SIGNAL AND IMAGE ENHANCEMENT BY A GENERALIZED FORWARD-AND-BACKWARD ADAPTIVE DIFFUSION PROCESS

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ABSTRACT

Signal and image enhancement in the presence of noise is considered in the context of the scale-space approach. A modified dynamic process, based on the action of an adaptive diffusion equation, is presented. The nonlinear diffusion coefficient is locally adjusted according to image features such as edges, textures and moments, and, as such, can also reverse its sign, i.e. switches from a forward to a backward (inverse) diffusion process according to a given set of criteria. This results in a generalized forward-and-backward adaptive diffusion process that enhances features such as transients and singularities in the one-dimensional case, and edges in images, while locally denoising smoother segments of the signal or image. Advantages afforded by the generalized adaptive diffusion process are illustrated by examples of both one-dimensional signals and images.

1. ENHANCEMENT BY DIFFUSION PROCESSES

The scale-space approach and partial differential equations (PDE) techniques have been extensively applied to signal and image processing over the last decade. As Witkin [1] had pointed out, the diffusion process (or heat equation) is equivalent to a smoothing process with a Gaussian kernel. Indeed, the fundamental solution of the standard, linear, diffusion equation

$$(1)\quad u_t = c\nabla^2 u$$

is a Gaussian function with a characteristic spread (standard deviation) that is proportional to $t^{1/2}$. A major drawback of such a linear framework is its uniform filtering of local signal features and noise. This was addressed by Perona and Malik (P-M) [2], who proposed a process known as nonlinear anisotropic diffusion, where diffusion can take place with a variable conductance in order to control the smoothing effect as follows:

$$(2)\quad u_t = \mathrm{div}(c(|\nabla u|)\nabla u) \qquad , c > 0$$

where C is a decreasing function of the gradient.

A close inspection of the P-M diffusion process reveals that it is isotropic but non-homogeneous [10]. Genuine anisotropic processes were suggested recently by Sochen, Kimmel, Malladi [8]. The application of the Beltrami diffusion equation to resolution enhancement of colored images was discussed by Sochen and Zeevi [13].

The conductance coefficient in the Perona-Malik process was chosen to be a decreasing function of the gradient of the signal. This operation selectively lowpass filters regions that do not contain large gradients (singularities as a step jump or an edge in the case of an image). However, as was proven by Catte et al. [3], this results in an ill-posed diffusion equation. Catte et al. therefore proposed a regularized version, where the coefficient is a function of a smoothed gradient.

According to the "Minimum-Maximum" principle no new local minima or maxima should be created at any time in the 1D case, in order not to produce artifacts in the diffused signal. Moreover, the values of the global minimum and maximum along the evolution of the signal in time are bounded by that of the initial state U_0(at t=0) in any dimension. These conditions are obeyed by the P-M and most other anisotropic diffusion processes introduced in image processing. This guaranties stability of the PDE and avoids explosion of the nonlinear diffusion process.

In signal enhancement/restoration, we do not want to restrict ourselves to the global minimum and maximum of the initial signal. On the contrary, we would like the points of extrema to be emphasized (if they indeed represent signal singularities and are not generated by noise). Therefore, a different approach should be considered.

As we want to emphasize large gradients, we would like to move "mass" from the lower part of a "slope" upwards. This process can be viewed as moving back in time along the scale space, or reversing the diffusion process [10]. Mathematically we can simply change the sign of the diffusion (conductance) coefficient to negative. Note that this is different than what was defined as "inverse diffusion" in previous studies (e.g. [3],[6]). There, in places where the derivative of the flux c·grad(u) was negative, it was defined as inverse diffusion, because one can write the diffusion equation near that point as:

$$(3)\quad u_t = -du_{xx} \qquad , d > 0$$

Although it has the form of an inverse diffusion process, it is weaker since it does not have the important inverse diffusion property of moving signal or image "particles", using our metaphorical language, upward along the slope of the gradient. With positive coefficient C, this could never happen, and therefore the minimum-maximum principal is not violated. Thus, signal enhancement requires further modification of the diffusion process. Specifically, to deblur an image and enhance singularities and edges, negative diffusion coefficient must be incorporated into the process.

The question is, can we simply use a linear inverse diffusion? This is obviously a highly unstable process. As mentioned earlier, the linear forward diffusion is analogous to convolution with a Gaussian kernel. Hence, the linear backward (inverse) diffusion is analogous to a Gaussian deconvolution, where the noise amplification explodes with frequency. Application of such a deconvolution process results in oscillations that grow with time until the original signal is completely lost.

To deal with this problem, three major issues must be addressed: The explosive instability, noise amplification and oscillations.

One way to overcome the inherent instability is by diminishing the value of the inverse diffusion coefficient at high gradients. In this way, after the singularity exceeds a certain gradient threshold it no longer affects the process. We can also terminate the diffusion process after a limited time, before reaching saturation.

To reduce the effect of noise, which after some pre-smoothing, can be regarded as having mainly medium to low gradients, we eliminate the inverse diffusion force at low gradients.

To reduce oscillations, we suppress them the minute they are introduced, by combining a forward diffusion force that smoothes low gradients. This smoothes also some of the original noise. However, low gradients of the signal, like those that are characteristic of certain textures in images, are also affected and smoothed out by this force.

Thus, we basically need two opponent diffusion forces acting simultaneously on the signal - a backward force (at medium gradients, where singularities are expected), and a forward one, used for stabilizing oscillations and reducing noise. Actually, we can combine these two forces in one complex forward-and-backward diffusion force with a diffusion coefficient (which is a function of the gradient) that varies continuously from positive to negative values.

In order to avoid smoothing out important features of the image such as textures, we should ideally have a local feature detector that will slow down the diffusion process in the vicinity of important features.

2. NEW CONDUCTANCE COEFFICIENT

We further extend and generalize the nonlinear PDE-based filtering method, and apply it as a combined feature-based enhancement and denoising mechanism. We minimize the effect of noise, which is inherently a byproduct of signal enhancement, by our generalized forward-and-backward diffusion processes. Moreover, important features are not filtered out by the forward diffusion process, enabling a different image processing mechanism to enhance them at a later stage, whenever it is necessary.

We propose a general feature enhancer-denoiser: Let

$c(x, y) = c(f_1, f_2, ... f_n)$, where the local feature estimators $f_i \equiv f_i(x, y)$, $i = 1,..,n$ can be selected from a broad range of choices introduced in the fields of image processing and computer vision like: edge detectors (already introduced implicitly under the gradient criterion), noise estimators, texture, scale, orientation, local power-spectrum, moments estimators etc. The logic dictating the conductance coefficient *c* should be as follows: Forward diffuse features that should be filtered out because they are corrupted by noise and are of no importance to the image nature; backward diffuse features that should be enhanced, and avoid diffusion where either diffusion processes (forward or backward) would distort important features.

In cases where there is some *a priori* knowledge of the type of images to be processed, the diffusion process could be much better controlled.

To illustrate this feature-dependant diffusion, consider the example of an urban scene primarily comprised of buildings. In this case one would like to preserve most vertical and horizontal lines and edges, significant wall textures and additional dominant edges at all orientations. To incorporate these requirements into our diffusion process, let us define by the symbols $e_e(x,y)$, $e_t(x,y)$, $e_{vl}(x,y)$, $e_{hl}(x,y)$ the local estimators that stand for edges, wall textures, vertical-lines and horizontal-lines, respectively. An appropriate conductance coefficient for the process is, in this case, given by:

$$(4) \quad c(e_e, e_t, e_{vl}, e_{hl}) = \frac{1}{1 + w_e e_e + w_t e_t + w_{vl} e_{vl} + w_{vh} e_{vh}},$$

where w_x denotes the relative weight required to balance the desired effect of each estimator. In this simplified example, it is clear that the diffusion process will slow down considerably whenever at least one of the weighted estimators is much larger than 1 $(w_x e_x >> 1 \quad , x \in \{e,t,vl,vh\})$. In other areas of the image a stronger forward diffusion will reduce the noise.

A more detailed and explicit example of our generalized inhomogeneous diffusion is an enhancement and denoising process based on an edge indicator function. Let s be an edge indicator. The adaptive process' diffusion coefficient is then defined as follows:

$$(5) \quad c(s) = \begin{cases} 1-(s/k_f)^n & ,0 \le s < k_f \\ \alpha\{((s-k_b)/w)^{2m}-1\} & ,k_b - w < s < k_b + w \\ 0 & ,\text{otherwise} \end{cases}$$

The parameter $k_f < k_b - w$ is essentially the limit of gradients to be smoothed out, where k_b and w define the range of the backward diffusion, and should take values of gradients that we want to emphasize. In our formula the range is symmetric, and we restrain the width from overlapping the forward diffusion area.

The parameter alpha controls the ratio between the backward and forward diffusion. If the backward diffusion process is too dominant, the stabilizing forward process does not avoid oscillations. One can avoid the development of new singularities in very smooth areas in the 1D case by bounding the maximum flux permissible in the backward diffusion to be less than the maximum of the forward one [for a proof see [11]]. Formally we say:

$$(6) \quad \max_{s<kf}\{s \cdot c(s)\} > \max_{kb-w<s<kb+w}\{s \cdot c(s)\}$$

In the case of our proposed coefficient, we get a simple formula for alpha, which just obeys this inequality by:

$$(7) \quad \alpha = k_f / 2k_b \quad \text{, for any} \quad 0 < w < k_b - k_f$$

In practical applications, this bound can usually be increased up to a double value without experiencing large instabilities. The exponent parameters n, m were chosen to be n=4, m=1.

The smoothed version of (5) is:

$$(8) \quad c_\sigma(s) = c(s) \otimes G_\sigma(s),$$

where $\otimes$ denotes convolution and G_σ is a Gaussian with standard deviation σ.

As an edge indicator one may use the absolute value of the gradient. A more robust version is obtained by convolving it with a Gaussian, i.e. $s = |\nabla I|$ or $s = |G_\rho \otimes \nabla I|$.

The diffusion coefficient has to be continuous and differentiable. In the discrete domain, (5) could suffice (although it is only piecewise differentiable), whereas (8) can fit the general continuous case. Other formulae with similar nature may also be proposed.

One way of choosing the parameters of the diffusion coefficients in the discrete case, is by calculating the mean absolute gradient (MAG). Local adjustment of the parameters, can be done by calculating the MAG value within a window. The parameters change gradually along the signal, and enhancement is accomplished by different thresholds in different locations. This is indeed required in the cases of natural signals or images (Fig. 2), due to their nonstationary structure.

There are a few ways to incorporate regularity into this PDE-based approach. One can replace the proposed conductance coefficient Eq. (5) by the regularized one, Eq. (8). Given an *a priory* information on the smallest scale of interest, it is possible to smooth smaller scales in a noisy signal by preprocessing. As we enhance the signal afterwards, this smoothing process does not affect the end result that much and enables operation in a much noisier environment.

3. RESULTS

We present results obtained by implementing the gradient dependant forward-and-backward process described above.

A blurred and noisy step edge (Fig. 1b) was processed, assuming the availability of prior information regarding the noise power and the approximate size of the original step. Enhancement of the step is obtained, while simultaneously denoising the rest of the signal (Fig. 1e).

The second example illustrates simultaneous denoising and enhancement of an image without any prior information about the structure of the image and/or the characteristics of the noise (Fig. 2).

4. CONCLUSION

Examples such as those presented in the previous section indicate that the generalized forward-and-backward diffusion process can accomplish simultaneously the conflicting tasks of enhancement and denoising. The process is controlled by a set of selected local feature estimators. This type of adaptive process allows a sophisticated adjustment of the diffusion strength locally, and can thereby overcome a major drawback characteristic of many image-processing procedures: while improving certain areas of the image, other segments are often degraded. Our adaptive scheme can often avoid this phenomena.

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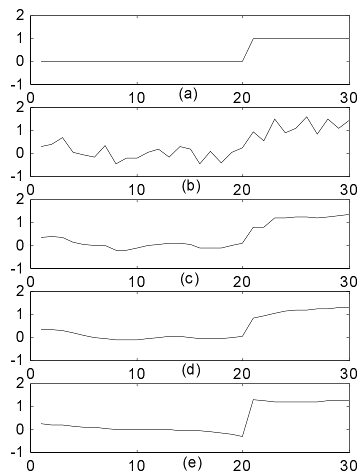

Figure 1. (a) Original step, (b) Blurred signal contaminated by white Gaussian noise (SNR=5dB), (c-e) Diffusion process after iterations: 20, 40, 160, respectively.

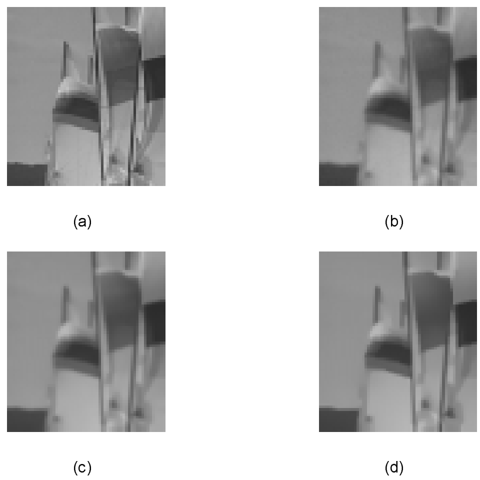

Figure 2. (a) Original part of sailboat image, (b) Blurred image contaminated by noise , (c-d) Diffusion process after iterations: 10, 40, respectively.