RESOLUTION ENHANCEMENT BY FORWARD-AND-BACKWARD NONLINEAR DIFFUSION PROCESSES

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ABSTRACT

A resolution enhancement scheme is presented. It consists of interpolation followed by processing with an adaptive nonlinear forward-and-backward diffusion process that enhances edges while suppressing interpolation ringings and denoising smooth areas.

keywords: *super-resolution, image interpolation, anisotropic diffusion, image enhancement, scale-space.*

1. INTRODUCTION

Traditional resolution enhancement studies use multiple images of the same place or object, taken from slightly different angles or locations. After proper registration, a higher resolution image can be obtained. Examples of such super-resolution (SR) procedures can be found in a study conducted by NASA on satellite images [1] and in video context (on a series of movie frames) [5].

Our aim is to achieve SR from a single image. For that we use a special anisotropic diffusion process, which is intended for image enhancement, and noise reduction.

Anisotropic diffusion processes have been shown to be effective denoising mechanisms that preserve edges. Perona and Malik (P-M) [4], proposed for this purpose a nonlinear diffusion equation of the form

$$I_t = \nabla \cdot (c(|\nabla I|)\nabla I); \qquad c > 0 \quad , \tag{1}$$

where c is a decreasing function of the gradient, such as

$$c(s) = \frac{1}{1+\left(\frac{s}{k}\right)^2} \quad , \tag{2}$$

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where k is a gradient threshold parameter. Subsequently, other nonlinear diffusion schemes were proposed (see [6] for an overview).

In [2] we proposed a signal enhancement process that combines forward and backward (FAB) diffusion processes, acting simultaneously on the signal. It is capable of enhancing important features, while reducing the noise level of other regions. In this paper we show how how the FAB process can serve as a postprocessing stage of interpolation, for resolution enhancement purposes.

2. FORWARD-AND-BACKWARD DIFFUSION PROCESSES

The proposed diffusion coefficient that controls the anisotropic diffusion process is formulated as follows:

$$c(s) = \begin{cases} 1-(s/k_f)^n & ,0 \le s \le k_f \\ \alpha\left[((s-k_b)/w)^{2m}-1\right] & ,k_b - w \le s \le k_b + w \\ 0 & ,\text{otherwise} \end{cases} \tag{3}$$

and its smoothed version is:

$$c_\sigma(s) = c(s) * g_\sigma(s) \quad , \tag{4}$$

where g_σ is a Gaussian with standard deviation σ and $*$ denotes convolution. Exponent parameters n,m , were chosen to be 4 and 1 respectively, and k_f smaller than $k_b - w$.

The P-M and FAB diffusion coefficients are compared in Fig. (1). Note the negative values that are introduced over a certain range of relatively high gradients of the FAB coefficient, intended for selective sharpening of a specific range of gradients (this range should include the edges to be enhanced).

The coefficient c has to be continuous and differentiable. In the discrete domain, (3) could suffice (although it is only piece-wise differentiable). (4) can fit

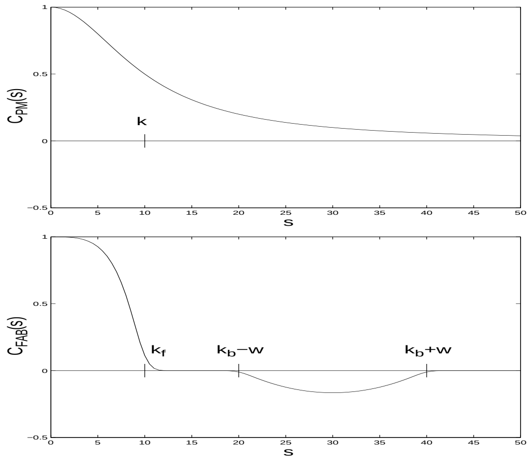

Figure 1: $c(s)$ of Eq. (2) (top) and Eq. (4) (bottom). The P-M and FAB diffusion coefficients as a function of the gradient magnitude.

the general continuous case. Other formulas with similar nature are currently under investigation.

As compared with the P-M equation (2), where an edge threshold" k is the sole parameter, the FAB process has a parameter for the forward force k_f, two parameters for the backward force (we defined them by the center k_b and width w), and the relations between the strength of the backward and forward forces (ratio we termed alpha). We therefore discuss some rules for determining these parameters. Essentially k_f - is the limit of gradients to be smoothed out, and is similar in nature to the role of the k parameter of the P-M diffusion equation. k_b and w define the backward diffusion range, and should take values of gradients that we want to emphasize. In our formula the range is symmetric, and we restrain the width from overlapping the forward diffusion area.

One way for determining parameters in the discrete case, without having any previous knowledge about the signal, is by calculating the mean absolute gradient (MAG). For instance, $[k_f, k_b, w] = [2, 4, 1] * MAG$. Local adjustment of the parameters, can be done by calculating the MAG value in a window. The parameters [$k_f(x,y)$,$k_b(x,y)$,$w(x,y)$] change gradually along the signal, and enhancement is accomplished by different thresholds in different locations. This is indeed required in cases of natural signals or images, due to their non-stationary structure. Usually a minimum value of forward diffusion should be kept, so smooth large areas would not get noisy.

The last parameter α controls the ratio between the backward and forward diffusion. For reasons of stability (see [2]) we require

$$\max_{s<k_f} \{s \cdot c(s)\} > \max_{k_b - w < s < k_b + w} \{s \cdot c(s)\}. \qquad (5)$$

For our proposed c, we get a simple inequality

$$\alpha \leq k_f / 2k_b \qquad \text{,for any } 0 < w < k_b - k_f. \qquad (6)$$

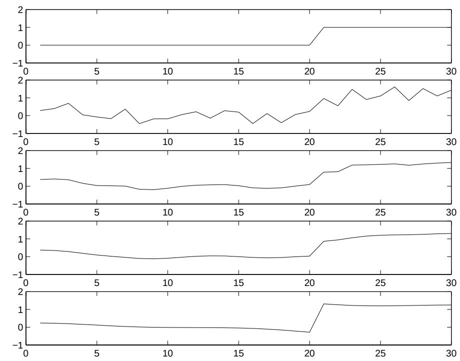

Figure 2: FAB process of a step signal, Gaussian noise: (a) Original step, (b) Blurred signal contaminated by white Gaussian noise (SNR=5dB), (c-e) Diffusion process after time: 1, 2, 8, respectively.

Figure 3: FAB processing of a parrot image - with local parameter adjustment: (a) Blurred image, (b) After FAB process.

Examples of processing of 1D and 2D signals by the proposed FAB method are shown in Figs. (2) and (3) respectively.

3. SUPER-RESOLUTION BY THE FAB PROCESS

We use the term super-resolution (SR) to describe processing wherein the resolution of an image is increased by using side information. The processed image should not only be with more pixels, but more importantly be characterized by a wider band than the original image.

We used the *forward-and-backward* diffusion process for a super-resolution-type mechanism conducted of two main elements - an interpolator and an enhancer-denoiser as shown in figure (4).

We aim to achieve SR from a single image, by exploiting the common characteristics of natural images. Our assumption is that images can basically be split into regions falling to one of the following three categories: smooth areas, edges and texture regions. At this point we simplify our model and regard only images without significant textures, that is - they can be represented by piecewise-smooth functions separated by edges.

The proposed scheme takes a low resolution image as an input with possibly some prior knowledge about the scene. The improvement is done in two steps: first we interpolate the image to the new desired size. Our implementation used cubic B-spline interpolation, but other methods could also be used. The first step gives good results for smooth areas, but edges gets "smeared". The interpolated image sometimes also have ringing effects, with low oscillations. The purpose of the second step is to enhance the edges and denoise the interpolation byproducts. We use the *forward-and-backward* diffusion process for this task. In our application the process' parameters k_f, k_b, w were locally adjusted according to the mean gradient criterion.

3.1. Application Example

In the following example we show how the SR process works. We consider a narrow-band communication system that can pass only very low resolution images in reasonable speed (like cellular phones).

Our original image is a high resolution scene. We down-sample it by 4 in each direction and send a low resolution "blocky" image that is 1/16 of the original size. At the receiving end we apply the super-resolution process as described above enlarging the image back to its original size.

A large improvement can be gained by sending a little more information in addition to the image itself, such as suitable parameters of the FAB process, places where enhancement should be avoided or strengthened etc. When the information sender has the original high resolution image, it can find much more easily the optimal parameters for the task.

In Fig. 5 an image in the different phases of the super-resolution process is shown. The low resolution image (b) is interpolated (c), where smooth regions are improved, but edges are smeared. The FAB processing (d) sharpens the edges and smoothes interpolation ringings (most noticeably at the mountain skyline). In our scheme we assumed the availability of side information dictating where enhancement should be avoided. These regions are typically blurry and fuzzy in the first place, like clouds for instance. Whereas in the case of global enhancement (d) the clouds were also sharpened, selective enhancement (avoided here above a certain height) has much more natural appearance (e). [Note that the clouds are fuzzy, whereas the mountain below is more crisp and sharp].

It should be emphasized that this process does not come to replace ordinary image compression. It can be used as an additional tool, that improves the overall performance in terms of bandwidth and the final result for the end user. However, compression and SR processes can be treated separately only in lossless image compressions. In the case of lossy processes, compression artifacts affect the SR performance (the FAB process serves in these cases also for compression noise reduction).

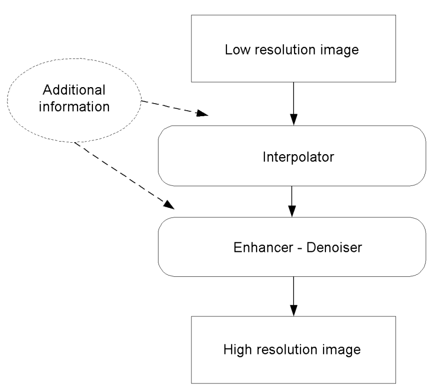

Figure 4: Super-resolution basic scheme

4. CONCLUSION

Resolution enhancement of a single image using interpolation and a forward-and-backward nonlinear diffusion post-processing provides edge enhancement and suppression of ringings. Good results were obtained in experiments on the basic model. Little additional side information further improves the overall performance.

5. REFERENCES

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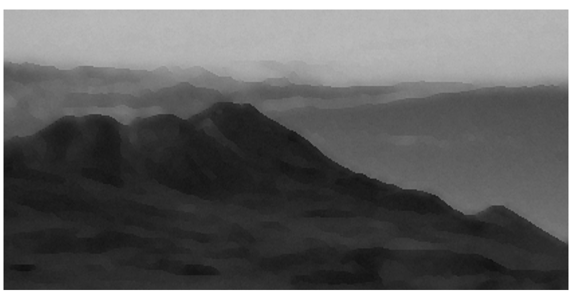
(a)

(b)

(c)



(d)

(e)

Figure 5: Super-resolution process example: (from top down) (a) Original high resolution image (b) Low resolution input image (1/16 of the original size) (c) Interpolated image (d) After FAB process (e) After FAB process with additional information: avoiding enhancement above a certain line (the clouds region is not enhanced).

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