Power Grid Analysis Based on a Macrocircuit Model

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Outline

- Review of Conventional Model
- Basic Microcircuit Model
- Macrocircuit Model
- Feedback Between the Power Grid and Current Consumer
- Summary – Advantages and Potential Applications
Power Distribution Networks

\[ V_{dd} - IR_p - L_p \frac{dI}{dt} \]

\[ V_{gnd} + IR_g + L_g \frac{dI}{dt} \]
Power Grid Voltage Drop

- Slows the circuit – performance and functionality compromised
- Excessive voltage drop – logic errors
- Power grid design – **low voltage drop in all of the nodes**
Decoupling Capacitors

Lowers the supply voltage fluctuations
Decoupling Capacitors

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Decoupling Capacitors

Lowers the supply voltage fluctuations
Current Consumers

- Billions of transistors in a modern IC
- Every logic gate consumes current from the power supply
- Transistors are non-linear devices
Conventional Current Consumer Model – Ideal Current Source

- Simulate individual circuit blocks, including transistors and parasitic elements within the power interconnect
- Replace each block by an ideal current source – a linear device
Conventional Power Grid Model

\[ V_{dd} - I(t)R_p - L_p \frac{dI(t)}{dt} \]

\[ V_{eqd} + IR_p + L_p \frac{dI(t)}{dt} \]

\[ 10^5 \]

\[ I_{Load} \]

\[ R_p \]

\[ L_p \]

\[ C_{dec} \]

\[ 10^4 \]
# Conventional Current Consumer Model – Characterization

<table>
<thead>
<tr>
<th>Old Model</th>
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<tbody>
<tr>
<td>Intuitive</td>
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<tr>
<td>Know every node’s voltage</td>
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Power Supply Time Scale

Clock Time Scale

psec, nsec

msec
Power Supply Time Scale

Clock Time Scale

msec

psec, nsec
Power Supply Time Scale

Clock Time Scale

Power Supply Time Scale
msec

Clock Time Scale
psec, nsec
Microcircuit Model
Microcircuit Model
Microcircuit Model

Conventional Model
Microcircuit Model

Power Supply Time Scale

Clock Time Scale

$V_{grid+}$

$R_1$

$C_1$

$V_1(t)$

$R_2$

$C_2$

$V_2(t)$

$V_{grid-}$

$V_{grid+}$

$R_{eff}(t)$

$C_{eff}(t)$

$V_{grid-}$
Effective Impedance

- **Effective capacitance** – accurately represents average energy stored in the element
- **Effective resistance** – accurately represents average power dissipated by the element
Determining the Effective Impedance

- Determine the effective impedance at the maximum switching rate of the element
- Determine the actual effective impedance of the element
  - A fraction of the maximum switching rate
Maximum Switching Rate

- Steady state assumption
- A non-trivial solution exists
Effective Impedance at Maximum Switching Rate

Based on delivered charge and energy dissipation for every cycle

\[ Q = \int_{0}^{T} dt I(t) \]

\[
E_1 \equiv \exp \left[ -\frac{T}{2R_1(C_1 + C_2)} \right], \quad E_2 \equiv \exp \left[ -\frac{T}{2R_2(C_1 + C_2)} \right]
\]
Effective Impedance at Maximum Switching Rate

- Based on delivered charge and energy dissipation for every cycle

\[ Q = \int_{0}^{T} dt I(t) \]

\[ C_0 = \frac{Q}{V_{CC}} = (C_1 + C_2) \frac{(1 - E_1)(1 - E_2)}{1 - E_1 E_2} \]

\[ E_1 \equiv \exp \left[ -\frac{T}{2R_1 (C_1 + C_2)} \right], \quad E_2 \equiv \exp \left[ -\frac{T}{2R_2 (C_1 + C_2)} \right] \]
Effective Impedance at Maximum Switching Rate

- Based on delivered charge and energy dissipation for every cycle

\[ W_D = \frac{1}{R_1} \int_0^{T/2} V_1^2(t)dt + \frac{1}{R_2} \int_{T/2}^T V_2^2(t)dt \]

\[ E_1 \equiv \exp \left(-\frac{T}{2R_1(C_1 + C_2)}\right), \quad E_2 \equiv \exp \left(-\frac{T}{2R_2(C_1 + C_2)}\right) \]
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\[
R_0 = \frac{V_{CC}^2}{W_D} \cdot T = \frac{1 - E_1E_2}{(C_1 + C_2)(1 - E_1)(1 - E_2)} \cdot T
\]

\[
E_1 \equiv \exp \left[ -\frac{T}{2R_1(C_1 + C_2)} \right], \quad E_2 \equiv \exp \left[ -\frac{T}{2R_2(C_1 + C_2)} \right]
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**Effective Impedance at Maximum Switching Rate**

- Based on delivered charge and energy dissipation for every cycle

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R_0 = \frac{V_{CC}^2}{W_D} \cdot T = \frac{1 - E_1 E_2}{(C_1 + C_2)(1 - E_1)(1 - E_2)} \cdot T = \frac{T}{C_0}
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\]
Effective Impedance at Maximum Switching Rate

- Based on delivered charge and energy dissipation for every cycle

\[ C_0 = \frac{Q}{V_{CC}} = (C_1 + C_2) \frac{(1-E_1)(1-E_2)}{1-E_1E_2} = C_1 + C_2 \]

\[ R_0 = \frac{V_{CC}^2}{W_D} \cdot T = \frac{1-E_1E_2}{(C_1 + C_2)(1-E_1)(1-E_2)} \cdot T = \frac{T}{C_0} \]

\[ E_1 = \exp \left[ -\frac{T}{2R_1(C_1 + C_2)} \right], \quad E_2 = \exp \left[ -\frac{T}{2R_2(C_1 + C_2)} \right] \]
Effective Impedance at Maximum Switching Rate

- Based on delivered charge and energy dissipation for every cycle

\[ R_0 C_0 = T \]

\[ C_0 = C_1 + C_2 \]
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Macrocircuits

- Expandable to numerous parallel microcircuits

\[ R_0 = R_{0,1} \parallel R_{0,2} \parallel ... \parallel R_{0,N} \]

\[ C_0 = C_{0,1} + C_{0,2} + ... + C_{0,N} \]

- For N microcircuits in parallel (N~10^6)
Macro Circuits – Effective Impedance

- Actual switching rate is less than the maximum rate
- Activity function of the macro circuit is considered

\[ C_{\text{eff}}(t) = C_0 \cdot \alpha(t), \quad R_{\text{eff}}(t) = R_0 \cdot \frac{1}{\alpha(t)} \]

- 1 nF
- 10-100 Ω
- 1 msec
Power Grid Analysis

\[
\dot{Q}_c(t) + \frac{1}{\tau(t)} Q_c(t) = \frac{V_0}{R_0}
\]

\[
\tilde{C} = C_0 + C_{\text{eff}}(t)
\]

\[
\frac{1}{\tau(t)} \equiv \frac{1}{\tilde{C}(t)} \left[ \frac{1}{R_{\text{eff}}(t)} + \frac{1}{R_0} \right]
\]

\[
Q_c(t) \equiv \tilde{C}(t) V_R(t)
\]
Power Grid Analysis

\[ \tilde{C} = C_0 + C_{\text{eff}}(t) \]

\[ \frac{1}{\tau(t)} \equiv \frac{1}{C(t)} \left[ \frac{1}{R_{\text{eff}}(t)} + \frac{1}{R_0} \right] \]

\[ Q_c(t) \equiv C(t)V_R(t) \]

\[ Q_c(t) = Q_c(0)e^{-\int_0^t dt' \frac{1}{\tau(t')}} + \frac{V_0}{R_0}e^{-\int_0^t dt' \frac{1}{\tau(t')}} \cdot \left[ \int_0^t dt'' \frac{1}{\tau(t'')} \right] \]
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Feedback Between the Power Grid and Current Consumer

Negative feedback – decrease voltage drop

Ideal current source model ignores this interaction
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Ideal current source model ignores this interaction
Comparison Between the Models – Feedback Issue

Voltage Drop vs. delta R

Ideal Current Source Model
Macro Circuit Model

Dependence on the Grid Resistance

Ideal Current Source Model
Macro Circuit Model
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Summary – Advantages and Potential Applications
## Summary

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<th>New Model</th>
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<tr>
<td>Intuitive</td>
<td>Less intuitive</td>
</tr>
<tr>
<td>Know every node’s voltage</td>
<td>Can’t know every node voltage drop</td>
</tr>
<tr>
<td>Significant computational time</td>
<td>More time efficient than the conventional model</td>
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<tr>
<td>Active supply as a consumer – misleading</td>
<td>Based only passive elements</td>
</tr>
<tr>
<td>Ignores effect of voltage drop on the current consumer</td>
<td>Considers the interactions between the power grid and the element</td>
</tr>
<tr>
<td>Complex</td>
<td>Complex reduced</td>
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![Diagram](image_url)
Thank You!