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# Generalized Laplacians and Curvatures for Image Analysis and Processing

Eli Appleboim[1] Emil Saucan[2] Gershon Wolansky[2] and Yehoshua Y. Zeevi[1]

1- EE. Dept. 2- Math. Dept.
Technion Haifa Israel

**Abstract.** Newly developed combinatorial Laplacians and curvature operators for grayscale, as well as color images are tested on 2D synthetic and natural images. This novel approach is based upon more general concepts developed by R. Forman and is inspired by the Bochner-Weitzeböck formula which is an essential identity in Riemannian Geometry. After the presentation of the operators as they operate on images we further demonstrate the implementation of them as diffusion kernels. The differences between the various Laplacians we define, are illustrated by these examples as each of the operators is shown to be adequate for different type of image processing tasks such as sharpening anomaly detection smoothing and denoising.

## 1 Introduction and Related Works

Diffusion methods, and in particular those based upon the Laplacian in general, and the heat equation in particular, belong by now to the basic repertoire of methods available to the (geometric) Image Processing community (see, e.g. [25], [1], [13], [23], [24], [27] and references therein). As such, the relevant literature is by far too extensive for us to attempt here even an incipient bibliography.

Also, curvature analysis is of great importance in Image Processing, Computer Graphics, Computer Vision and related fields, for example in applications such as reconstruction, segmentation and recognition (e.g. [6], [14], [22], [28], [13], [1]). The conventional approach embraced in most studies implements curvature estimation of a polygonal (or, more generally, polyhedral) mesh or a grid, approximating the supposedly smooth ($\mathcal{C}^2$) image under study. The curvature measures of the mesh converge in this case to the classical, differential, curvature measure of the investigated surface.

Stimulated by Perelman's seminal work on the Ricci flow [18], [19], and by its application in the proof of Thurston's Geometrization Conjecture, and, implicitly of the Poincaré Conjecture ([17]), resulting in discrete versions of the Ricci flow and related flows ([5], [10], [15]), Ricci curvature (and Ricci flow) penetrated the main stream of Imaging and Graphics, starting with the works of Gu et al (see, e.g. [29]).

Ricci curvature measures the deviation of the manifold from being locally Euclidean in various tangential directions. More precisely, it appears in the second term of the formula for the $(n-1)$-volume $\Omega(\varepsilon)$ generated within a solid angle (i.e. it controls the growth of measured angles).