

# The Generalized Stochastic Likelihood Decoder: Random Coding and Expurgated Bounds

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## Abstract

The likelihood decoder is a stochastic decoder that selects the decoded message at random, using the posterior distribution of the true underlying message given the channel output. In this work, we study a generalized version of this decoder where the posterior is proportional to a general function that depends only on the joint empirical distribution of the output vector and the codeword. This framework allows both mismatched versions and universal (MMI) versions of the likelihood decoder, as well as the corresponding ordinary deterministic decoders, among many others. We provide a direct analysis method that yields the exact random coding exponent (as opposed to separate upper bounds and lower bounds that turn out to be compatible, which were derived earlier by Scarlett *et al.*). We also extend the result from pure channel coding to combined source and channel coding (random binning followed by random channel coding) with side information available to the decoder. Finally, returning to pure channel coding, we derive also an expurgated exponent for the stochastic likelihood decoder, which turns out to be at least as tight (and in some cases, strictly so) as the classical expurgated exponent of the maximum likelihood decoder, even though the stochastic likelihood decoder is suboptimal.

**Index Terms** Stochastic decoder, likelihood decoder, random coding exponent, expurgated exponent, random binning, source-channel coding.

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