In Memoriam: Moshe Zakai (1926–2015)

We are deeply saddened by the loss of a dear friend. Moshe Zakai was an extraordinarily talented man who made a major difference in the life and career of those who collaborated with him as well as many of his students. Born in 1926 in Sokólka, Poland, Zakai came to Israel (then Palestine) as a child and passed away on November 27, 2015, in his hometown Haifa.

The advanced status of statistical communication theory and Information Theory in Israel stems historically from the fact that about a century ago, long before the 1948 establishment of the state of Israel, the Technion in Haifa and the Hebrew University in Jerusalem were founded. The Electrical Engineering department at the Technion excelled in classical power engineering and electromagnetic theory.

After the establishment of the State of Israel in 1948 there was an urgent need to quickly develop a technical leadership at the scientific department of the ministry of defence. This led to adopting an innovative policy of encouraging and financially supporting young promising engineers to get a Ph.D. from first league universities abroad. Moshe Zakai was one of the first to benefit from this policy.

From 1956 to 1958 Zakai did graduate work at the University of Illinois and was awarded the PhD in Electrical Engineering. He then returned to the scientific department as head of the communication research group. He encouraged Jacob Ziv, who showed interest in the newly established Information Theory, and later Israel Bar-David, who was interested in radar problems, to use the same program and go to MIT for their D.Sc. degrees.

In 1965 Moshe Zakai and Israel Cederbaum, an expert on graph theory and networks, migrated from the scientific department to the Technion EE department; they were later joined by Jacob Ziv and Israel Bar-David. This migration, initiated and led by Cederbaum and Zakai, completely renovated the department, turning the Technion into an advanced academic center excelling in Statistical Communication, Information Theory, Computer Engineering and Probability Theory.

Moshe Zakai was a strong proponent of exploiting modern advanced mathematical tools in the study of communication and radar theory. Motivated by his interests in these topics, he became deeply interested in stochastic differential equations. Soon, together with Eugene Wong, he realized that there was a serious obstacle in applying Itô's theory: white noise is not physical, and Itô's solution was not continuous in the input (in the sense that driving a stochastic differential equation with an approximation of white noise does not yield a solution that is close to Itô's solution).

Together, Wong and Zakai, in a ground-breaking 1965 paper, showed how to resolve this problem: an extra term (now called the *Wong-Zakai correction*) has to be added to the "physical" equation, and with this correction term continuity is restored. This observation now opened the door to rigorous applications of Itô calculus in communication and control on the one hand, and to new developments in the theory of stochastic processes on the other. To some extent, one could interpret Martin Hairer's recent theory of regularity structures (for which he received the Fields medal in 2014) as a suitable way to introduce Wong-Zakai corrections in the setup of nonlinear stochastic *partial* differential equations.

Another topic to which Zakai made a seminal contribution is the theory of nonlinear filtering. Filtering deals with extracting a signal from a noisy observation of it, by computing the conditional distribution of the signal given the observations. In the setup of Gaussian processes, the problem was solved in the 40s by Wiener and Kolmogorov (It is worthwhile to note that Wiener was motivated by control applications, stemming from the WWII effort.) Later, Kalman devised a recursive filter that computed the optimal (linear) filter; Kalman's filter was a crucial element in the development of modern control, radar and communication systems. However, it did not perform well in non-Gaussian situations, where one needed to evaluate the optimal *nonlinear* filter.

The mid 60's saw a flurry of activity in addressing this challenge, and various representations of the optimal filter were derived. However, none of those could be computed effectively, as it required solving an infinite system of coupled stochastic differential equations. Zakai's major insight in his fundamental 1969 paper was to realize that by focusing on an un-normalized version of the conditional density, one could obtain a single bilinear stochastic partial differential equation (the *Zakai Equation*), from which the filter could be easily computed (and which reduces to the Kalman filter in the Gaussian case). Zakai's equation has been the basis for all progress in filtering theory; in particular, modern approaches to compute the filter using genetic algorithms ("particle filters") effectively compute the solution to Zakai's equation.

Already early in his career, and motivated by communication and radar applications, Zakai derived with Jacob Ziv the *Ziv-Zakai* bound on parameter estimation error, exploiting some information theoretic tools in the process. Later, they extended these to the filtering setup. Zakai returned to this problem throughout his career, each time applying new tools. In particular, he derived with his student Ben-Zion Bobrovsky bounds on the filtering problem based on an infinite dimensional extension of the Cramer-Rao bound, and later, with his students Eddy Mayer-Wolf and Ofer Zeitouni, he used ideas from Malliavin's calculus to improve on those.

After a foray with Eugene Wong into the study of multi-parameter stochastic processes, Zakai completed in the last two decades of his professional life his transition to a full time probabilist, working on the Malliavin calculus and its application in the study of changes of measure. In a nutshell, the Malliavin calculus (introduced by Malliavin in 1979) studies the smoothness of the law of Wiener functionals, and in particular of solutions to Itô's equations with respect to perturbations of the driving white noise. Malliavin's original motivation was to give a probabilistic proof of Hörmander's criterion for the regularity of solutions of parabolic partial differential equations. Zakai was one of a handful of probabilists who started working on Malliavin's calculus shortly after its introduction. Very early on, he introduced a different, more geometric, approach, summarized in his influential 1985 paper. He then embarked with Süleyman Üstünel and others on a program to apply these ideas to the study of anticipative changes of measures on Wiener space. Their joint book from 2000 summarizes the theory and is the standard reference for the study of transformations on Wiener space.

Zakai continued his research program well into retirement. In 2005, using Malliavin's calculus as a working tool, Zakai came back to his information theoretic roots and extended the Guo-Shamai-Verdu relations between information, filtering and smoothing in the general Gaussian channel to the abstract Wiener space setup, in a way closing the loop between his early and later research interests.

Zakai's work was recognized by many awards, including the IEEE control society prize and Israel's Rothschild prize. He was a Fellow of the IEEE, Fellow of the Institute of Mathematical Statistics, foreign member of the US National Academy of Engineering and a member of the Israel Academy of Sciences and Humanities. He was a strong proponent of employing sophisticated mathematical tools in engineering, and his influence in that direction helped shape the revolution in EE departments worldwide in the 60s and 70s. His voice and mentorship will be sorely missed by his many colleagues, students and friends.

Eddy Mayer Wolf, Ofer Zeitouni, and Jacob Ziv