Channel Probing in Communication Systems:
Myopic Policies Are Not Always Optimal

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Joint work with Eytan Modiano and Isaac Keslassy
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Opportunistic Communication

- The quality of wireless channels fluctuates over time

- Objective: Transmit over channels which are in “good” state.
  - “Good” channels yield high throughput
  - Opportunistically selecting channels improves system throughput

- Opportunistic communication requires knowledge of the channel states
  - Transmitter needs to obtain this information (CSI)
  - Obtains CSI via channel probing

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Control Information

- **General Question**: How much information is necessary to effectively control the network.
  - How often should the transmitter obtain information?
  - What information should the transmitter obtain?
Control Information

- **General Question:** How much information is necessary to effectively control the network.
  - How often should the transmitter obtain information?
  - What information should the transmitter obtain?

- **Channel Probing Problem:**
  - How often to probe?
    - Last part of the talk
  - What channels to probe?
    - First part of the talk
Previous Work

• Many works looking at channel probing problem
  – See [JMMM ’11], [GMS ‘06], [CP ‘06], [CL ‘07], and references within
Previous Work

• Many works looking at channel probing problem
  – See [JMMM ’11], [GMS ‘06], [CP ‘06], [CL ‘07], and references within

• [Ahmad, Liu, Javidi, Zhao, Krishnamachari; 2009]
  – Channel states vary between ON state and OFF state.
  – Probe one channel in every slot
  – MUST transmit on the probed channel
  – Policy that probes the channel most likely to be ON is optimal
  – Myopic Policy: A policy maximizing immediate reward (greedy policy).

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Channel States

- Transmitter and Receiver connected through multiple channels

- Channel states are independent of one another

Fig. 1: Opportunistic Communication Example. Receiver sends channel state information to the transmitter, who makes a decision over which of $M$ channels to transmit.

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Channel States

- Transmitter and Receiver connected through multiple channels

- Channel states are independent of one another

- Assume channel states are ON or OFF:
  - Transmissions across an ON channel are successful
  - Transmissions over an OFF channel are dropped
Channel States

- Transmitter and Receiver connected through multiple channels

- Channel states are independent of one another

- Assume channel states are ON or OFF:
  - Transmissions across an ON channel are successful
  - Transmissions over an OFF channel are dropped

- Channel States vary over time:
  - Positive channel memory
  - \( \pi = \text{the steady state probability of being in the ON state} \)

\[
S_i(t): \quad 1 - p \quad \text{OFF} \quad 1 - q \quad \text{ON}
\]
Channel Probing

- Every T slots, the transmitter chooses a channel to probe
  - This is the only way to learn channel state information (CSI)
  - CSI is relevant for multiple time slots

- *Belief* of channel $i$: the probability that channel $i$ is ON given the history of all channel probes.
  - If channel $i$ was probed $k$ slots ago and was in state $s$.

\[ x_i = P(S_i(t) = \text{ON} | S_i(t-k) = s) \]
Channel Probing

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- **Belief** of channel $i$: the probability that channel $i$ is ON given the history of all channel probes.
  - If channel $i$ was probed $k$ slots ago and was in state $s$,
    \[ x_i = P(S_i(t) = \text{ON} | S_i(t-k) = s) \]

- Transmitter transmits over channel with highest belief
  - Expected throughput: \[ \max_i x_i(t) \]
  - Transmitter will transmit over the same channel until new channel probe.
  - Transmitter is not restricted to transmit over the probed channel
    • This restriction was present in previous work [Ahmad et. al, ‘09]

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Infinite Channel System

• We are interested in systems with a large number of channels.

• Infinite channel simplification:
  – When you probe a channel and it’s OFF, it is effectively removed from the system.
  – There always exists a channel that hasn’t been probed for an infinitely long time
    • Belief of such a channel = \( \pi \)

• What is the optimal channel probing policy?
  – Assume fixed probing instances.

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Probe Best Policy

- **Probe best policy** [Ahmad et al. ‘09]: At each probing instance, probes the channel with the highest belief.

- **Observation**: under the probe best policy, at most one channel has belief larger than $\pi$.

- **Example**: (order channels in descending order of belief)
  - Assuming $T = 1$ for illustration, but all intuition holds for $T > 1$.

\[
egin{align*}
\pi_1 & > \pi \\
\pi & \\
\pi & \\
\vdots & \\
\end{align*}
\]

Probe best channel

If that channel is still ON

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Probe Best Policy

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\[
\begin{align*}
  x_1 &> \pi \\
  \pi &
\end{align*}
\]

Probe best channel

\[
\begin{align*}
  \pi \\
  \pi \\
  \vdots
\end{align*}
\]

If that channel is still ON

\[
\begin{align*}
  1 - q \\
  \pi \\
  \pi \\
  \vdots
\end{align*}
\]
Probe Best Policy

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- *Example:* (order channels in descending order of belief)
  - Assuming $T = 1$ for illustration, but all intuition holds for $T > 1$.

  \[
  x_1 > \pi
  \]

  \[
  \pi \quad \text{Probe best channel} \quad \text{If that channel is now OFF}
  \]

  \[
  \pi
  \]

  \[
  \vdots
  \]

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Probe Best Policy

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- **Example:** (order channels in descending order of belief)
  - Assuming $T = 1$ for illustration, but all intuition holds for $T > 1$.  

  $$
  \begin{array}{c}
  x_1 > \pi \\
  \pi \\
  \pi \\
  \vdots \\
  \end{array}
  \quad \text{Probe best channel} \quad \begin{array}{c}
  p \leq \pi \\
  \pi \\
  \pi \\
  \vdots \\
  \end{array}
  $$

  If that channel is now OFF

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Probe Best Policy

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  – Assuming $T = 1$ for illustration, but all intuition holds for $T > 1$.

```
x_1 > \pi
\pi
\pi
\vdots
```

Probe best channel

If that channel is now OFF

```
p \leq \pi
\pi
\pi
\vdots
```

```
\pi
```

```
\pi
```

```
\vdots
```

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Probe Best Policy

- As long as the transmitter probes OFF channels, the state stays the same.

\[
\begin{array}{c}
\pi \\
\pi \\
\pi \\
\vdots
\end{array}
\]

Probe best channel → If that channel is OFF
Probe Best Policy

- As long as the transmitter probes OFF channels, the state stays the same.

\[
\begin{array}{c}
\pi \\
\pi \\
\pi \\
\vdots \\
\end{array}
\quad \xrightarrow{\text{Probe best channel}} \quad \begin{array}{c}
\text{If that channel is OFF} \\
\pi \\
\pi \\
\vdots \\
\end{array}
\quad \begin{array}{c}
\pi \\
\pi \\
\vdots \\
\end{array}
\begin{array}{c}
p \leq \pi \\
\pi \\
\vdots \\
\end{array}
\]
Probe Best Policy

- As long as the transmitter probes OFF channels, the state stays the same.

- This repeats until an ON channel is found.

- Renewal Channel Process: renewal occurs upon OFF channel probe

- A renewal occurs when the ordered belief vector is $\mathbf{x} = (\pi, \pi, \ldots)$
  - If an ON channel is found, that channel is probed until found OFF $\rightarrow$ renewal

- Use renewal-reward theory to compute average throughput.

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Probe Best Policy Discussion

• Advantages to Probe Best Policy
  – Probing the channel with the highest belief maximizes the immediate probability of finding an ON channel
  – Maximizes Immediate Throughput (greedy).

• Disadvantages to Probe Best Policy
  – When an OFF channel is found, the transmitter has no knowledge of which channel to probe to find an ON channel.
  – Until an ON channel is found, the transmitter sends packets over a channel with belief $\pi \rightarrow$ Low expected throughput.
Immediate Reward

• Assume we sort the channels by belief (high to low)

\[ E[Reward \mid \text{Probe Ch. } i] = \Pr(\text{Ch. } i \text{ is ON}) E[Reward \mid \text{Ch. } i \text{ is ON}] \]

\[ + \Pr(\text{Ch. } i \text{ is OFF}) E[Reward \mid \text{Ch. } i \text{ is OFF}] \]

• Probe the best channel:

\[ E[Reward] = x_1 \cdot 1 + (1 - x_1) \cdot x_2 \]

\[ = x_1 + x_2 - x_1x_2 \]
Immediate Reward

- Assume we sort the channels by belief (high to low)
  \[- \left( x_1, x_2, x_3, \ldots \right) \]
  \[ E[\text{Reward} | \text{Probe Ch. i}] = \Pr(\text{Ch. i is ON})E[\text{Reward} | \text{Ch. i is ON}] \]
  \[ + \Pr(\text{Ch. i is OFF})E[\text{Reward} | \text{Ch. i is OFF}] \]

- Probe the best channel:
  \[ E[\text{Reward}] = x_1 \cdot 1 + (1 - x_1) \cdot x_2 \]
  \[ = x_1 + x_2 - x_1 x_2 \]

- Probe the 2\textsuperscript{nd} best channel:
  \[ E[\text{Reward}] = x_2 \cdot 1 + (1 - x_2) \cdot x_1 \]
  \[ = x_1 + x_2 - x_1 x_2 \]
Immediate Reward

- Assume we sort the channels by belief (high to low)
  \[-(x_1, x_2, x_3, \ldots)\]
  \[E\text{[Reward] | Probe Ch. } i = \text{Pr}(\text{Ch. } i \text{ is ON})E\text{[Reward] | Ch. } i \text{ is ON} + \text{Pr}(\text{Ch. } i \text{ is OFF})E\text{[Reward] | Ch. } i \text{ is OFF}\]

- Probe the best channel:
  \[E\text{[Reward]} = x_1 \cdot 1 + (1 - x_1) \cdot x_2\]
  \[= x_1 + x_2 - x_1x_2\]

- Probe the 2\text{nd} best channel:
  \[E\text{[Reward]} = x_2 \cdot 1 + (1 - x_2) \cdot x_1\]
  \[= x_1 + x_2 - x_1x_2\]

- Probe the k\text{th} best channel:
  \[E\text{[Reward]} = x_k \cdot 1 + (1 - x_k) \cdot x_1\]
  \[= x_1 + x_k - x_1x_k\]

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Probe Second Best Policy

- **Probe Second Best Policy**: The policy that probes the channel with the second highest belief.
- **Intuition**:
  - Under the *probe second best policy*, there can be two channels with belief greater than $\pi$.

\[
\begin{align*}
x_1 &> x_2 \\
x_2 &> \pi \\
\pi &
\end{align*}
\]

 Probe 2\textsuperscript{nd} best channel \quad If that channel is still ON
Probe Second Best Policy

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• Intuition:
  
  – Under the *probe second best policy*, there can be two channels with belief greater than $\pi$.

  $x_1 > x_2$
  
  $x_2 > \pi$
  
  $\pi$

  Probe 2\textsuperscript{nd} best channel

  If that channel is still ON

  $x'_1 > \pi$

  $1 - q$

  $\pi$

  $\vdots$

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**Probe Second Best Policy**

- **Probe Second Best Policy**: The policy that probes the channel with the second highest belief.

- **Intuition**:
  - Under the *probe second best policy*, there can be two channels with belief greater than \( \pi \).

\[
x_1 > x_2 \\
x_2 > \pi \\
\pi \\
\vdots
\]

Probe 2\textsuperscript{nd} best channel

If that channel is still ON

\[
x_1' > \pi \\
1 - q \\
\pi \\
\vdots
\]

\[
x_1' > \pi \\
1 - q \\
\pi \\
\vdots
\]

\[
x_1 > \pi \\
1 - q \\
\pi \\
\vdots
\]

\[
x_1' > \pi \\
1 - q \\
\pi \\
\vdots
\]

\[
x_1 > \pi \\
1 - q \\
\pi \\
\vdots
\]

\[
x_1' > \pi \\
1 - q \\
\pi \\
\vdots
\]

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Probe Second Best Policy

- **Probe Second Best Policy**: The policy that probes the channel with the second highest belief.
- When a channel is OFF, the transmitter has a channel with high belief to transmit over until a new ON channel is found.

\[
x_1 > x_2
\\[x_2 > \pi\]
\[\pi\]
\]

Probe 2\textsuperscript{nd} best channel

If that channel is still OFF
Probe Second Best Policy

- **Probe Second Best Policy**: The policy that probes the channel with the second highest belief.
- When a channel is OFF, the transmitter has a channel with high belief to transmit over until a new ON channel is found.

\[ x_1 > x_2 \]
\[ x_2 > \pi \]
\[ \pi \]

Probes 2\textsuperscript{nd} best channel

\[ x_1' > \pi \]
\[ p \leq \pi \]
\[ \pi \]

If that channel is still OFF

\[ \vdots \]

\[ \vdots \]
Probe Second Best Policy

- **Probe Second Best Policy**: The policy that probes the channel with the second highest belief.

- When a channel is OFF, the transmitter has a channel with high belief to transmit over until a new ON channel is found.
Probe Second Best Policy

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- When a channel is OFF, the transmitter has a channel with high belief to transmit over until a new ON channel is found.

\[
\begin{align*}
    x_1' & > \pi \\
    \pi & \quad \text{Probe 2}^{\text{nd}} \text{ best channel} \\
    \pi & \quad \text{If that channel} \\
    & \quad \text{is still OFF}
\end{align*}
\]
Probe Second Best Policy

- *Probe Second Best Policy*: The policy that probes the channel with the second highest belief.
- When a channel is OFF, the transmitter has a channel with high belief to transmit over until a new ON channel is found.

\[
x'_1 > \pi \\
\pi \\
\pi \\
\vdots
\]

Probe 2\text{nd} best channel

\[
\pi \quad \text{If that channel is still OFF} \quad \pi
\]

\[
x''_1 > \pi \\
p \leq \pi \\
\pi \\
\vdots
\]

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Probe Second Best Policy

- **Probe Second Best Policy**: The policy that probes the channel with the second highest belief.
- When a channel is OFF, the transmitter has a channel with high belief to transmit over until a new ON channel is found.

\[
x'_1 > \pi \\
\pi \\
\pi \\
\vdots
\]

Probe 2\textsuperscript{nd} best channel

\[
x''_1 > \pi \\
p \leq \pi \\
\pi \\
\vdots
\]

If that channel is still OFF

\[
x''''_1 > \pi \\
\pi \\
\pi \\
\vdots
\]

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Renewal Theory analysis

- A renewal occurs upon two consecutive ON channel probes
- Eventually, two consecutive probes will be ON:

\[
\begin{align*}
&x_1 > x_2 \\
x_2 > \pi \\
\pi
\end{align*}
\]

Probe 2nd best channel

Probed Ch. is ON

\[
\begin{align*}
&1 - q \\
x_1' > \pi \\
\pi
\end{align*}
\]

Probe 2nd best channel

Probed Ch. is ON

\[
\begin{align*}
&1 - q \\
(1 - q)^2 + pq \\
\pi
\end{align*}
\]


- From this state, the time to arrive to this state again is i.i.d.

- We can use renewal-reward theory to calculate average throughput.
  - Function of \( p, q, \) and \( T. \)
Policy Comparison

- **Theorem:** For fixed probing times $T$, the *probe second best* policy has a **higher expected throughput** than the probe best policy.

  Numerically, for $p=q=0.05$:

  ![Throughput graph](image)

  - Note that in the case where the transmitter must send over the probed channel, the probe second best policy has smaller immediate reward.
  - Probe best policy shown to be optimal in this case [*Ahmad et al. ’09*].
Other Policies

• *Round robin policy*: Probes the channel for which the transmitter has the least knowledge.
  – In an infinite channel system, the belief of the probed channel is always $\pi$, while the transmitter will send over the channel that was last found to be in an ON state.
Other Policies

- **Round robin policy**: Probes the channel for which the transmitter has the least knowledge.
  - In an infinite channel system, the belief of the probed channel is always $\pi$, while the transmitter will send over the channel that was last found to be in an ON state.

- **Theorem**: The round robin policy has the *same expected throughput* as the probe best policy.
  - This policy maximizes the amount of knowledge the transmitter has about all channels
  - Probe best policy is greedy, and has very little knowledge of the rest of the channels (other than the best).
  - However, both policies perform the same in terms of expected throughput.
Dynamic Probing Times

• Suppose there is a fixed cost associated with probing
• Question: How often should the transmitter probe a channel?
Dynamic Probing Times

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• Question: How often should the transmitter probe a channel?
• Results:
  1) For a fixed probing interval, can compute optimal probing interval for probing policies discussed previously.
Dynamic Probing Times

• Suppose there is a fixed cost associated with probing
• Question: How often should the transmitter probe a channel?
• Results:
  1) For a fixed probing interval, can compute optimal probing interval for probing policies discussed previously.
  2) When interval length can vary from probe to probe, for probe best and round robin policies:
     • If probed channel is OFF, immediately probe again
     • If probed channel is ON, wait a predetermined interval before probing again
     – Optimal probing interval under probe second best policy is unknown

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Conclusion

• Considered channel probing policies, where a transmitter probes a channel, and then chooses which channel to transmit over.

• Using renewal theory, computed average throughput for the probe best policy, probe second best policy, and round robin policy.

• Probe second best policy outperforms the probe-best policy, which was previously shown to be optimal for a slightly different model.
Looking Forward

• What about an optimal policy?
  – **Conjecture**: The *probe second best* policy is the optimal probing policy for fixed probing intervals $T$.
  – Simulation results / numerical results supporting claim.
  – Proof of Optimality is still under investigation.
Looking Forward

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• Fundamental Limits
  – This talk: focused on optimal channel probing strategies
  – What is the theoretical minimum amount of information exchange required?
  – How do channel probing policies perform in comparison with this fundamental limit?