IMAGE ENHANCEMENT SEGMENTATION AND DENOISING BY TIME DEPENDENT NONLINEAR DIFFUSION PROCESSES

Guy Gilboa, Yehoshua Y. Zeevi

Department of Electrical Engineering,
Technion - Israel Institute of Technology
Technion city, Haifa 32000, Israel
e-mail: gilboa@tx.technion.ac.il, zeevi@ee.technion.ac.il

Nir Sochen

Department of Applied Mathematics,
University of Tel-Aviv
Ramat-Aviv, Tel-Aviv 69978, Israel
e-mail: sochen@math.tau.ac.il

ABSTRACT

We present two nonlinear diffusion processes with time-dependent diffusion coefficients. Both processes converge to nontrivial solutions, eliminating the need to impose an arbitrary diffusion stopping time, otherwise required in the implementation of most nonlinear diffusion processes.

The two schemes employ nonlinear cooling mechanisms that preserve edges. One scheme is intended for general denoising, whereas the other is targeted for enhancement or segmentation of images.

key words: *nonlinear diffusion, time-dependent diffusion, anisotropic diffusion, image denoising, image enhancement.*

1. INTRODUCTION

The Nonlinear diffusion processes have been widely used over the past decade in edge preserving denoising. Perona and Malik [6] proposed a nonlinear diffusion equation in the form of:

$$I_t = \nabla \cdot (c(|\nabla I|)\nabla I), \qquad c > 0 \tag{1}$$

with c being a decreasing function of the gradient, such as

$$c(s) = \frac{1}{1 + \left(\frac{s}{k}\right)^2} \quad , \tag{2}$$

where k is a gradient threshold parameter. Some drawbacks and limitations of the original model have been mentioned in the literature [2], [12], [4]. Catte et al. [2] have proved the ill-posedness of the Perona-Malik (P-M) diffusion scheme and proposed a regularized version, where the coefficient is a function of a smoothed gradient:

$$c_\sigma(s) = \frac{1}{1 + \left(\frac{G_\sigma * s}{k}\right)^2} \quad , \tag{3}$$

where G_σ is a Gaussian of standard deviation σ, and $*$ denotes convolution.

Weickert et al. [11] showed how the stability of the P-M equation could be explained by spatial discretization, and proposed a generalized regularization formula in the continuous domain [7].

Various modifications of the original scheme were presented [9], attempting to overcome issues of stability [2],[7], adding directionality [3] or removing high gradient (impulsive) noise [8]. Yet, most schemes still converge to a trivial solution (i.e. the average value of the image) and therefore require the implementation of an appropriate stopping mechanism in practical image processing. As there still does not exist a widely accepted analytical method or even a heuristic one, it is in many cases done manually, by inspection. This is certainly unacceptable for most image processing applications. An approach that will converge to a desired solution or, at least, change very slowly in the vicinity of such a solution, will therefore considerably enhance the applicability of diffusion-type processes.

Two additional parameters have to be typically specified in a nonlinear diffusion denoising process: a threshold k for gradient preserving, and a regularizing parameter σ, of pre-convolving the image or the local gradient, needed in order to compute the local diffusion coefficient. These parameters are usually computed according to some *a priori* knowledge or estimation of the important gradients to be preserved (for k) and the characteristics of the noise involved (for σ).

We present two methods that implement two different nonlinear cooling mechanisms. The PDE cools down fast to a frozen state, where diffusion slows down considerably. These processes create a natural way of combining the parameters in an implied way, and cre-

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ate new processes with time dependent diffusion coefficients that perform well on various types of images.

2. COOLING DOWN THE SYSTEM

In most noise models, such as white (Gaussian or uniform) noise, as well as impulsive noise, the noise has theoretically unbounded gradient. Discretization bounds the measured gradients, yet the gradient criterion by itself is not enough for a good distinction between signal and noise. A common way to add some more robustness to this criterion is to pre-smooth (lowpass filter) the image beforehand and reduce most of the large gradients originating from noise. If the filter is relatively weak, edge localization is not affected considerably. After the initial linear filtering, we would like to preserve the strong edges left. Therefore, in terms of diffusion, we would like to have an initial short interval of basically linear diffusion, followed by a nonlinear edge-preserving diffusion. Moreover, strong edges should be less affected with time and the processing should converge to a steady state.

Going back from image-processing to physical processes, we know that the diffusion coefficient c is a monotonic function of the temperature [1], where, as temperature increases, molecules move more rapidly and the diffusion coefficient increases.

For the purpose of denoising, we would like some nonlinear cooling, that depends on the gradient, where large gradients cool faster and are preserved. We propose two time-dependent diffusion coefficients:

$$c_1(s,t) = \frac{1}{1+\left(\frac{s}{k(t)}\right)^2}, \quad \text{where} \quad k(t) = \frac{1}{\epsilon + \alpha_1 t} \tag{4}$$

and

$$c_2(s,t) = \frac{1}{1+\left(\frac{s}{k}\right)^{\alpha_2 t}} \quad . \tag{5}$$

2.1. Threshold Freezing

In equation (4) the gradient threshold $k(t)$ decreases with time, allowing lower and lower gradients to take part in the smoothing process. At time $t \to 0$ we have effectively a linear diffusion, where for $|\nabla I| << 1/\epsilon$ we have $c(|\nabla I|, t \to 0) \to 1$. As time advances, $k(t)$ decreases and nonlinear diffusion takes place. Only smoother and smoother regions are smoothed further, where gradients above $k(t)$ can get somewhat enhanced due to local inverse diffusion. The scheme converges to a steady state where for $t \to \infty$ we get $c(|\nabla I| > 0, t \to \infty) = 0$, meaning - no diffusion is taking place. In real applications, where one has 256 gray levels, for instance, whenever $k(t) << 1$ the scheme converges, and no significant changes occur even before that.

The scheme depends only on a single parameter: the cooling rate which is determined by α_1. As α_1 increases, the cooling is faster, less noise is being filtered but edges are better preserved. Here all three independent parameters of threshold k, filter width σ and stopping time are combined in one parameter in a natural way, creating a new denoising process that can perform well on many natural images.

2.2. Slope Freezing

The diffusion coefficient of equation (5) describes a selective freezing process, where the threshold k stays constant, but the slope is getting steeper with time (as seen in Fig. (1)). This scheme is intended for segmentation, where piecewise constant solutions are preferred. At time $t \to 0$ the diffusion is like a linear one with $c = 0.5$. With time, the diffusion coefficient, for gradients below k, is getting closer to 1. For gradients above k the diffusion coefficient is getting closer to 0.

To gain further insight into the properties of the diffusion process with time-dependent diffusion, it is instructive to consider the one-dimensional case where the process creates "walls" whenever diffusion almost stops at gradients above k. Between these walls the diffusion is strong (approaching linear diffusion) and noisy regions, but textured also, are quickly smoothed out, resulting in a local constant average value. This type of processing is somewhat similar to the shock filters proposed by Osher and Rudin in [5], though here the edge detector is a simple gradient threshold and not a second-derivative zero crossing.

The two-dimensional case is considerably more complicated: gradients above k form walls that guard regions from being diffused. But in this case the walls are not as stable. If the base of the wall forms a closed curve, it will stay closed from then onwards. If it is not a closed curve, there will be a connection between the inside and outside (with gradients below k that diffuse fast) - and the walls will ultimately dissolve. For this phenomenon we join the term "bleeding".

Regarding stability, both schemes are ill-posed in the sense of local inverse diffusion that can take place during the image evolution. Though, no significant instabilities are observed beside staircasing effects, as in the P-M process. The initial time duration, where close to linear diffusion takes place (in both proposed schemes), regularizes these processes. Therefore it performs well also in noisy environments (Fig. 4).

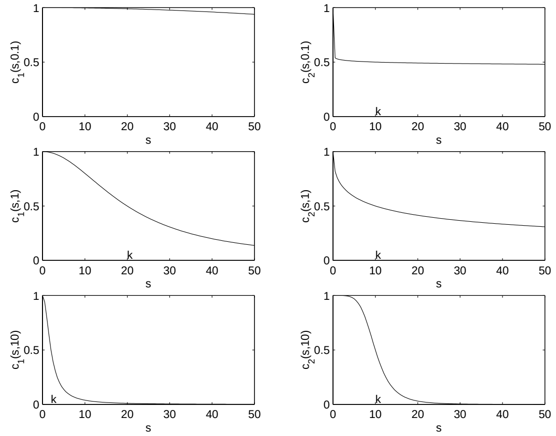

Figure 1: Plots of the diffusion coefficients at times: 0.1,1,10. Left: c_1 , $\alpha_1 = 0.05$, right: c_2 , $\alpha_2 = 0.5, k = 10$.

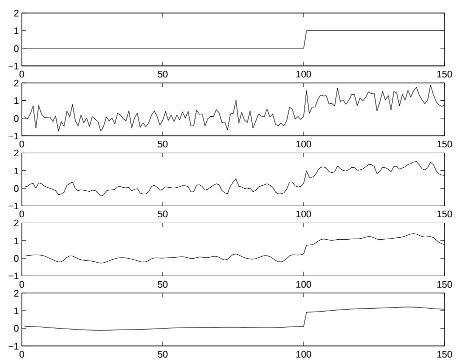

Figure 2: Denoising of a step signal by the slope-freeze process. From top down: original step signal; signal contaminated by white Gaussian noise (4 dB); sharpened and denoised by the slope-freeze process ($\alpha_2 = 0.5, k = \frac{1}{3}$) at times: 1, 2.5, 25.

3. EXAMPLES

Figures 2 and 3 depict typical results of the slope and threshold freeze processes (in 1D and 2D), respectively. To further illustrate the application of our time dependent diffusion processes, we compare the denoising of an MRI image (Fig. 4) by four nonlinear processes: P-M (eq. 2), regularized P-M (eq. 3), the threshold-freeze (eq. 4) and the slope-freeze (eq. 5).

In order to make a fair comparison, the stopping time had to be carefully chosen for the P-M schemes. This was done by calculating the mean-squared-error (MSE) at each iteration, and choosing the time of minimum MSE. The MSE (normalized by the original im-

Figure 3: Peppers image contaminated by white Gaussian noise (5 dB), denoised by the threshold-freeze process, from left: Original; Noisy image; Denoised image ($\alpha_1 = 0.04$).

age variance) is defined by:

$$MSE(t) = \frac{\int\int_{\Omega} |I_{orig}(x,y) - I(x,y,t)|^2}{\int\int_{\Omega} |I_{orig}(x,y) - E[I_{orig}]|^2} \quad , \qquad (6)$$

where I_{orig} is the original image (without the noise). Note that this way of finding the optimal time is practical only for simulations, where one has the original noisy-free image. It cannot be used in real applications where a noisy image is the input. Also, the MSE criterion for judging images is not the best, but it is general and straitforward.

The MSE of the four denoising schemes are presented in Fig. 5. One can clearly see that the two proposed schemes, with time dependent coefficients, are much more stable, and remain so for a very long time (the converging solution is not much worse than the one at time of minimum MSE). The P-M processes, on the other hand, are much more sensitive to the choice of the stopping time.

In our implementation of the diffusion schemes A slight change in the time dependent coefficient processes was made in we used incremental time steps in order to improve the accuracy and computational efficiency. Since at the beginning of the processing, the diffusion coefficient is very sensitive to time, small time steps were taken initially and then grew with time. [4 different time steps were used: 0.025,0.050,0.125,0.250]. Each of the first 3 step sizes was used for 10 iterations, and the rest of the process was executed using the maximum step (0.25). For example, to reach time $t = 10$ we needed 62 iterations.

4. CONCLUSION

The nonlinear diffusion processes with time-dependent diffusion coefficients proposed in this study provide new stable and efficient tool for image processing. The threshold-freeze method is most suitable for applications as a general denoising scheme, whereas the slope-freeze method is intended for sharpening and segmentation.

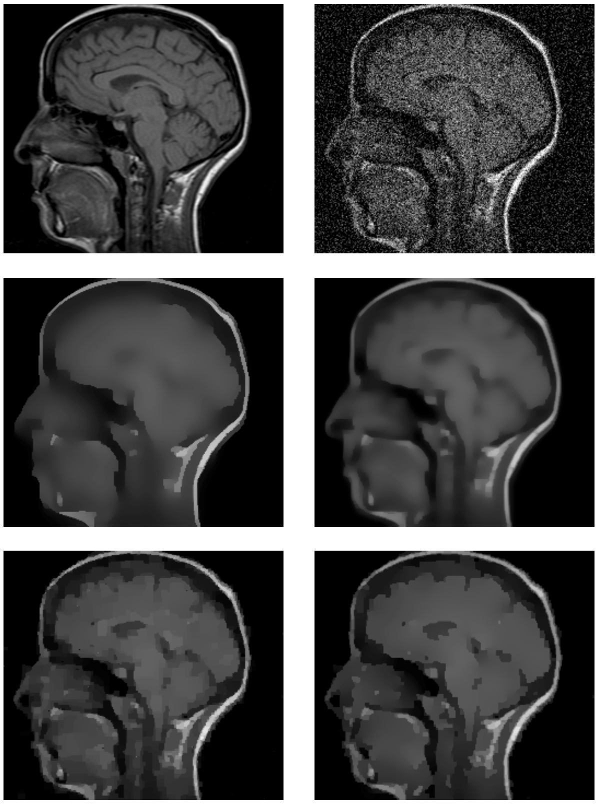

Figure 4: Head MRI image contaminated by white Gaussian noise (0 dB SNR), denoise by four nonlinear schemes. From Top left: Original; Noisy image; P-M process; Regularized P-M process; Threshold freeze with c_1, $\alpha_1 = 0.06$; Slope freeze with c_2, $\alpha_2 = 0.5$. Stopping times: 51.5, 14, 50, 50, respectively; $k = 5$.

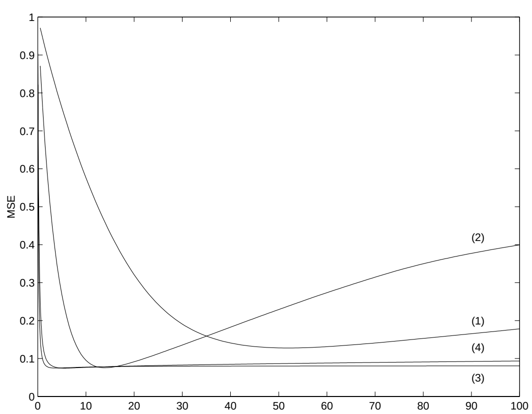

Figure 5: Normalized MSE of the four denoising processes plotted as a function of time. (1) PM, (2) Regularized PM, (3) Threshold freeze with c_1, (4) Slope freeze with c_2.

Main advantages inherent in these processes are the nontrivial converging solutions, and the reduction in the number of required parameters, especially the stopping time is no longer an issue. The implementation is fairly simple, yielding a fast convergence. The methods are robust and perform well even in the case of low SNR. The idea of cooling can be extended to other kinds of nonlinear diffusion processes.

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