Latency Optimized Mapping of Logic Functions for Memristor Aided Logic (MAGIC)

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Technical Report – Latency Optimized Mapping of Logic Functions for Memristor Aided Logic (MAGIC)

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Abstract-This document describes in detail the optimization problem proposed in [1].

I. LATENCY OPTIMIZATION PROBLEM

The latency optimizing problem of an in-memory Boolean function has two degrees of freedom: the locations of data and the execution time of different logic gates. The following variables are defined for each logic gate:

- \( \{R_{A_j}, C_{A_j}\}, \{R_{B_j}, C_{B_j}\}, \{R_{E_j}, C_{E_j}\} \) - Location (coordinates of memory cells) of the inputs and the output of NOR gate \( j \).
- \( \{R_{A_k}, C_{A_k}\}, \{R_{E_k}, C_{E_k}\} \) - Location (coordinates of memory cells) of the inputs and the output of NOT gate \( k \).
- \( T_i \) - Clock cycle in which gate \( i \) (either NOT or NOR) is executed.

Therefore, each NOR gate and each NOT gate has 7 and 5 variables, respectively.

![Figure 1 - Inputs and outputs of NOR and NOT gates](image)

The execution of a given Boolean function is finished when the operations of all gates are completed, thus \( \max_j(T_j) \) is the latency of a specific mapping, where \( 0 < j \leq \#\text{gates} \). Therefore, the latency (in clock cycles) of the best mapping is the minimum latency out of all different mappings and is

\[
\text{Latency}_{\text{best mapping}} = \min \left\{ \max_j(T_j) \right\}
\]

\( 0 < j \leq \#\text{gates} \) (1)

The legal mappings are limited by location, connectivity and timing, and are restricted by the following constraints:

1) Location constraints:
- Every input and output (I/O) has to be mapped to a memory cell, thus the coordinates of each I/O are limited by the physical size of the memory.
  \[
  \forall x_j \in \{A_j, B_j, E_j\} : \left( 0 < C_{x_j} \leq \text{Col}_{\text{num}} \right) \cap \left( 0 < R_{x_j} \leq \text{Row}_{\text{num}} \right)
  \]
- Two different outputs cannot be placed in the same memory cell (whereas the same input may be used for two different gates, therefore their inputs share coordinates). In such a configuration, reusing of a cell (after resetting) is not supported. Note that this constraint is not mandatory and is used to simplify the problem at the cost of potentially using more cells.
  \[
  \forall E_k, E_j : (C_{E_j} \neq C_{E_k}) \cup (R_{E_j} \neq R_{E_k})
  \]
• I/Os of each gate have to be located in the same row and different columns, or vice versa.

\[ \forall \text{gate } j: \left[ \left( C_{A_j} = C_{B_j} = C_{E_j} \right) \cap \left( R_{A_j} = R_{B_j} \neq R_{E_j} \right) \right] \cup \left[ \left( C_{A_j} \neq C_{B_j} \neq C_{E_j} \right) \cap \left( R_{A_j} = R_{B_j} = R_{E_j} \right) \right] \]

• The execution of different NOR or NOT gates simultaneously is possible only when they are aligned in the rows or in the columns.

\[ \forall \text{gates } j, k: \quad T_j \neq T_k \cup \left[ \left( \left( C_{A_h} = C_{B_h} \right) \cup \left( C_{A_j} = C_{B_j} = C_{E_j} \right) \right) \cap \left( R_{A_j} = R_{B_j} = R_{E_j} = R_{A_h} = R_{B_h} = R_{E_h} \right) \right] \cup \left[ \left( \left( R_{A_h} = R_{A_h} \right) \cup \left( R_{A_j} \neq R_{B_j} \right) \right) \cap \left( R_{E_j} = R_{E_h} \right) \right] \cap \left( C_{A_h} = C_{A_j} = C_{B_h} = C_{B_j} = C_{E_h} = C_{E_j} \right) \]

2) Connectivity constraints:
The following constraint is the only constraint which is determined by the connectivity of the gates, according to the netlist.

• Every output of gate \( h \) that is connected to an input of gate \( j \) has to be mapped to the same memory cell, and the execution of gate \( j \) has to be performed only after the execution of gate \( h \). This configuration does not support movement of data within the memory array and is not mandatory.

\[ \forall E_h, x \in \{ A_j, B_j \} \text{ that are connected: } \left[ \left( C_{E_h} = C_{x_j} \right) \cap \left( R_{E_h} = R_{x_j} \right) \right] \cap (T_h < T_j) \]

![Figure 2](image)

**Figure 2** – output of gate \( h (E_h) \), is connected to an input of gate \( j (A_j) \)

3) Timing constraints:

• The execution time of each gate is positive

\[ \forall \text{NOT gate } k, \text{NOR gate } j: T_j, T_k > 0 \]

• The execution of a NOR gate and a NOT gate have to be done during different clock cycles since their number of inputs is different

\[ \forall \text{NOT gate } k, \text{NOR gate } j: T_j \neq T_k \]

REFERENCES