An $L_1$-Method for the Design of Linear-Phase FIR Digital Filters

Liron D. Grossmann    Yonina C. Eldar

Abstract

This paper considers the design of linear-phase finite impulse response digital filters using an $L_1$ optimality criterion. The motivation for using such filters as well as a mathematical framework for their design is introduced. It is shown that $L_1$ filters possess flat passbands and stopbands while keeping the transition band comparable to that of least-squares filters. The uniqueness of $L_1$-based filters is explored, and an alternation type theorem for the unique frequency response is derived. An efficient algorithm for calculating the optimal filter coefficients is proposed, which may be viewed as the analogue of the celebrated Remez exchange method. A comparison with other design techniques is made, demonstrating that the $L_1$ approach may be a good alternative in several applications.

I. INTRODUCTION

Linear-phase finite impulse response (FIR) digital filters play an important role in many signal processing applications, for example, in multirate systems, image processing, and communication systems, to mention a few. Consequently, design methods for linear-phase filters have been intensively researched in the digital signal processing literature for over almost half a decade; see [1] and references therein.

The design of FIR filters has long been recognized as an approximation problem, where an ideal frequency response, usually a discontinuous function, is approximated by a finite number of smooth functions. Such an approximation problem usually consists of a tradeoff. On the one hand, the resulting filter should preserve the discontinuous behavior of the ideal response, i.e. sharp transitions. On the other hand, these filters are also required to be as flat as possible in the passbands and stopbands. It is widely known that these two requirements are contradictory [2].

The design process typically involves four steps [1]. First, defining a desired ideal frequency response. Second, choosing an allowed class of filters (e.g. length $N$ FIR filters). Third, establishing a measure of “goodness” between the allowed filter and the ideal one. Clearly, different optimality criteria lead to different filter behavior, so this step provides a convenient way to handle the inherent tradeoff of the design problem. Finally, developing a computational method to find the coefficients of the best linear-phase filter. The choice of optimality criterion is often dictated by the existence of an efficient algorithm for calculating the optimal filter.

During the past forty years, numerous techniques for designing digital FIR filters have been suggested. The majority of them rely on one or a combination of the following optimality criteria: least-squares ($L_2$), minimax ($L_\infty$) and maximally flat. A combined criterion for achieving a tradeoff between the least-squares and the minimax [3]. The use of $L_p$ norm, $2 \leq p \leq \infty$ has also been suggested as successful measure for replacing the $L_2$ and $L_\infty$ [4].