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Lower Bounds on Exponential Moments of the Quadratic Error in Parameter Estimation

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Abstract

Considering the problem of risk–sensitive parameter estimation, we propose a fairly wide family of lower bounds on the exponential moments of the quadratic error, both in the Bayesian and the non–Bayesian regime. This family of bounds, which is based on a change of measures, offers considerable freedom in the choice of the reference measure, and our efforts are devoted to explore this freedom to a certain extent. Our focus is mostly on signal models that are relevant to communication problems, namely, models of a parameter–dependent signal (modulated signal) corrupted by additive white Gaussian noise, but the methodology proposed is also applicable to other types of parametric families, such as models of linear systems driven by random input signals (white noise, in most cases), and others. In addition to the well known motivations of the risk–sensitive cost function (i.e., the exponential quadratic cost function), which is most notably, the robustness to model uncertainty, we also view this cost function as a tool for studying fundamental limits concerning the tail behavior of the estimation error. Another interesting aspect, that we demonstrate in a certain parametric model, is that the risk–sensitive cost function may be subjected to phase transitions, owing to some analogies with statistical mechanics.

Index Terms: risk–sensitive estimation, risk–averse estimation, robust estimation, Bayesian estimation, Cramér–Rao bound, unbiased estimation, tail behavior, phase transitions.