Technion - Israel Institute of Technology
Department of Electrical Engineering Spring 2009

Estimation and Identification in Dynamical Systems (048825)
Computer Exercise 1
Submit by 19/5/09

1 Introduction

In this exercise we shall implement a Kalman Filter for target tracking. We choose a standard "second order model", where a (target acceleration) is a white noise: $\frac{d^2}{dt^2}x =$ white noise. Thus, possible changes in target velocity are modeled as (white) noise. After discretization, we obtain

$$x(k+1) = F_1 x(k) + G_1 w(k) \tag{1}$$

where $x(k) = (x_p, x_v)^T$ is the state vector carrying the position and velocity information, $w(k)$ is a zero mean white Gaussian "acceleration noise" with covariance σ_w^2,

$$F_1 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix},\ G_1 = \begin{bmatrix} 0.5T^2 \\ T \end{bmatrix}$$

and T is the sampling interval.
Remark: Check the interpretation of these equations without the noise term. The complete equation will be derived in a future lecture.

We assume that position-only measurements are available at the sampling points, namely

$$z(k) = H_1 x(k) + v(k) \tag{2}$$

where $H_1 = [1\ 0]$ and $v(k)$ is a zero mean Gaussian measurement noise with covariance σ_v^2.

To deal with 2-D tracking we may assume a decoupled motion along each axis. Namely, the state space modeling of the the dynamics and measurement models will be formulated as,

$$X(k+1) = FX(k) + GW(k)$$
$$Z(k) = HX(k) + V(k),$$

where

$$X(k) = (x_p, x_v, y_p, y_v)^T,\ Z(k) = (z_x(k), z_y(k))^T$$

with $z_x(k)$ and $z_y(k)$ being the one dimensional measurements defined in Eq. (2). Similarly,

$$F = \text{diag}[F1,\ F1],\ G = \text{diag}[G_1,\ G_1],\ H = \text{diag}[H_1,\ H_1]$$

$V(k)$ and $W(k)$ are vector valued measurement and process noises, having i.i.d components. Under this setting, a 2-D tracking may be obtained using an independent filter for each axis.

2 Tasks

2.1 Trajectory and Measurements

With $x_p(0) = y_p(0) = 0$, generate the following trajectory of the target to be tracked.

1. Between $k = 0(sec)$ and $k = 200(sec)$ the target moves with a constant velocity $x_v = 300m/\sec,\ y_v = 0m/\sec$.

2. Between $k = 200(sec)$ and $k = 300(sec)$, the target performs a $270°$ maneuver with constant tangential velocity of $300m/\sec$. This corresponds to an acceleration of $\approx 1.4G$.

3. Between $k = 300(sec)$ and $k = 500(sec)$, the target moves with a constant velocity $x_v = 0m/\sec,\ y_y = 300m/\sec$.

Assuming $\sigma_v = 1000(m)$ (in both directions), generate noisy position observations every $T = 1(sec)$. Plot the trajectory and the observations (on the same x-y chart), and the velocity in both axes versus time.
Note: The trajectory and measurements will be fixed throughout the exercise!

2.2 Tracking

Implement a KF for estimation of the target state. State clearly what initialization you used.

1. Let $\sigma_w = 0.3(m/s^2)$. Plot on the same figure the estimated trajectory, the raw measurements, and the true trajectory for . On the same figure (different subplot) plot the estimation and measurement errors versus time. Repeat for the velocity. (Do not square the errors). Explain your results for all parts of the trajectory. In your explanation refer to the target model.

2. In item 1 above you have obtained a large estimation error at some part of the trajectory. Modify the filter process noise σ_w to reduce this phenomenon. Plot the position data (true and estimated trajectories and the raw measurements) as well as the corresponding errors. Explain your observations.

3. Set σ_w to its original value. Repeat the first task with the **filter parameter** σ_v reduced by three orders of magnitude. Plot the position data only. Explain your results. Discuss and compare the three experiments.
   Note: the measurements are still generated with the same standard deviation as before. It is the filter parameters that change.

4. Set σ_w and σ_v to their original values. Initialize your filter with the state vector $(0, -100, 0, -100)^T$. Set the initial error covariance to be the identity matrix. Repeat the experiment, plot and discuss your results.

5. Increase the initial error covariance such that the undesired phenomenon you've obtained in the previous part is reduced. Repeat the experiment, plot and discuss your results.

6. Now we shall consider the case of an imperfect sensor. Assume that the sensor detects the target with a known probability P_d. If at time k the target is not detected, there is no measurement provided to the filter at that time. Propose a (slightly) modified filter that is capable of dealing with such cases. Using previously generated measurements, test your filter for $P_d = 0.5$. Plot on the same figure the estimation results for the new filter and the corresponding estimates of the original filter (i.e. for $P_d = 1$). Explain and discuss the results.