Cramér–Rao Lower Bound on AoA Estimation Using an RF Lens-Embedded Antenna Array

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***Abstract*—In this letter, we investigate the Cramér–Rao lower bound (CRLB) on angle of arrival (AoA) estimation of an RF lens-embedded antenna array. We first derive an expression for the received signal, in terms of intrinsic lens characteristics, using an RF lens-embedded antenna array. As the AoA changes, the received signal through the RF lens also changes in amplitude. This differentiates the RF lens-embedded antenna array from a conventional uniform linear array (ULA) without an RF lens. Based on this property, we next derive the CRLB on AoA estimation and confirm that the CRLB of an RF lens-embedded antenna array may be better than the CRLB for a conventional ULA in a certain desired range. The results can be used as a guideline for designing RF lens-embedded antenna arrays in fifth generation localization.**

***Index Terms*—Angle of arrival (AoA) estimation, Cramér–Rao lower bound (CRLB), lens antenna.**

I. Introduction

With the emergence of fifth generation (5G) communications, the demands for high data rates and low latency are gradually increasing. To meet these demands, many researchers [1]–[3] have proposed and supported core technologies including massive multiple-input multiple-output (MIMO), cell densification, and millimeter wave (mmWave). In mmWave communications, the free space loss is so severe that the use of directive and/or beam-steerable antennas is required. The authors in [4]–[9] proposed a novel approach of applying a lens antenna to mmWave MIMO, which offers the advantages of high gain, narrow beam width, and low sidelobes in undesired directions. In addition, a number of studies have been conducted on lens antennas, such as research on lens antenna design [10], [11] and on reducing the signal distortion of a receiver using the angle of arrival (AoA) dependent energy-focusing property of a lens array [12].

Kela *et al.* in [13] showed that in ultradense networks beamforming that uses location information based on line-of-sight (LOS) was superior to beamforming that uses full-band channel state information. The improved performance was evident in terms of network throughput due to the presence of pilot contamination in 5G environments. In that work, the extended Kalman filter was used to estimate the location information and the received beamforming was aligned to the dominant LOS. The improvement of network mean throughput was proven regardless of the mobility of users.

In most practical cases, the location information is determined by the estimation of direction and distance from angle of departure, AoA, time of arrival, and time of departure. In particular, the 5G public-private-partnership automotive vertical white paper [14] proposed a highly accurate localization requirement. This demand was one of eight key performance indicators for the five different vertical industries—autonomous, e-health, factory, energy, and media sector. Therefore, to meet the requirements for 5G convergence industries, accurate estimation of AoA is important.

In this letter, we determine a model of the received signal after passing through an RF lens a for the phase array by exploiting optics. Based on this model, we then derive the Cramér–Rao lower bound (CRLB) on AoA estimation using a lens MIMO antenna, which recovers the location information. We then compare it with a conventional uniform linear array (ULA) without an RF lens, graphically demonstrating that there exists a certain range of AoAs in which antennas with an RF lens have potential for better performance than antennas without a lens in AoA estimation.

II. Lens Characteristics

We begin by describing the received signal caused by diffraction based on wave optics.[1] In particular, we use a simplified model to describe the wave characteristic of light.

In optics, a lens is a device that concentrates or disperses light using refraction phenomena. The characteristics of the spherical wave and the lens are used to analyze a signal passing through the lens [15]. Specifically, a planar wave is a summation of omnidirectional spherical waves and the received signal passing

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[1] Wave optics utilizes the wave property rather than the directivity.

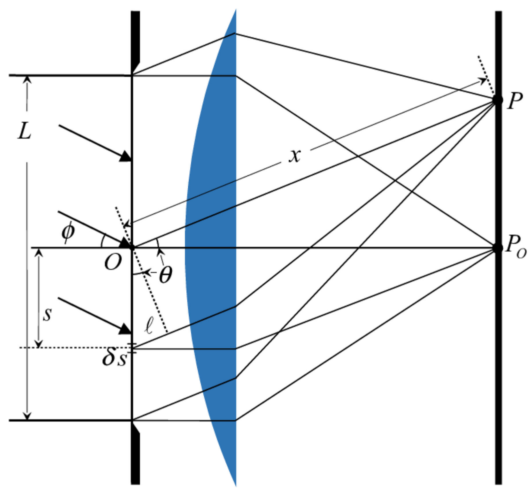

Fig. 1. Ideal lens system model description.

through the positive lens at the same angle is concentrated at one point. Fig. 1 shows a slit of width L, which is illuminated by a wave arriving at the lens in parallel from the left.

Let y be the received signal at the focal plane (the focal points are placed at distance x and angle θ from the origin O) through the entire slit and δs be the infinitesimal element of width on the slit at point s of which distance from origin is O. Then, δy_s is the partial received signal generated by all directional signal components passing through δs. The directional signal is obtained by splitting a planar wave into a sum of spherical waves.

When δs is placed on the origin, δy_s becomes δy_O and its amplitude is proportional to the length δs. The inverse of the square root of the distance for two-dimensional spherical waves [15], can be expressed as follows:

$$\delta y_O = \frac{p\delta s}{\sqrt{x}} e^{j2\pi(f_c\tau - x/\lambda)} \tag{1}$$

where p is the signal amplitude, x is the length between the origin and the focal plane, and f_c and λ are the frequency and the wave length, respectively.

Due to the characteristics of the lens, the signals received at the same angle are concentrated at one point. That is, the signal components received at 0° angle through the slit arrive in the same phase at P_O, and the received signal components passing through the slit at angle θ experience the same phase and distance attenuation. Therefore, the received signal at the focal plane for the component signal passing through the slit at angle θ can be readily expressed by noting that their path would be longer by $\ell = s\sin(\theta)$. This results in a phase difference in P, which varies according to the path difference as the position of δs changes

$$\delta y_s = \frac{p\delta s}{\sqrt{x}} e^{j2\pi\{f_c\tau - (x - s\sin(\theta))/\lambda\}}. \tag{2}$$

To obtain the entire received signal through the lens, the received signal through δs is integrated from one edge of the slit $s = -L/2$ to the other edge $L/2$. We assume symmetry of elements at s and $-s$, that is, $\delta y = \delta y_{-s} + \delta y_s$. Applying Euler's formula and the identity $\sin(\alpha) + \sin(\beta) = 2\cos(\frac{\alpha-\beta}{2})\sin(\frac{\alpha+\beta}{2})$, we obtain

$$\delta y = \frac{2p\delta s}{\sqrt{x}} \cos\left(\frac{2\pi s\sin(\theta)}{\lambda}\right) e^{j2\pi(f_c\tau - x/\lambda)}. \tag{3}$$

Integrating from $s = 0$ to $L/2$ results in the received signal, we get

$$\begin{aligned} y &= \frac{2p}{\sqrt{x}} e^{j2\pi(f_c\tau - x/\lambda)} \int_0^{L/2} \cos(2\pi s\sin(\theta)/\lambda)\,\delta s \\ &= \frac{pL}{\sqrt{x}} \mathrm{sinc}\{L\sin(\theta)/\lambda\}\, e^{-j2\pi x/\lambda} e^{j2\pi f_c\tau}. \end{aligned} \tag{4}$$

Suppose, next, that the signal arrives with an angle of ϕ. The expression in (4) is then modified by replacing $\mathrm{sinc}\{L\sin(\theta)/\lambda\}$ with $\mathrm{sinc}[L\{\sin(\theta) - \sin(\phi)\}/\lambda]$, leading to

$$y = \frac{pL}{\sqrt{x}} \mathrm{sinc}[L\{\sin(\theta) - \sin(\phi)\}/\lambda] e^{-j2\pi x/\lambda} e^{j2\pi f_c\tau}. \tag{5}$$

It is evident from (5) that ϕ affects only the amplitude, $\frac{pL}{\sqrt{x}}\mathrm{sinc}[L\{\sin(\theta) - \sin(\phi)\}/\lambda]$. Note that phases at array elements are only a function of the distance between the RF lens and the antenna array, independent of AoA. This is a key difference between the received signal through the lens and that received without a lens; it motivates us to derive a new CRLB of the lens-assisted antenna array.

We note that, normally, the set of focal points constitutes an arc, so that (5) is directly applicable. However, if we consider a *linear lens* then the focal points are on a line. The resulting signal can be derived by adjusting x and θ to place all focal points on a straight line.

III. Signal Model

Consider a receiver using an RF lens. Only the signals through the RF lens can be received by the antenna elements and each antenna element is on the focal plane of the lens. In this letter, we consider the following two types of lenses. First, *linear lens*, in which the focal plane is assumed to be linear; and second, *arc lens*, in which the focal plane is on an arc. Both cases are shown in Fig. 2. Regardless of the shape of the antenna array, we assume that the distance between the center of the antenna and the lens is the focal length f. For a *linear lens*, antenna elements are placed away from the lens at a distance of f. The array antenna has an odd number of elements N, equispaced by d with a signal coming from direction ϕ in the form of a plane wave, which has the same size lens aperture L.

In an *arc lens*, an arc array antenna is used. Each element of the focal arc antenna is placed on the focal arc [12] by moving the linear array antenna in a direction perpendicular to the slit. In a practical situation, the length of the lens is longer than the length of the antenna by approximately 10 λ at both ends to maximize the power focusing effect, resulting in higher accuracy. In this letter, however, the length of the RF lens and the antenna array are considered the same because we assume a power reception equal to ULA, thus, allowing for a fair comparison of the bounds.

The index of each antenna element is set from $-(N-1)/2$ to $(N-1)/2$. Based on (5), we can define the amplitude $\mathbf{A}(\phi)$ of an $N \times N$ diagonal matrix with its diagonal elements in

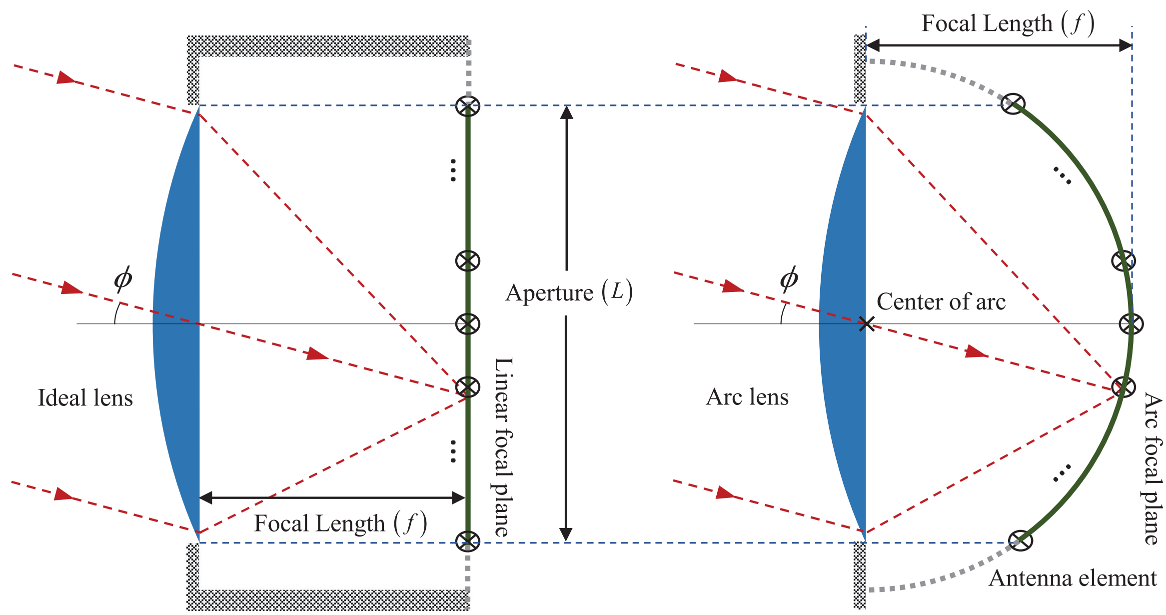

Fig. 2. Linear and arc lens models.

terms of the derived form of the received signal through the RF lens. Using the independent property between the amplitude and the phase to AoA, the signal received through the lens can be modeled as follows:

$$\mathbf{x} = p\mathbf{A}(\phi)\,\mathbf{s}e^{jb} + \mathbf{n}. \tag{6}$$

Here, $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is an $N \times 1$ column of the received signal vector, p is the transmit signal amplitude, b is the phase arrived at a lens that is contained in τ in the expression of (5) since it depends on the time difference between transmitter and receiver, and $\mathbf{n}$ is additive white Gaussian noise with covariance ${\sigma_n}^2\mathbf{I}$, where $\mathbf{I}$ denotes the identity matrix. From (5), $\mathbf{A}(\phi)$ can be derived as

$$\mathbf{A}(\phi)_{n,n} = \frac{L}{\sqrt{x}}\mathrm{sinc}\left[\frac{L}{\lambda}\left\{\frac{dn}{x} - \sin(\phi)\right\}\right], \tag{7}$$

where $x = \sqrt{d^2n^2 + f^2}$ for the linear antenna array and $x = f$ for the arc antenna array. Note that (6) is not applicable to a ULA since the phase of the received signal of the ULA differs for each antenna element, unlike RF lens antennas. If an *arc lens* is used, then $\mathbf{s} = s\left[1 \cdots 1\right]^{\mathrm{T}}_{N\times 1} e^{-j2\pi x/\lambda}$ is the transmitted symbol, where $e^{-j2\pi x/\lambda}$ is the phase induced by the lens characteristics. This value is determined by the distance between the lens and the antenna elements. When using a *linear lens*, the transmitted symbol becomes

$$\mathbf{s} = s\left[e^{-\frac{j2\pi\sqrt{d^2\left(-\frac{N-1}{2}\right)^2 + f^2}}{\lambda}} \cdots e^{-\frac{j2\pi\sqrt{d^2\left(\frac{N-1}{2}\right)^2 + f^2}}{\lambda}}\right]^{\mathrm{T}}_{N\times 1}.$$

IV. CRLB Analysis

In this section, we derive the CRLB of the error variance in estimating the AoA with and without an RF lens. The three parameters—amplitude, phase, and AoA of the received signal—are mapped to p, b, and ϕ, respectively, and defined in the vector $\boldsymbol{\gamma} = \left[p\ b\ \phi\right]$. In both cases, let $\mathbf{v}$ be the received signal except Gaussian noise. The total received signal is, thus, $\mathbf{x} = \mathbf{v} + \mathbf{n}$. The probability density function, given the parameter vector $\boldsymbol{\gamma}$, is given by:

$$\boldsymbol{f}_{\mathbf{x}}(\mathbf{x}|\boldsymbol{\gamma}) = Ce^{-(\mathbf{x}-\mathbf{v})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{x}-\mathbf{v})} \tag{8}$$

where $\mathbf{R} = \sigma_n^2\mathbf{I}$ and C is a normalization constant.

A. CRLB of Linear Array

Consider the received signal from a conventional ULA. The phase of the received signal changes according to the AoA. The phase difference between antenna elements due to the AoA is $e^{jd\cos\phi}$ when the antenna spacing is d. The received signal without Gaussian noise is, therefore, $\mathbf{v} = p\mathbf{s}(\phi)e^{jb}$, where $\mathbf{s}(\phi)$ is the steering vector

$$\left[z^{-\frac{N-1}{2}}\ z^{-\frac{N-3}{2}}\ \cdots\ z^{\frac{N-3}{2}}\ z^{\frac{N-1}{2}}\right]^{\mathrm{T}}_{N\times 1}. \tag{9}$$

Here, z is $e^{jkd\cos\phi}$ where k is the index of the time sample. Since all the nondiagonal terms in the Fisher information matrix are zero, the corresponding CRLB is readily derived as

$$\mathrm{CRLB}_{\text{no lens}}(\phi) = \frac{6\sigma_n^2}{p^2N\left(N^2-1\right)k^2d^2\cos^2\phi}. \tag{10}$$

B. CRLB Analysis of Lens-Assisted Antenna Array

By setting $\mathbf{v} = p\mathbf{A}(\phi)\,\mathbf{s}e^{jb}$, the log likelihood of (8) without an additional constant is

$$\boldsymbol{g}(\boldsymbol{\gamma}) = \frac{p}{\sigma_n^2}\Big[e^{-jb}\mathbf{s}^{\mathrm{H}}\mathbf{A}(\phi)\,\mathbf{x} + e^{jb}\mathbf{x}^{\mathrm{H}}\mathbf{s}\mathbf{A}(\phi) - p\mathbf{s}^{\mathrm{H}}\mathbf{A}^2(\phi)\,\mathbf{s}\Big]. \tag{11}$$

Since some of the off-diagonal terms in the Fisher information matrix are nonzero, the derivations of all elements are needed, as shown below

$$\begin{aligned}
\mathbb{E}\left[\frac{\partial^2 \boldsymbol{g}}{\partial p^2}\right] &= -\frac{2}{\sigma_n^2}\sum_n \mathbf{A}^2(\phi)_{n,n} \\
\mathbb{E}\left[\frac{\partial^2 \boldsymbol{g}}{\partial b^2}\right] &= -\frac{2p^2}{\sigma_n^2}\sum_n \mathbf{A}^2(\phi)_{n,n} \\
\mathbb{E}\left[\frac{\partial^2 \boldsymbol{g}}{\partial \phi^2}\right] &= -\frac{2p^2}{\sigma_n^2}\sum_n \left\{\frac{\partial}{\partial \phi}\mathbf{A}(\phi)_{n,n}\right\}^2 \\
\mathbb{E}\left[\frac{\partial^2 \boldsymbol{g}}{\partial p \partial \phi}\right] &= -\frac{2p}{\sigma_n^2}\sum_n \mathbf{A}(\phi)_{n,n}\frac{\partial}{\partial \phi}\mathbf{A}(\phi)_{n,n} \\
\mathbb{E}\left[\frac{\partial^2 \boldsymbol{g}}{\partial p \partial b}\right] &= \mathbb{E}\left[\frac{\partial^2 \boldsymbol{g}}{\partial b \partial \phi}\right] = 0.
\end{aligned} \tag{12}$$

The summation indices omitted in (12) are from $-(N-1)/2$ to $(N-1)/2$. The CRLB for AoA estimation with a lens-embedded antenna array is described in (13).

Theorem 1. (CRLB for AoA estimation of lens antenna): The error variance for the AoA estimation problem of lens antenna is lower bounded by $\mathrm{CRLB}_{\text{lens}}(\phi)$ in (13)

$$\mathrm{CRLB}_{\text{lens}}(\phi) = \left[\frac{\sigma_n^2}{2p^2}\sum_n \mathbf{A}^2(\phi)_{n,n}\right] \Big/ \left[\left\{\sum_n \mathbf{A}^2(\phi)_{n,n}\right\} \times \sum_n\left\{\frac{\partial}{\partial\phi}\mathbf{A}(\phi)_{n,n}\right\}^2 - \left\{\sum_n \mathbf{A}(\phi)_{n,n}\frac{\partial}{\partial\phi}\mathbf{A}(\phi)_{n,n}\right\}^2\right]. \tag{13}$$

Proof: Except for the constant term, the determinant of the Fisher matrix is the denominator of (13). To check for the existence of the bound, it is necessary to confirm that the denominator is not zero. Since $\mathbf{A}(\phi)$ and $\partial\mathbf{A}(\phi)/\partial\phi$ are diagonal matrices of size $N \times N$, they are expressible in vector form of size $N \times 1$, which is equivalent to $\mathbf{u}(\phi)$ and $\mathbf{v}(\phi)$. Then, the denominator of (13) is

$$\{\mathbf{u}(\phi)\cdot\mathbf{u}(\phi)\}\{\mathbf{v}(\phi)\cdot\mathbf{v}(\phi)\} - \{\mathbf{u}(\phi)\cdot\mathbf{v}(\phi)\}^2. \tag{14}$$

By the Cauchy–Schwarz inequality, (14) is always greater than zero. ■

C. Graphical Description of CRLB

In this section, by exploiting (10) and (13), we compare CRLBs of antennas with and without an RF lens. When the AoA is 0°, the focal point is at the center of the MIMO array. The lens aperture is equal to the size of the antenna, L is $20\,\lambda$, and the focal length f is $20\,\lambda$, where SNR is set to be 0 dB. The antenna spacing is $\lambda/2$; thus, the number of antennas is 41. Fig. 3 shows three cases of the CRLB. The first case represents the ULA without an RF lens; the second case represents an *arc lens* where the set of focal points of the lens is an arc and the antenna is also on the arc; the third case is a *linear lens* where the set of focal points of the lens is linear and the antenna is also in its linear form.

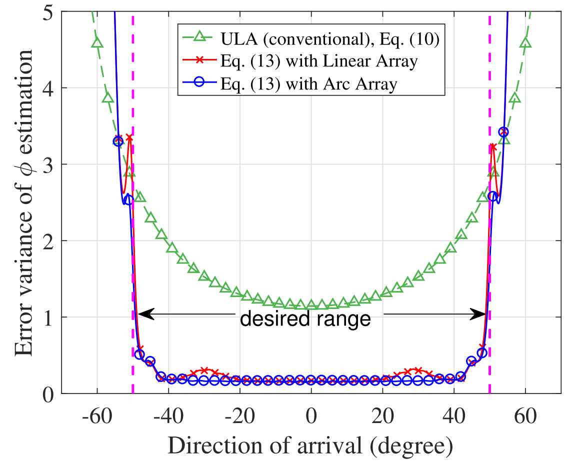

Fig. 3. CRLB comparison with lens antenna and ULA. Note that the desired range increases as the lens size increases.

In Fig. 3, the desired range is indicated by the dotted line, which implies the desired range within which the performance using the RF lens is superior to that using the conventional ULA. The range is affected by the focal length and aperture and can be determined by comparing (10) and (13). The range decreases as the focal length becomes longer and the aperture value becomes smaller. In this case, the CRLB of an RF lens-embedded antenna array outperforms a conventional ULA for a desired range of AoA that is from $-50°$ to $50°$.

Through this evaluation, we can confirm that there is no significant difference in performance between the *arc lens* and the *linear lens*. In our particular scale, within a certain range according to the focal length, the CRLB of the RF lens-embedded antenna array outperforms the conventional ULA by as much as ten times, as shown in Fig. 3.

V. Conclusion

In this letter, we investigated the CRLB for an RF lens-embedded antenna array using an optics-based diffraction model. We demonstrated that with respect to the AoA estimation the RF lens antenna can achieve much better performance than ULA. Note that the results are based on equally spaced antenna. As we have shown in this letter, the information of the AoA only exists in the amplitude among the received signals. In other words, the performance of AoA estimation depends on the acquisition of amplitude. What is important here, then, is the position of each antenna element, as it is a spatial sampling of the amplitude. The CRLB-achieving solution is still an open problem and we leave this for future work.

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