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Burst-Erasure Correcting Codes with Optimal Average Delay

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Abstract

The objective of low-delay codes is to protect communication streams from erasure bursts by minimizing the time between the packet erasure and its reconstruction. Previous work has concentrated on the constant-delay scenario, where all erased packets need to exhibit the same decoding delay. We consider the case of heterogeneous delay, where the objective is to minimize the average delay across the erased packets in a burst. We derive delay lower bounds for the average case, and show that they match the constant-delay bounds only at a single rate point 0.5. We then construct codes with optimal average delays for the entire range of code rates. The construction for rates under 0.5 achieves optimality for every erasure instance, while the construction for rates above 0.5 is optimal for an infinite number, but not all, of the erasure instances. The paper also studies the benefits of delay heterogeneity within the application of sensor communications. It is shown that a carefully designed code can significantly improve the temporal precision at the receiving node following erasure-burst events.

Keywords

Low-delay codes, erasure codes, burst erasures, average delay, codes for sensor communications

I. Introduction

There are many practical scenarios where a communication system needs to reconstruct corrupt or lost data with minimal delay. These scenarios are common in communication devices with small buffers, and in systems interacting with the physical world. When delay becomes a major concern, one needs to explicitly introduce it to the coding model. A very elegant coding model involving decoding delay has been introduced by Martinian [1], which in particular showed that MDS codes, a common panacea for erasures, are *not* optimal when burst-erasure correction is needed with low delay. The paradigm developed by Martinian – its constructions and bounds – was the basis for several follow-up works promoting different scenarios of low-delay communication: [2] (flexible construction), [3] (multiple bursts), and [4] (multi-user). Additional works addressing low-delay coded communications have appeared in [5], [6], [7], [8], [9], [10], [11], [12], [13].

The prior work has concentrated on the case where every packet in the stream needs to exhibit the same delay. There are many practical scenarios where this restriction is not necessary. For example, in many control networks (e.g. automotive networks), the nodes not only forward data, but also perform computations on it. In such networks it is preferable to obtain part of the data very early, and start the computation while additional packets are being reconstructed. For such scenarios we are considering in this paper *heterogeneous delay*, and seek to minimize the *average delay* of reconstruction, calculated over the packets erased in a burst-erasure instance.

As it turns out, there is a big gap between the achievable delays in the average and constant regimes. It is possible to reduce the delay considerably if one lifts the constant requirement. In particular, in Section III we derive bounds for the average case, and show that they only match the constant case at a single rate point $R = 0.5$. The bounds mark the fundamental limits to the *average* decoding delay given the code rate R and erasure burst length B. The bound is divided to three rate regions, compared to only two regions in the bound for constant delay [1]. Analytical and constructive reasonings about heterogeneous delay are simplified by introducing a new definition of delay we call *recovery delay*. We then move in Section IV to construct codes that match the average-delay bounds for the entire range of code rates. One construction for rates $R \geq 0.5$ achieves optimality for an infinite number of burst instances characterized by their phase with respect to the construction. Another construction for rates $R < 0.5$ achieves optimality for every burst instance. The constructions are given as infinite families of codes with fairly flexible parameters. We first characterize the parameter families that allow average-delay optimality, and focus on them in the constructions. Rates that allow optimality are of the form $m/(m+1)$ for $R \geq 0.5$, and $1/(m+1)$ for $R < 0.5$ (m an integer). For $R \geq 0.5$ we construct codes with optimal average delay for any choice of m, and for any burst length B. We choose to present this as two constructions: one when m and B are co-prime, and another for general m and B. The special case of m and B co-prime enjoys a more regular structure and simpler proofs, thus winning the main attention in the paper.

The new proposal of low average-delay codes is principally motivated by sensor communications in delay-sensitive control applications. In such applications, measurement data packets are transmitted to a remote node over a lossy channel, and the

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