationarity

References

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Thank you