Generalized SURE for Exponential Families: Applications to Regularization

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Abstract

Stein's unbiased risk estimate (SURE) was proposed by Stein for the independent, identically distributed (iid) Gaussian model in order to derive estimates that dominate least-squares (LS). In recent years, the SURE criterion has been employed in a variety of denoising problems for choosing regularization parameters that minimize an estimate of the mean-squared error (MSE). However, its use has been limited to the iid case which precludes many important applications. In this paper we begin by deriving a SURE counterpart for general, not necessarily iid distributions from the exponential family. This enables extending the SURE design technique to a much broader class of problems. Based on this generalization we suggest a new method for choosing regularization parameters in penalized LS estimators. We then demonstrate its superior performance over the conventional generalized cross validation approach in the context of image deblurring. The SURE technique can also be used to design estimates without predefining their structure. However, allowing for too many free parameters impairs the performance of the resulting estimates. To address this inherent tradeoff we propose a regularized SURE objective. Based on this design criterion, we derive a wavelet denoising strategy that is similar in spirit to the standard soft-threshold approach but can lead to improved MSE performance.