Distributed Network Protocols
Lecture Notes[1]

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[1]Thanks are due to Lior Shabtay for producing and solving many of the problems

Contents

Preface

This report contains Lecture Notes for the Distributed Network Protocols course that I have taught at the Technion some time ago. The course is at the senior-undergraduate / first-year-graduate level. Its pre-requisite is an introductory course in Computer Networking.

Most of the presented material is in reasonable form, but some parts are still in preliminary stages. This is the case in particular with the Minimum Spanning tree and the Session Management chapters. I will periodically update my website home page with updated versions.

Chapter 1

Introduction

Chapter 2

DATA-LINK CONTROL PROTOCOLS

Data Link Control (DLC) Protocols are protocols that use error detection and retransmission mechanisms to protect data sent over a noisy communication media from transmission errors. The media can be any lower layer transmission facility, like one link or a sequence of links, a local area network (LAN) using an arbitrary medium access control (MAC) mechanism, a logical connection or a Virtual Channel in an ATM network. The main role of the DLC protocol is to accept data at one end of the media and ensure its delivery at the other end in the same order as accepted, without losses or duplicates. As long as the media does not fail, all data should be delivered at the receiving end in finite time. If data accepted by a DLC protocol cannot be transmitted because of media failure, appropriate notification should be submitted to the higher layers. DLC protocols which guarantee these properties are said to provide *data reliability*. As demonstrated in later chapters, higher level protocols normally rely on the fact that the DLC provides data reliability on each of the links of the network.

The most commonly used DLC Protocols are the bit-oriented DLC Protocols such as HDLC [ISO81], [SDL80], [IBM70], ADCCP [Car82], LAP-B (Link Access Protocol - Balanced) used in X.25 [Sta92] or LAPD (Link Access Protocol - D-channel) defined in CCITT recommendation I.441/Q.921 for ISDN [Sta92]. In these protocols there are three situations that may result in undetected transmission errors: i) undetected frame errors, ii) improper operation of the DLC Protocol, iii) incorrect initialization of the DLC Protocol.

There is no way to ensure that undetected transmission errors will never occur under any circumstances. There is always the possibility that a logically correct program will execute improperly due to hardware or system errors. Moreover, any error detection scheme has an inherent probability of undetected errors. Consequently, all the work on reliable DLC Protocols is directed towards minimizing the probability of undetected transmission errors.

The issue of detecting frame errors has received great attention in the literature and powerful cyclic redundancy coding (CRC) schemes are currently used to minimize the probability of undetected frame errors. This subject is not addressed in the present work.

The issue of proper operation of the DLC Protocol after initialization is addressed in Sections 2.2 and 2.3. We provide a rigorous definition of the concepts of *data reliability* and *synchronization* at initialization and prove that sliding-window DLC-protocols ensure data reliability, provided that they are synchronized at initialization and all frame errors are detected. The issue of correct initialization of the DLC Protocol is addressed in Section 2.4.

2.1 The Model

The configuration of two DLC processes connected by a transmission media is given in Figure 2.1. *Data source* is a generic name for some device, process or higher layer that produces data strings which have to be transmitted over a communication media to a *data sink*. We shall assume that the data strings are packetized and shall refer to them as *packets*. Furthermore, we assume that the packets are queued in a buffer at the data source and that they are transferred *in order* to the DLC process *at times dictated by the DLC Protocol.* At the other end, the packets are delivered by the DLC process to the data sink at times dictated by the DLC Protocol.

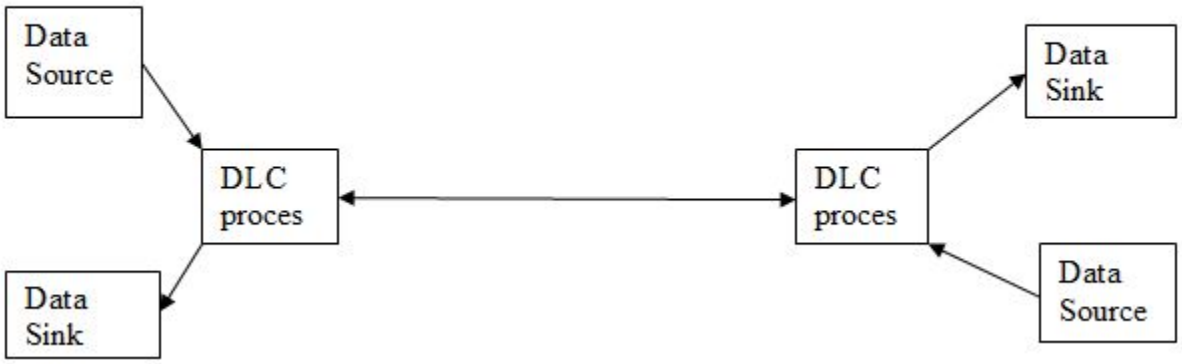


Figure 2.1: The Model

A bit-oriented *Data Link Control Protocol* is a pair of processes, one at each station, that operate together using some type of *acknowledgement* and *retransmission* scheme to ensure data reliability over an error-prone transmission media. A DLC process accepts packets from the data source, transforms them into *information frames* by appending any necessary control, sequencing, framing and error detection information and transfers them to the lower layer. In addition, it receives incoming frames from the lower layer, checks them for correctness, converts them back into packets, and, if the DLC protocol dictates so, passes them on to the data sink.

The DLC processes are served by a point-to-point FIFO-preserving communication media between two communicating stations. Frames delivered by one DLC to the media may be lost, may arrive at the other end in error (e.g., because of transmission noise), or may arrive correctly after some finite but unknown delay. We assume that all errors are detected by the DLC and the frames received in error are discarded. FIFO-preserving media means that nondisrupted frames arrive in the same order as sent and a frame cannot be in the media if frames sent at a later time have arrived already. We do not require that a bound on the transmission delay is known a priori. Finally, we assume that when the transmission media is operational, the probability that a unit is lost or received in error is strictly less than 1. Examples of FIFO-preserving media are one communication link or a sequence of links, a local area network (LAN) using an arbitrary medium access control (MAC) mechanism, a logical connection or a Virtual Channel in an ATM network, for which lower layer protocols or the physical properties ensure FIFO. A TCP connection in the Internet operates over a non-FIFO-preserving media, since the IP layer allows packets of a TCP connection to be sent on different routes.

The protocol used by DLC processes to transmit data reliably over the error-prone transmission media

are referred to as the DLC Protocol. Each such protocol is composed of two stages: *Initialization stage and Connected stage.* The purpose of the Initialization stage is to synchronize the two DLC processes and clean the channel of old information, while in the Connected stage the processes exchange information data. Whenever a node comes up, the DLC process enters Initialization stage. Normally, one also considers a third stage, the Disconnection stage. However, if the later comes as result of a disconnection request and the connection is never reestablished, that stage is irrelevant for our purposes. If Disconnection comes as a result of a failure and the connection must be reestablished, the Disconnection stage is incorporated in the Initialization stage of the reestablished connection.

2.2 The Alternating Bit Protocol

CHECK IF CHANGES ARE NEEDED

The simplest DLC protocol is the Alternating Bit Protocol suggested in [BSW69]. For purposes of illustration, we shall describe it here in an environment when there are no failures and assume that the two processes are synchronized at some initial time when the system is devoid of any old frames. The basic model assumptions are:

a) There are no failures in the system.

b) The probability of error or loss of a transmission unit in the media is strictly less than 1.

c) There is an initial time when the two DLC's are synchronized, i.e. the variables VS and VR defined below are $VS = VR = 0$.

d) At initial time there are no frames in the system.

The DLC processes A and B hold binary variables VS and VR respectively, whose values at initialization are assumed to be $VS = VR = 0$. The DLC processes fetch packets from their local data source, at times dictated by the protocol, and transform them into frames by attaching to each a bit, called the *alternating bit* . The bits carried by frames sent by DLC A and B will be denoted by NS and NR respectively. The corresponding frames will be denoted by A_{NS} and B_{NR}. A frame that arrives correctly from DLC A to DLC B and carries bit NS will be denoted by A_{NS}. Such a frame may be delivered by DLC B to the local data sink or may be discarded, as dictated by the protocol. DLC B discards any frame received in error from A. This event at DLC B will be denoted by A_e. Similarly, B_{NR} and B_e will denote respectively the receipt of a correct frame carrying bit NR and the receipt of an error-corrupted frame. In addition, both DLC A and DLC B contain timers that expire periodically.

The Alternating Bit Protocol is specified in Table 2.1 and summarized in Fig. 2.2. The notation T/A next to a state transition denotes the fact that T is the trigger for that transition and A is the action to be taken *after* transition. The triggers will generally be the receipt of particular frames or a timeout. The alternating Bit Protocol starts with $VS = VR = 0$ and DLC A accepting the first packet from the source. It attaches to it $NS = 0$ and sends it to the other side. If this frame arrives correctly, DLC B fetches the first packet from its data source, assigns 1 to it and sends it over. If it does not, DLC B sends a dummy frame B_0, which, upon arrival, correctly or not, forces retransmission of A_0. The activity at A is as follows: when it receives B_{NR} with $NR \neq VS$, it considers the last sent frame acknowledged, flips VS, fetches a new packet from the local data source assigns to it the new VS and sends it over. In addition, it delivers the received B_{NR}, after deletion of the NR bit, to the local data sink. One can prove that the dummy frame is never delivered to the local data sink. If it receives B_{NR} with $NR = VS$ or B_e (a B frame with error) or the timer expires, it resends A_{VS}. The activity at B is similar, except that $NS = VR$ signifies acknowledgement of the last frame, while $NS \neq VR$ (as well as an error in the received frame or the expiration of the timer) triggers retransmission of B_{VR}. The alternating bit sent by B has a double meaning: the bit attached to the data frame sent by B to A, as well as the bit that B expects to see in the next correctly received frame from A. Similarly, the bit sent by A to B has a double meaning: the appropriate bit for data from A to B, as well as an acknowledgement for having correctly received the data frame with this bit from B. For example, the bit 1 in the first A_1 sent by A to B indicates that this frame contains the second packet fetched by DLC A from its source, and also that A has received correctly B_1. Note that the algorithms of A and B are not symmetric and the meaning of the bit is different. The bit NR, sent by B to A is the next expected bit from A, whereas the next expected bit from B is not NS, but its 2-complement $\overline{NS}$.

Algorithm for A

```
Initialization (VS = 0)
      A_0 ← first packet accepted from data source;
      send A_0;
      start timer;


A1    receive B_NR or B_e or timer expires
A2    {   if  (received B_NR) {
A3            if  (NR ≠ VS) {
A4                  deliver payload of B_NR to local sink;          /*B_NR is not dummy*/
A5                  VS ← ~VS;
A6                  A_VS ← next packet accepted from local source;
                  }
A7            else  discard received packet;
              }
A8            send A_NS with NS = VS;
A9            reset timer;
      }
```

Algorithm for B

```
Initialization ( VR = 0 )
      B_0 ← dummy frame
      start timer


B1    receive A_NS or A_e or timeout
B2    {   if  (received A_NS) {
B3            if  (NS = VR) {
B4                  deliver payload of A_NS to local sink;
B5                  VR ← ~VR;
B6                  B_VR ← next packet accepted from local source;
                  }
B7            else  discard received frame;
              }
B8            send B_NR with NR = VR;
B9            reset timer;
      }
```

Table 2.1: The Alternating Bit Protocol

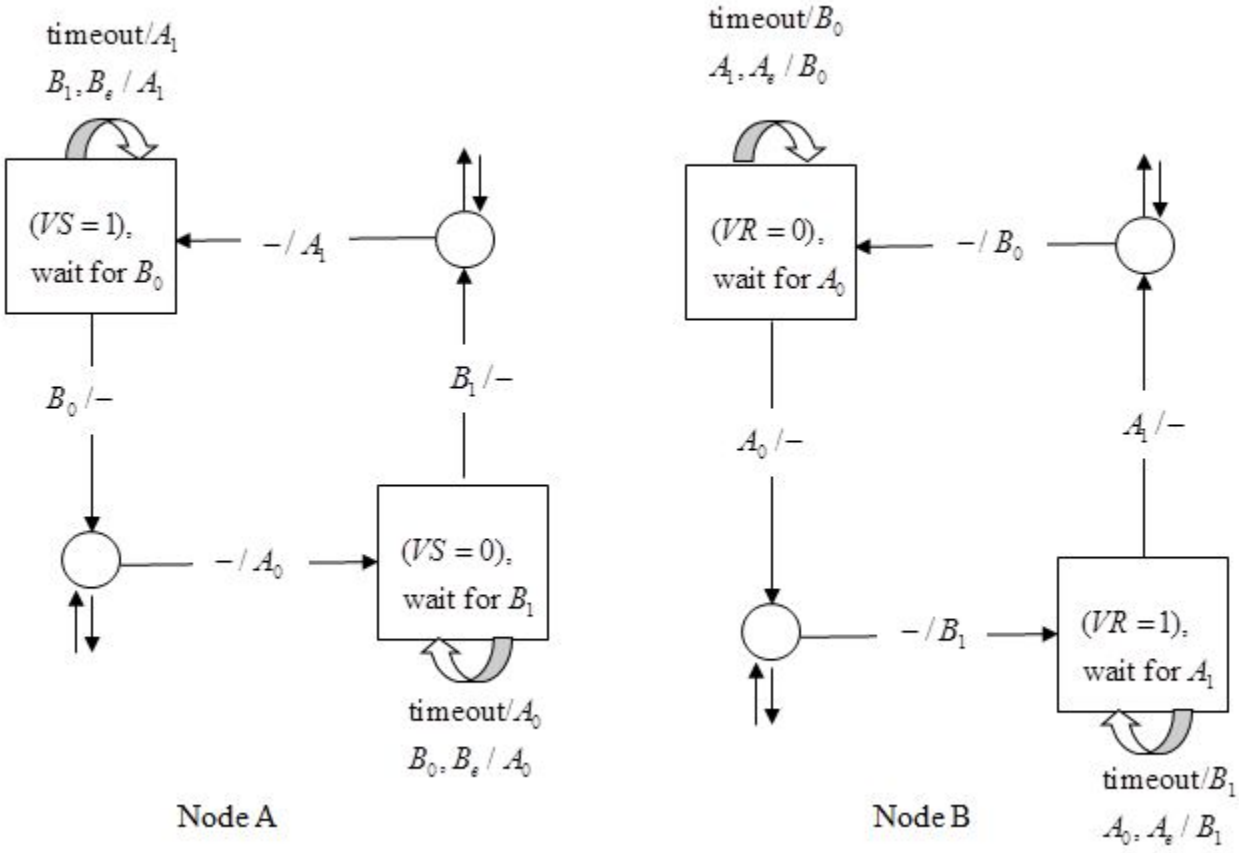


Figure 2.2: The Alternating Bit Protocol

Reliability of a DLC protocol will be fully defined in Sec. 2.3, but in the context of *no media failures*, it consists of the following properties. We say that a packet is *considered acknowledged* when it is replaced in the sending buffer by a new packet, i.e. in <A6> or in <B6> in the algorithm[1].

(1) FIFO: Packets are delivered to the data sink in the same order as received by the DLC from the corresponding data source with no gaps or duplicates.

(2) Confirm: All considered acknowledged packets have been delivered to the corresponding data sink.

(3) Delivery: All packets produced by the data source are considered acknowledged within finite time.

Note that **Delivery** and **Confirm** imply that all packets produced by the data source are delivered to the data sink in finite time. Reliability of the Alternating Bit Protocol has been investigated extensively and has been proved by many methods []-[]. We shall provide here the proof in a descriptional manner.

Theorem 2.1 *The Alternating Bit Protocol ensures data reliability.*

Proof:

Proof of FIFO and Confirm

We shall concentrate here on the proof for data flowing from the data source at A to the data sink at B. Afterwards we shall indicate how a similar proof can be applied for data flowing in the opposite direction.

Packets fetched by DLC A from the local data source will be numbered for identification purposes by consecutive increasing numbers $P(0), P(1), P(2), P(3), \ldots$, where $P(0)$ is the first packet fetched at initialization. Recall that at initialization $VS = 0$, the first packet $P(0)$ is assigned bit $NS = 0$ and VS is flipped whenever a new packet is fetched from the data source. Therefore the bit assigned to $P(I)$ is I mod 2.

When a frame B_{NR} is received by A with $NR \neq VS$, we shall say that the received frame is labeled *active.* The packet that is considered acknowledged (and is replaced in the local buffer) when an active frame is received, will be referred to as *correlated* with the active frame. The *FIFO* and *Confirm* properties can now be restated as follows:

Lemma 2.2 *Suppose a frame B_{NR} arrives at A and is labeled active. At the time when the frame was sent by B, all packets up to and including the correlated packet and only those, have been delivered to the data sink in order, with no duplicates and no gaps.*

Proof:

Notes i) and ii) below follow directly from the algorithms and will be used in the proof:

i) No frame containing packets prior to and including $P(I-1)$ can be sent by DLC A after having fetched packet $P(I)$ from the local data source; consequently no such frame can be received by B after any frame containing $P(I)$.

ii) No frame containing packets following and including $P(I+1)$ can be sent by A before the time when the frame containing packet $P(I)$ is replaced in the local buffer.

In order to describe the sequence of events that may occur between the two DLC's, we will use a timing diagram with two parallel time axes (one for each DLC) as shown for example in Figure 2.3. An arrow drawn from one axis to the other represents a frame sent by one station and received by the other. Each arrow is labeled with the corresponding message type. Now consider Figure 2.3 where NR_1, NR_2 denote

[1]The notation $< \cdot >$ indicates the appropriate line in the Algorithms.

the bits attached to two consecutive active acks. From the algorithm, $NR_2 = \overline{NR}_1$ and let t_1, t_3 and t_2, t_4 denote the respective arrival and departure times. Let $P(I)$, $P(I+1)$ be the packets correlated with the two consecutive active acks. The variable VS at time t_1- has the value $I \mod 2$ and hence $\overline{NR}_1 = I \mod 2$.

The induction step consists of showing that, if all packets up to and including $P(I)$ and only those have been delivered in order, with no duplicates and no gaps until time t_2, the same is true for all packets up to and including $P(I+1)$ until time t_4. This amounts to proving that during $[t_2, t_4]$, the only packet delivered to the sink at B is $P(I+1)$ and only once. Consider the interval of time from t_2 to t_4. First note that, since $P(I+1)$ is replaced in the buffer at time t_3, ii) above implies that no frame containing packets following and including $P(I+2)$ can arrive at B during the interval $[t_2, t_4]$. Also, i) above, says that after any frame containing $P(I)$ arrives at B, no frame containing $P(I-1)$ or preceding packets can arrive at B. Since by the induction assumption, some frame containing $P(I)$ has arrived at B before t_2, no frame containing $P(I-1)$ and preceding packets can arrive at B during $[t_2, t_4]$. Hence in the considered interval, the only frame with $NS = NR_1$ that can arrive is the one containing $P(I+1)$. Applying ii) again, if the frame containing $P(I+1)$ arrives, no frame containing $P(I)$ and preceding packets can arrive at B afterwards. The conclusion is that the only frame with bit $NS = NR_1$ that can arrive at B during the interval $[t_2, t_4]$ is the one containing $P(I+1)$ and if it arrives, no frame with $NS = \overline{NR}_1$ can arrive afterwards. Since VR at time t_2 is NR_1 and at time t_4 is $NR_2 = \overline{NR}_1$, the variable VR is flipped in $[t_2, t_4]$ and in view of the above it is flipped exactly once. This event can occur only when $P(I+1)$ is delivered to the data sink at B, completing the induction step.

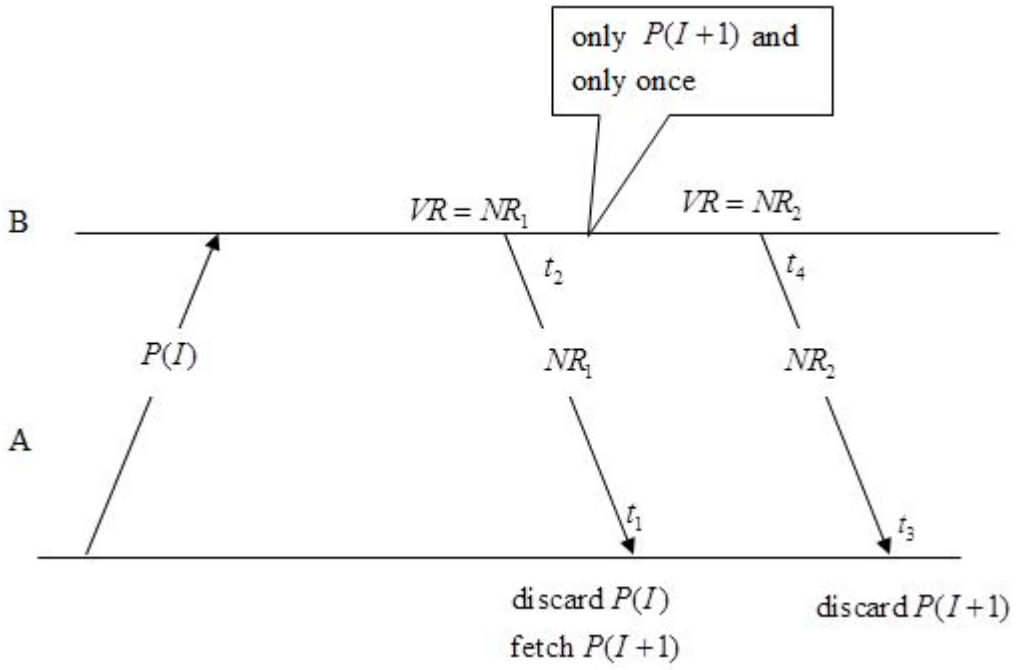


Figure 2.3: Diagram for proof of Lemma

It remains to prove the initial step of the induction. Since at initialization there are no frames in the system, the first received active frame is sent after that time, at time t' say, and let NR' be its assigned bit. Since upon receipt of the first active frame $VS = 0$, holds $NR' = 1$. By ii) above, only the frame containing $P(0)$ can be received by DLC B before NR' is sent. Finally the same argument as before shows that $P(0)$

has been indeed accepted and only once, completing the induction for the direction from A to B. qed

The induction step for the flow of data from B to A is identical to the one for the other direction with R and S, as well as $=$ and $\neq$, interchanged. The difference is only in the initialization part, since receipt of the first A_0 at B does not necessarily signal that the dummy frame B_0 was received at A. Consequently, the initialization step is for $P_B(1)$, the first frame fetched from the data source[2].

Proof of Delivery

Suppose that packet $P(I)$ is the first packet that is never considered acknowledged. This means that $P(I)$ is never replaced in the sending buffer at DLC A. From Lemma 2.2 follows that when packet $P(I)$ is fetched from the data source, all packets up to and including $P(I-1)$ have already been delivered to the data sink. Therefore DLC B will not change VR until packet $P(I)$ is correctly received. Since there is a timer at A, the frame containing $P(I)$ will be sent an infinite number of times. We have assumed that the loss or error probability of a frame is strictly less than 1, hence the frame will eventually arrive correctly, causing $P(I)$ to be delivered to the data sink at B. At that time, VR is changed to $(I+1) \mod 2$. DLC B also has a timer and will send B_{NR} with $NR = (I+1) \mod 2$ an infinite number of times. Again, one of these ACK's will arrive correctly at the source DLC, causing $P(I)$ to be considered acknowledged, contradicting the assumption. This completes the proof of the Theorem. qed

Problems

Problem 2.2.1 Prove that the comment in <A4> is correct namely that if $(NR \neq VS)$, then the received frame cannot be the dummy frame.

Problem 2.2.2 Suppose that assumption c) on page 12 is changed to $VS = 0$, $VR = 1$, at initial time. Will the Alternating Bit protocol still work? Prove reliability or give counterexample.

Problem 2.2.3 Suppose assumption d) on page 12 does not hold at initial time. Does reliability of the Alternating Bit protocol still hold? Prove or give counterexample.

Problem 2.2.4 Consider the Acknowledge Bit Protocol as given in Table 2.2. It is a symmetric protocol, except for the initialization steps.

a) Explain the name of the protocol.

b) Show that this protocol is reliable if fewer than two successive errors occur (in opposite directions).

c) Give an example of an unreliable execution of the protocol.

Problem 2.2.5 In a new version of the Alternating Bit Protocol, DLC B runs the same algorithm as DLC A (Table 2.1), with B and A interchanged, NR and NS interchanged and VR and VS interchanged. Only the initialization step for B stays the same as in Table 2.1 (to prevent both DLC's from sending the first frame). CHECK

Is this procedure reliable? Prove or give counterexample.

Problem 2.2.6 State the induction step for DLC B in Lemma 2.2.

Problem 2.2.7 Consider an Alternating Bit Protocol that uses an extra bit for verification (e.g. one bit is alternated for every new packet fetched from the local source, while a second bit is turned on whenever a frame is received error-free). Thus the frame header contains two bits instead of one.

a) Specify this protocol formally.

[2] See Problems 2.2.5, 2.2.6.

Algorithm for A

```
Initialization (VS = 1)
{     A_1 ← first packet accepted from data source;
      send A_1;
}


A1        receive B_NR or B_e or timeout
A2        {    if   (received B_NR) {
A3                   VS ← 1;
A4                   if  (NR = 1) {
A5                        if  (B_NR not dummy) deliver it to local sink (after deleting NR);
A6                        discard A_VS;
A7                        A_VS ← next packet accepted from local source;
                     }
A8                   else   discard received frame;
               }
A9             else   VS ← 0;
A10            send A_NS with NS = VS; reset timer;
          }
```

Algorithm for B

```
Initialization (VR = 0)
      B_0 ← dummy frame;


C1        receive A_NS or A_e or timeout
C2        {    if   (received A_NS) {
C3                   VR ← 1;
C4                   if  (NS = 1) {
C5                        deliver A_NS local sink (after deleting NS);
C6                        discard B_VR;
C7                        B_VR ← next packet accepted from local source;
                     }
C8                   else   discard received frame;
               }
C9             else  VR ← 0;                                   problem here???
C10            send B_NR with NR = VR; reset timer;
          }
```

Table 2.2: The Acknowledge Bit Protocol

b) What are the advantages and disadvantages of this protocol compared with the Alternating Bit Protocol?

Problem 2.2.8 Assume that the communication media never fails. Is it possible to design a reliable alternating-bit protocol (or any other data-link protocol that ensures reliability) without using timeouts? If Yes, write the protocol code. If Not, explain why.

Problem 2.2.9 Change the alternating bit protocol, so the code for A and B will be exactly the same. Hint: use a randomized initialization algorithm.

Problem 2.2.10 What happens in the Alternating Bit Protocol when only one of the sides needs to send data? How can this problem be fixed? Write a version of the protocol that fixes this problem (you may use one more bit for every packet).

Problem 2.2.11 In the definition of Fig. 2.2 it is specified that the action is taken *after* transition. What happens if it is taken *before* transition?

2.3 Sliding-Window DLC Procedures

The Alternating Bit Protocol is an extremely simple protocol. It uses one bit in each direction with a dual purpose: i) the assigned bit for the data frame and ii) acknowledgement for the data flowing in the other direction. One main disadvantage of that protocol stems from the fact that the flows of data in both directions are interdependent. Another disadvantage is poor utilization of the media, since only one frame can be outstanding at any given time. The solution to the first disadvantage is to separate the two flows, by using one bit for the sequence number and another bit for acknowledgments. This is the original Alternating Bit protocol, proposed by Lynch [Lyn68]. For the data flow from A to B, we would define two variables VS_A and VR_B at DLC A and DLC B respectively. The first is the bit assigned to the current packet at A, the other is the bit expected by B in the next frame. For the data flowing in the other direction, there would be separate variables VS_B and VR_A, with the corresponding meaning. A data frame from A to B would carry sequence number $NS_A = VS_A$. An acknowledgement frame from A to B (for a data frame from B to A) would carry $NR_A = VR_A$. This is in contradistinction with the model of [BSW69], described in Sec.2.2, where always holds $VS_A \equiv VR_A$. Here we separate the two. Since acknowledgement frames are normally short and the overhead for such frames is large, it is customary to piggyback, whenever possible, the acknowledgement from A to B (for data frames from B to A) on data frames flowing from A to B. However the two protocols are still independent and the data rates in both directions can be different. In particular, if there is need to send an ack and there is no data frame going in that direction, the protocol normally does allow special ack frames.

The solution to the disadvantage of poor media utilization is to use more than one bit for sequence numbers. The protocol where the two directions work independently can easily be generalized to multiple-bit sequence numbers. The protocols described in this section represent exactly this generalization. Another generalization included in this section is that the media is allowed to fail and recover, whereas in order to simplify the presentation, in Sec.2.2 we have assumed that the two nodes are synchronized externally at initial time and the media stays up forever afterwards. We define *a general class* of DLC procedures to which we shall refer as *sliding-window DLC procedures.* As the protocols for the two data flows are independent, our description will focus on the interaction required between the two DLC's to transmit data from station A to station B. To facilitate our discussion, the DLC at station A will be referred to as the *sender DLC* and the DLC at station B will be referred to as the *receiver DLC*. Except for some minor notational differences, all known bit-oriented DLC procedures (e.g., HDLC, SDLC, LAP-B, LAPD) are members of the class of sliding-window DLC procedures.

In a sliding-window DLC procedure, the sender DLC maintains a *send counter number,* denoted[3] by VS, and the receiver DLC maintains a *receive counter number,* denoted by VR. The range of VS and VR is between 0 and $W-1$, where W is some fixed integer. The quantity $(W-1)$ is called the *window size;* the Alternating Bit protocol uses $W = 2$ and the HDLC protocol uses $W = 8$ or $W = 128$. *Initialization* of the counter numbers is performed by the Link Initialization Procedure and will be discussed later. For now it suffices to say that the relation $VS = VR$ must hold at initialization.

The sliding-window DLC protocol is specified in Table 2.3 and is described in the following paragraphs. Once the send counter number is initialized, the sender DLC is allowed to accept $(W-1)$ packets from the data source. These packets are assigned consecutive *sequence numbers* from VS to $(VS+W-2) \bmod W$. The sender DLC transforms each accepted packet into an information frame by appending a control header containing the assigned sequence number NS. An information frame is stored by the sender DLC from the

[3]To avoid confusion, we point out that the quantity VS here is different from $V(S)$ of HDLC [ISO81]. However, for our purpose it is more convenient to use this notation.

time the corresponding packet is accepted until it is *considered acknowledged,* at which time it is discarded.

When a frame with sequence number $NS = VR$ is received correctly at the receiver DLC, it is delivered, without the control header, to the data sink as a packet and VR is incremented $\mod W$. Frames received in error or with sequence number $NS \neq VR$ are discarded and no action is taken. The receiver DLC also has some mechanism to periodically send an *information ACK frame* containing *acknowledgement number* $NR = VR$ to the sender DLC. Whenever an information ACK frame with acknowledgement number $NR \neq VS$ arrives at the sender DLC, the variable VS is repeatedly incremented $\mod W$, until it reaches NR. In addition, when VS is incremented from value K to $(K+1) \bmod W$, the stored frame that carries sequence number K is *considered acknowledged* and discarded. Then a new packet is accepted from the data source and is assigned sequence number $(K-1) \bmod W$. Observe that at any time, the sender DLC stores $(W-1)$ frames, with sequence numbers from VS to $(VS+W-2) \bmod W$. We do not specify here the times when the sender DLC is allowed to send its stored frames or when the receiver DLC sends the acknowledgement. However, the sender DLC is required to periodically send out the information frame with sequence number $NS = VS$. Similarly, the receiver DLC is required to send out periodically an acknowledgement. The timers at the sender and receiver DLC's implement this requirement.

Algorithm for sender DLC (DLC at A)

```
A1    upon entering Connected mode
A2    {   VS ← 0;
A3        A_0 − A_{W−2} ← first (W − 1) packets accepted from data source;
A4        send A_0 and afterwards A_1, ..., A_{W−2};
A5        start timer;
      }
A6    upon receiving ACK_NR or ACK_e or timeout
A7    {   if (received ACK_NR) {
A8            while (VS ≠ NR) {
A9                discard A_VS and consider it acknowledged;
A10               VS ← (VS + 1) mod W;
A11               A_{(VS−2) mod W} ← next packet accepted from local source;
              }
          }
A12       send A_NS with NS = VS and afterwards
              with NS = (VS + 1) mod W, ..., (VS + W − 2) mod W;
A13       reset timer;
      }
```

Algorithm for receiver DLC

```
B1    upon entering Connected mode
B2    {   VR ← 0;
      }
B3    upon receiving A_NS
B4    {   if (NS = VR) {
B5            deliver payload of A_NS to local sink ;
B6            VR ← (VR + 1) mod W;
          }
B7        else discard received frame;
      }
B8    periodically, send ACK_VR;
```

Table 2.3: The sliding-window DLC Protocol

We now note that different sliding-window DLC implementations employ various additional optimization techniques to achieve efficient link utilization. One example that was mentioned before is to append the ACK frame to information frames being sent in the opposite direction. Other examples include using NACK frames (selective reject), and/or checkpointing, and saving information frames with $NS \neq VR$ for later use. Although important from the performance point of view, these techniques will not be discussed here.

In addition to the normal operation described above, DLC procedures must include mechanisms for detecting media failures, mechanisms for detecting media recoveries and an initialization protocol that will allow the resumption of normal operation. The most common failure detection mechanisms are to declare the media as failed if frames are not acknowledged after a given number of transmissions or after a predetermined time and if frames do not arrive from the other side for a certain period. Whenever a DLC detects a failure, it declares any unacknowledged information frames as possibly lost and forwards appropriate notification to the higher layers. It then invokes an initialization protocol that first *probes* the channel periodically for detection of media recovery and then *synchronizes* the system for resumption of normal operation. When the initialization protocol terminates, the DLC resumes normal operation. We shall say that a DLC is in *Connected State* when it performs normal operation and that it is in *Initialization Mode* otherwise (i.e., when it is executing the initialization procedure). The basic model assumptions are:

a) The communication media can be either operational or failed. While operational, the probability of frame error or loss in the media is strictly less than 1. While failed, no frames traverse it.

b) The communication media works with a FIFO discipline. Error-free frames arrive at a DLC in the same order as sent by the other DLC. A frame cannot be in the media if frames sent after it have already arrived.

c) At each node there is a failure detection mechanism with the property that if the communication media is failed for sufficiently long time, the mechanism detects the failure in finite time (not necessarily the same instant at both nodes). Note that the failure detection mechanism is allowed to be wrong in one sense: the media may be operational, but bad enough to make the failure detection mechanism declare the media failed. In particular, it may happen that the mechanism at one node will detect failure, but the one at the other node will not.

d) If a DLC is in Connected state and a failure is detected or if the Link Initialization protocol dictates so, the DLC enters Initialization Mode. At that time, it clears its buffer of any stored frames, declares any unacknowledged information frames as possibly lost and forwards appropriate notification to the higher layers.

e) A DLC in Initialization Mode discards any received information frames or information ACK's and does not accept packets from its source.

One comment is in place here regarding part of the assumption d) above. One may think that another possibility would have been to keep unacknowledged information frames in the DLC buffer until the media comes up again and continue operation afterwards as if nothing has happened. This cannot be done however. One should realize that the frames normally contain information of higher layers, for example a session that happens to use the considered media. If a link of the session fails, one cannot freeze an entire session to wait for the media to recover. The normal procedure is to either kill the session and possibly restart it afterwards or to reroute the session on a different path and require a higher layer protocol to take care of the non-disruptive path change. In any case, if and when the link under consideration comes up again, the information in the old frames is meaningless.

We next discuss the notion of *reliability* of a DLC procedure. DLC protocols must ensure that the DLC layer provides sufficient reliability properties to the higher layer to ensure their proper operation. The

question is what set of properties can be considered as providing a "sufficient set". An attempt to define such a set has been made in [BS88], but as shown presently, it turned out that the original statement of the definition of a reliable DLC procedure is not sufficient.

The following definition formalizes the notion of reliability for a DLC procedure.

Definition: A bit-oriented DLC procedure is said to ensure *data reliability* if it satisfies the following properties:

1. **Follow-up:** If a DLC enters Initialization Mode at some time when the other DLC is in Connected state, then the latter will also enter Initialization Mode in finite time.

2. **Crossing:** If a DLC enters Initialization Mode at some time t_1, there is a time t after t_1 but before the DLC next enters Connected State, such that the other DLC is also in Initialization Mode and no packet accepted by the sender DLC at either end before time t can be delivered to the corresponding data sink after time t.

3. **Deadlock-Free:** There exists a value T_1 such that if (a) both DLC's are in Initialization Mode at some time t and (b) during the interval of length T_1 after t there are no channel errors and (c) the delay for all frames (queueing+propagation) is bounded, then at time $t + T_1$ both DLC's are in Connected State. The DLC's stay in Connected State if there are no media failures afterwards.

4. **FIFO:** Suppose that a DLC delivers to its data sink a packet that has been accepted at time t by the other DLC from the corresponding data source. Then all data packets accepted by the other DLC since it last entered Connected Mode until time t, have been delivered to the data sink without errors, in order, with no gaps or duplicates.

5. **Confirm:** Whenever a DLC is in Connected State, all packets accepted from its data source since it last entered the Connected State, and considered acknowledged, have been delivered to the corresponding data sink.

6. **Delivery:** Suppose that a DLC enters Connected State and stays there forever afterwards. Then all packets produced by that DLC's data source and accepted by the DLC after it entered Connected state are considered acknowledged within finite time.

The need for the **Follow-up** property is obvious. In particular it disallows the situation where one DLC stays forever in Connected state and the other is in Initialization Mode. The **Crossing** property relaxes and formalizes the usual notion of a "correct global initial state" that we have used in Sec. 2.2 (see e.g. [SL83]), where both DLC's are in Connected State with sequence number 0 and the channel is empty of frames. The generalization takes into consideration the case when one DLC enters Connected State and starts sending frames before the other enters Connected State, so that strictly speaking there is no instant when the system is in a "correct global initial state". In this situation we still think of the DLC procedure as reliable, provided it satisfies the property indicated above under **Crossing**. The **Deadlock-Free** property says that if the channel works properly, the DLC's are not deadlocked in Initialization Mode. **FIFO** states that the sequence of packets delivered to the data sink is a prefix of the sequence received from the data source. **Confirm** states that packets that are considered acknowledged by the source DLC have indeed been delivered to the data sink. The **Delivery** property ensures that the DLC procedure is not the cause for nondelivery of data. It does not allow the possibility that the media is operational and is not declared failed by the failure detection mechanism, but the DLC procedure is stagnated in a situation where packets are

not delivered or not considered acknowledged. Observe that **Delivery** and **Confirm** ensure that under the conditions stated in the Delivery property, all packets are delivered to the data sink in finite time. Note also that **FIFO** and **Confirm** ensure proper delivery of packets corresponding to frames that are *considered acknowledged*. At any instant there are $(W-1)$ packets that have been accepted from the data source but are not yet acknowledged. Such packets may or may not be delivered to the data sink (if the DLC enters Initialization Mode), but the **FIFO** property says that whatever is delivered to the sink, is delivered in sequence, whether it is considered acknowledged or not. In particular, if a DLC enters Initialization Mode, it should notify the higher layers that these packets have not been acknowledged and consequently, may have been lost. This is in accordance with model assumption d).

We reiterate here that packets containing data, as well as control messages belonging to protocols of levels higher than the DLC layer, are considered by the DLC layer as data packets. Reliable transmission of data consists of fulfilling the 6 properties above. Regarding control messages of higher-level protocols, most such protocols assume a DLC protocol on each link that ensures reliability. One of the basic questions asked very often in the design and validation of higher level protocols, is what are the precise properties one can expect from the DLC. Unfortunately, in many works those properties are loosely stated, and the statement differs from work to work. We believe that the 6 properties stated above provide a precise and unifying definition of DLC data reliability. In an environment without failures, it is very easy to establish the properties that should be required from the protocol to be considered reliable (see Sec. 2.2). However, it is not that easy to define reliability in a system where failures may occur. The definition of reliability in [BS88] has been an attempt to include all necessary requirements in the definition. However, it turned out during the period since [BS88] was published, that important features were left out or misstated. In particular, in that work the requirement of **FIFO** for unacknowledged packets was not included and the **FIFO** and **Delivery** properties were stated incorrectly. The present definitions are an attempt to restate the properties in a better and hopefully final form, but given the past experience, it is very doubtful that this will indeed be the final word in this respect.

In the sequel we will show that any sliding-window DLC procedure ensures data reliability if properly synchronized at initialization. However, we must first formalize the notions of synchronization and of a Link Initialization procedure.

The procedure that synchronizes the DLC's when the system first comes up and resynchronizes them after a media or node failure will be referred to as the *Link Initialization (LI) Procedure* [BS88]. In particular, whenever a node comes up, it enters Initialization Mode and performs the LI procedure. Also, if a node is up and the failure detection mechanism declares the media failed, the node enters Initialization Mode and performs the LI procedure. When the LI procedure is invoked, the DLC enters Initialization Mode and, if the media is operational again, it communicates with the other DLC through special *LI-Control frames* (the equivalent of Unnumbered Frames in HDLC [Car82], [ISO81]). After appropriate frame exchange, the LI procedure should bring both DLC's into the Connected State. The following definition formalizes the notion of synchronization for a DLC procedure.

Definition: An LI procedure working in conjunction with a bit-oriented DLC procedure is said to *ensure synchronization* if it satisfies the following four properties:

1. **Follow-up:** If a DLC enters Initialization Mode at some time when the other DLC is in Connected state, then the latter will also enter Initialization Mode in finite time.

2. **Clear:** If a DLC enters Initialization Mode at some time t_1 and returns to Connected state at time t_2, there is a time t between t_1 and t_2 when the other DLC is also in Initialization Mode and the system is devoid of any information frames and any information ACK frames.

3. **Reset:** If a DLC enters Connected state while the other DLC is already in Connected state, then the send (VS) and receive counter numbers at one DLC are identical to the receive (VR) and send counter numbers, respectively, at the other DLC.

4. **Deadlock-Free:** There exists a value T_1 such that if (a) both DLC's are in Initialization Mode at some time t and (b) during the interval of length T_1 after t there are no channel errors and (c) the delay for all frames (queueing + propagation) is bounded, then at time $t + T_1$ both DLC's are in Connected State.

The following Theorem demonstrates the relationship between a sliding-window DLC procedure, its associated LI procedure, and the notion of data reliability.

Theorem 2.3 *If a sliding-window DLC procedure is initialized by an LI procedure that ensures synchronization, then the DLC procedure ensures data reliability.*

Proof: Consider a sliding-window DLC procedure with an LI procedure that ensures synchronization. We need to prove that the DLC procedure has the **Crossing, FIFO, Confirm** and **Delivery** properties.
Proof of Crossing: The *Clear* property of the LI procedure defines a time t when there are no information frames or information ACK frames in the system and when both DLC's are in Initialization Mode. Therefore, at that time, there are no information frames at all in the entire system. This means that no frame containing any packet accepted from any source before time t exists in the system. Moreover, when it enters Initialization Mode, the DLC cleans its buffer of stored frames containing previously accepted packets. Therefore, no frames containing packets accepted from any data source before time t can arrive to any DLC from the media after that time, hence the *Crossing* property.
Proof of FIFO, Confirm and Delivery:

Consider the time t defined in the Clear property of the LI procedure and suppose that after time t some DLC is the first to go to Connected State. The Clear property implies that until time t_0 when the other DLC also goes to Connected State, no information or information ACK frames can be accepted at either DLC from the other. This is because (a) the second DLC is still in Initialization Mode and will discard any information frames, and (b) the first DLC will not receive before t_0 any information or ACK frames because the second does not send any. The *Reset* property says that at time t_0 the counter numbers are synchronized, and without loss of generality we can take them to be 0.

As said before, we look at information data flowing in one direction only, from A to B. Packets accepted by station A from the local data source will be numbered for identification purposes by consecutive increasing numbers $P(0), P(1), \ldots$ (not modulo W), where $P(0)$ is the first packet accepted after entering Connected state. Hence the sequence number assigned to $P(I)$ is I mod W. First observe that the sliding-window DLC mechanism dictates that a frame can be considered acknowledged (and discarded) by A only as a result of receiving an ACK frame and no more than $(W - 1)$ frames can be discarded as a result of receiving a given ACK frame. Also, consecutively discarded frames contain consecutive packets. An ACK frame whose receipt at the sender DLC results in discarded information frames will be referred to as an *active ack.* Observe that an ack is labeled as active or not active only upon being received by A. More precisely, an ACK with number NR received by A is active if $NR \neq VS$, and not active otherwise. The packet corresponding to the last frame that is discarded when an active ack is received will be referred to as *correlated* with the active ack. Lemma 2.4 provides the proof of the **Confirm** property and of the **FIFO** property for packets that are considered acknowledged. **FIFO** for packets that are not considered acknowledged is proved in Lemma 2.5.

Lemma 2.4 *Suppose an ACK arrives at the sender DLC and is labeled active. At the time when the ACK was sent by the receiver DLC, all packets, starting with $P(0)$ and ending with the correlated packet, and only those, have been delivered to the data sink in order, with no duplicates and no gaps.*

Proof: Notes a) and b) below follow directly from the sliding-window DLC properties and will be used in the proof:

a) No frame containing packets prior to and including $P(I-(W-1))$ can be sent by the sender DLC after accepting packet $P(I)$ and consequently no such frame can be received by the receiver DLC after any frame containing $P(I)$.

b) No frame containing packets following and including $P(I+(W-1))$ can be sent by the sender DLC before the time when the frame containing packet $P(I)$ is discarded.

Now consider Figure 2.4 where NR_1, NR_2 denote the ACK numbers of two *consecutive* active acks and let t_1, t_3 and t_2, t_4 denote the respective arrival and departure times. From the time just after ACK_{NR_1} is processed and until just before ACK_{NR_2} is received, the variable VS does not change and for this value of VS holds $VS = NR_1$ and $VS \neq NR_2$, hence $NR_1 \neq NR_2$. Let $P(I)$, $P(K)$ be the packets correlated with the two consecutive active acks. We have $0 < K - I \leq W - 1$; $I \bmod W = (NR_1 - 1) \bmod W$; $K \bmod W = (NR_2 - 1) \bmod W$.

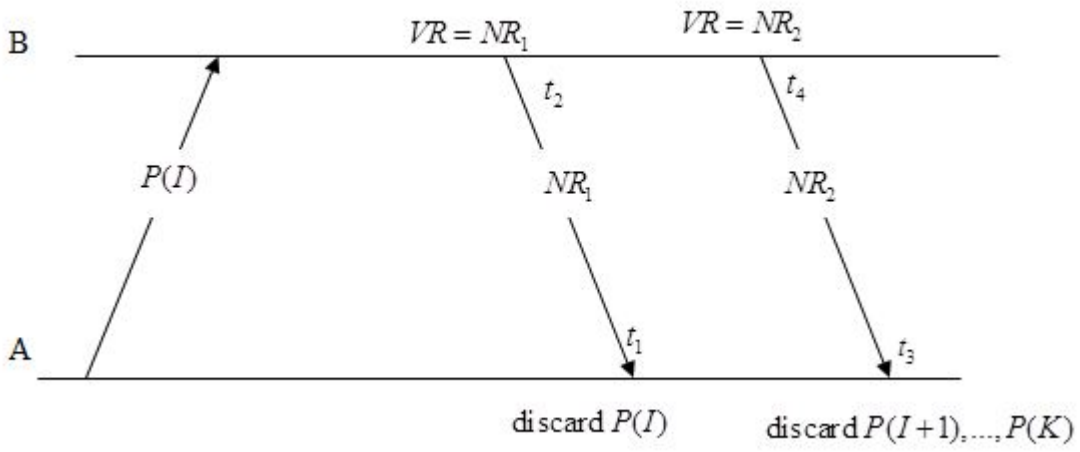


Figure 2.4: Diagram for proof

The induction step consists of showing that, if all packets up to and including $P(I)$ and only those have been delivered in order, with no duplicates and no gaps until time t_2, the same is true for all packets up to and including $P(K)$ until time t_4. This amounts to showing that during $[t_2, t_4]$, the packets $P(I+1), \cdots, P(K)$ and only those packets are delivered to the sink at B exactly once and in order. First note that, since $P(I+1)$ is discarded at time t_3, b) above implies that no frame containing packets following and including $P(I+W)$ can arrive at the receiver DLC during the interval $[t_2, t_4]$. Also, a) above, says that for all J holds that, after any frame containing packet $P(J)$ arrives at the receiver DLC, no frame containing $P(J-(W-1))$ or preceding packets can arrive at the receiver DLC afterwards. Now, take J to be $I, I+1, ..., I+(W-1)$. We obtain that in the considered interval the following must be true: the only frame with send sequence number $NS = NR_1$ that can arrive is the one containing $P(I+1)$; if the frame containing $P(I+1)$ arrives, the only frame with $NS = (NR_1 + 1) \bmod W$ that can arrive afterwards contains $P(I+2)$; and so on, if the frame containing $P(I+(W-2))$ arrives, the only frame with $NS = (NR_1 + (W-2)) \bmod W$ that can arrive afterwards contains $P(I+(W-1))$ and if the latter arrives, no frame with $NS = (NR_1 + (W-1)) \bmod W$ can arrive afterwards. Since the receiver DLC accepts frames only with consecutive send sequence

numbers modulo W, only $P(I+1)$ to $P(I+(W-1))$ can be accepted in this interval, in order and with no gaps or duplicates. Since VR at time t_4 is NR_2, the packets that have in fact been accepted in the interval $[t_2, t_4]$ are $P(I+1)$ up to and including $P(K)$, completing the induction step.

Now, since at time t_0, defined at the beginning of the proof of FIFO and Confirm, there are no information ACK frames in the system, the first received active ACK is sent after that time, at time t' say, and let NR' be its ACK number. The fact that $VS = 0$ at initialization, implies $NR' \neq 0$ and only frames containing $P(0)$ to $P(W-2)$ can be received by the receiver DLC before NR' is sent. Finally the same argument as before shows that the ones that have been indeed accepted are $P(0)$ to $P(NR'-1)$, in order and only once, completing the proof of the Lemma. qed

Proof of Delivery: Suppose the sender DLC enters Connected state and stays there forever afterwards. We need to show that in this situation all packets produced by the data source will eventually be considered acknowledged. From the Follow-up property follows that the receiver DLC cannot stay forever in Initialization Mode. From the Clear property follows that it cannot enter and leave Initialization Mode. Hence after some time the receiver DLC is in Connected state and stays there.

Let $P(I)$ be the first packet that is never considered acknowledged. This means that all frames up to and including $P(I-1)$ are delivered and considered acknowledged after a certain time. Hence $VS = I \bmod W$ and will never change, $VR = I \bmod W$ and the frame containing $P(I)$ will have sequence number VS. Since the source DLC has a timer and the frame with sequence number VS is sent every time the timer expires, the frame containing $P(I)$ will be sent an infinite number of times. Consider the situation after the receiver DLC is in Connected state and stays there. One of the basic model assumptions is that when the media is operational, the probability of frame loss or error is less than 1, and hence, if the frame is repeatedly sent, it will eventually arrive correctly at the receiver DLC. When this happens, packet $P(I)$ is delivered to the data sink, and VR becomes $(I+1) \bmod W$. The receiver also has a timer and will send ACK_{NR} with $NR = (I+1) \bmod W$,..., $(I+W-1) \bmod W$ an infinite number of times. Again one of these ACK'S will arrive correctly at the source DLC, causing $P(I)$ to be considered acknowledged. qed

Lemma 2.5 *Packets that are never considered acknowledged, but are delivered to the data sink, are delivered in order, with no duplicates and no gaps.*

Proof: The **Delivery** property ensures that if there are packets that are never considered acknowledged, then the sender DLC enters Initialization Mode. The **Follow-up** property ensures that the receiver DLC will also enter Initialization Mode. Suppose NR_2 in Fig. 2.4 is the last active ack sent by DLC B and received by DLC A before entering Initialization Mode. As in Lemma 2.4, let $P(K)$ be the correlated packet. We need to show that any packets $P(J)$, $J > K$ that are delivered to the data sink, are delivered in order, with no gaps or duplicates. The proof proceeds in a similar way as the induction step in the proof of Lemma 2.4. Note that, since $P(K+1)$ is never discarded, note b) in the proof of Lemma 2.4 implies that no frame containing packets following and including $P(K+W)$ can ever arrive at the receiver DLC. Also, note a) in the proof of Lemma 2.4 says that for all J holds that, after a packet $P(J)$ arrives at the receiver DLC, no frame containing $P(J-(W-1))$ or preceding packets can arrive at the receiver DLC. Now, take J to be $K, K+1, ..., K+(W-1)$. We obtain that after t_4 the following must be true: the only frame with send sequence number $NS = NR_2$ that can arrive is the one containing $P(K+1)$; if the frame containing $P(K+1)$ arrives, the only frame with $NS = (NR_2+1) \bmod W$ that can arrive afterwards contains $P(K+2)$; and so on, if the frame containing $P(K+(W-2))$ arrives, the only frame with $NS = (NR_2+(W-2)) \bmod W$ that can arrive afterwards contains $P(K+(W-1))$ and if the latter arrives, no frame with $NS = (NR_2+(W-1)) \bmod W$ can arrive afterwards. Since the receiver DLC accepts frames only with consecutive send sequence numbers modulo W, only $P(K+1)$ to $P(K+(W-1))$ can be accepted, in order and with no gaps or

duplicates, completing the proof of the Lemma. qed

Problems

Problem 2.3.1 Indicate the data reliability properties that do not hold if the window of the sliding-window DLC Protocol is W instead of $W-1$. Give examples.

Problem 2.3.2 Indicate the data reliability properties that do not hold if the media does not have the FIFO property. Give examples.

Problem 2.3.3 Consider the sliding-window DLC Protocol of Table 2.3, where instead of discarding information frames with $NS \neq VR$, the receiver saves them for later use. For example, if the sender sends frames 0, 1 and 2, but the first two are lost, then the receiver does not discard the third, but uses it later when the first two arrive correctly. Assuming that the Initialization Procedure ensures Synchronization, is this DLC Procedure reliable? If yes, prove. If not, indicate the necessary changes that will make it reliable, and prove reliability of the new protocol.

Problem 2.3.4 Explain why are the six properties of data reliability independent. Give examples when each of the properties does not hold while the other five do.

Problem 2.3.5 In some cases we want the receiver to have control of the window size (For example, it can reflect the number of free buffers currently available to the receiver). A good place to do it is on the acknowledge packet. Write the sender side of a protocol that ensures that no packet is sent beyond the allowed window.

Problem 2.3.6 The normal selective repeat DLC presented in the exercise session has the following drawback : When several contiguous packets are not received and the timeout at the sender expires, the latter sends only one packet and waits for the acknowledgement to see if the following packet have been received correctly. Only then it sends the one after it, and so on. In other words, there is no windowing for these packets. Discuss an efficient (with no unnecessary retransmission of packets) way to solve this problem. When does this drawback make the go-back-n protocol better than the selective-repeat protocol presented in the exercise session?

Problem 2.3.7 On a given full duplex link, the bit error rate is 1 per 1000 bits. The length of the DLC header together with the CRC is 50 bits. It contains an ACK field that has an error correcting code, so even in an erroneous frame, the ACK field can still be used. Each data packet sent is acknowledged immediately by an ACK frame or in the ACK field of a data frame. The CRC is useful ? for packets of length less than 10^5 bits.

a) What packet size is optimal for achieving maximum throughput?
b) Repeat a) when it is known that on the average case, the number of consecutive corrupted bits is 4.
c) Suppose that the line has bandwidth of 300 bits/sec. The line is full duplex. What size of window would you select for the DLC procedure ?
d) Now suppose that the delay on the line is 3 seconds. That means that a bit that was sent at time t on the line, will be received at time $t+3$. What size of window would you select now?
e) Specify the DLC retransmission timeout and the LI timeouts that should be selected for the system.
f) Specify a failure detection mechanism for the above system. Specify its parameters (like timeouts).
g) Suppose that a real-time low-rate source uses this link. In this case, throughput is of less importance, and instead it has delay demands : at least 99% of the data should be received correctly within 12 seconds. What parameters should be changed?

Problem 2.3.8 (Old statement of FIFO) This problem can be solved only after reading some of the chapters on Network Protocols). The statement of FIFO in [BS88] did not include requirement of FIFO for packets that are not considered acknowledged at the time of a failure detection. Give examples of Network Protocols that do not work with the old statement of FIFO.

2.4 Link Initialization Procedures

In this section we discuss the DLC Link Initialization (LI) Procedures. We have shown in Theorem 2.3 that any sliding-window DLC procedure ensures data reliability if it is initialized by an LI procedure that ensures synchronization. The best currently known LI procedures are the ones used by HDLC [Car82], [ISO81], [BC77], [Sta82]. However, as shown in Section 2.4.1, the HDLC LI Procedures fail to provide synchronization and allow inadvertent loss of data as a result of nothing more than unpredictably long media delays (e.g., due to long queueing caused by congestion); these scenarios assume no loss of memory or failure of any other type at the nodes. The problem is that, for proper operation, the HDLC LI procedures require that the time-out periods used in the procedure are larger than the maximal roundtrip delay of the control frames. This poses a major problem in the design of the time-out periods. As shown in the examples of Section 2.4.1, time-outs that are too short may result in lack of synchronization, non-reliable communication and inadvertent loss of frames. On the other hand, time-outs that are too long result in a long setup time, because the DLC process waits longer than necessary before retransmitting control frames that are lost. The time-out interval in the HDLC LI procedure has to be set to a large enough value to ensure that the media is free of old control frames before it expires, but such a value may be much larger than it is normal necessary. Moreover DLC procedures are often used on transmission media like satellite with on-board processing, gateways, local area networks, etc., for which it is hard to determine a priori bounds on the transmission delay. For such environments it is difficult to establish a time-out interval that is tight on one hand and not exceeded by the round-trip delay on the other hand. PERLMAN,103.89,TIMERS, LOW PROB.

In Section 2.4.2 we present the Link Initialization procedures suggested in [BS88], that work without the stringent requirement that the time-out interval exceeds the round-trip delay. Their main property is that DLC process synchronization and channel clearance are achieved under arbitrary channel delays for all cases of media malfunction, as long as there is no loss of memory in the nodes. The interesting fact is that the complexity of those procedures is smaller than the one of the HDLC LI procedures, so that the increased reliability is achieved at no extra expense. Moreover, the new procedures can be adapted to cope with node failures, provided only two bits of non-volatile memory exist in each of the stations (or only one non-volatile bit in the primary station for the Unbalanced mode). In fact it turns out that this is the best one can do because it has been shown in [LMF88] that there exists no LI procedure that ensures synchronization under node failures without the use of non-volatile memory.

As mentioned before, no procedure is completely failsafe against all types of failures. In particular, if the non-volatile memory fails, then loss of synchronization and inadvertent loss of data may occur even with the new LI procedures. However, the procedures of Section 2.4.2 considerably reduce the possibility of error as compared to the HDLC LI procedures.

In the remainder of this section we shall use finite state diagrams (see for example Figure 2.5) to describe the operation of various LI procedures. The notation T/A next to a state transition will denote the fact that T is the trigger for that transition and A is the action to be taken $after$ transition. The triggers will generally be the receipt of particular LI-Control frames or a time-out. We shall adopt the convention that the receipt of any frame other than those specified as triggers causes no action (i.e., the frame is disregarded and discarded). The term *reset* will denote the resetting of all counter numbers and the clearing of all memory associated with the communication system at the given node.

In order to describe the sequence of events that may occur between the two DLC processes, we will use a timing diagram with two parallel time axes (one for each DLC) as shown for example in Figure 2.6 An arrow drawn from one axis to the other represents a frame sent by one station and received by the other. An arrow that does not terminate on a time axis represents a lost frame (e.g., a frame that arrives in error and

is discarded). Each arrow is labeled with the corresponding frame type. LI-control frames are represented by solid arrows and information frames or information ACK frames are represented by dashed arrows. A_n represents an information frame with sequence number n and ACK_m represents a frame with acknowledge sequence number m.

2.4.1 The HDLC LI Procedure

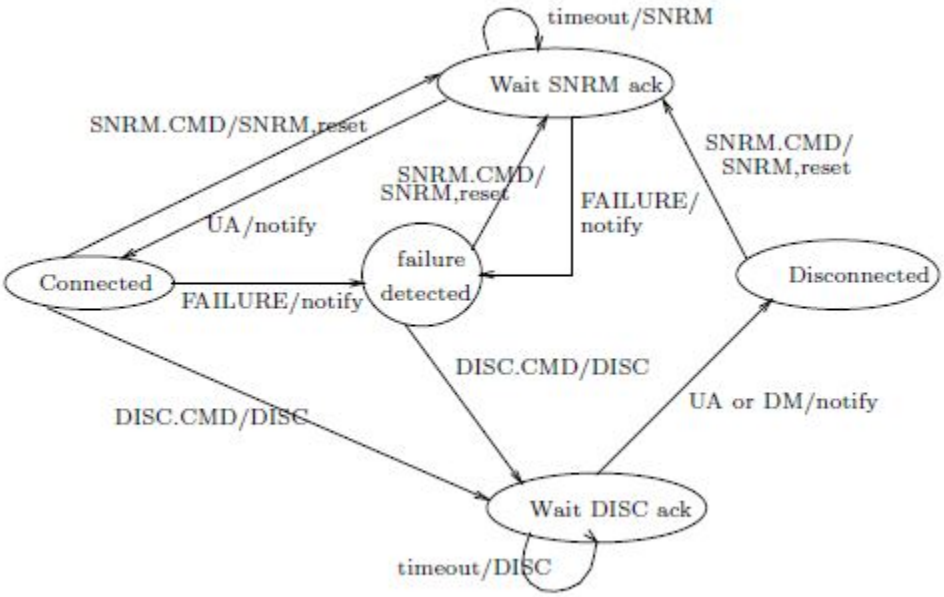
In this section we describe the Link Initialization Procedure used by HDLC in the Unbalanced Normal Response Mode and show that it does not ensure synchronization. The finite state diagrams describing the HDLC Link Initialization Procedure are shown in Figure 2.5 [BC77] (Figure 2.5 is the same as Sec. 5 in [BC77] except it also shows time-outs and exchange of messages with the higher layer). *Set normal response mode* (SNRM), *disconnect* (DISC), *unnumbered acknowledgement* (UA), and *disconnect mode* (DM) are the LI frames. *notify* means "Notify the Higher Layer[4]" and *reset* means "Reset sequence number." Note that the actual operation of this LI procedure is dependent on information obtained from a higher layer, allowing for various versions of operation. In particular, transition from the Failure Detected and Disconnected States is triggered by the receipt of instructions SNRM.CMD or DISC.CMD from a higher layer. However, we will show that all versions of this LI procedure do not ensure synchronization, independent of the actions taken by the higher layers.

Consider the possible sequence of frame exchanges shown in Figure 2.6(a)[5]. The sequence begins with the primary station entering Wait-Disc ack state and the secondary in Disconnected state, a common configuration. The primary sends a DISC message to which it receives a UA ack. The primary then enters Wait-SNRM Ack. After sending an SNRM frame, the primary times-out. When the timer expires, another SNRM is sent. Normally the timer is set such that if a UA is not received within its range, there is a good chance that the SNRM or the UA has been lost. However, as indicated at the beginning of this section, in some situations it may be hard or inefficient to guarantee that this is always the case. The situation considered in Figure 2.6(a) is where the timer expires twice before the UA for the first frame is received. Upon receiving the UA, the primary enters *Connection Mode*, and in the scenario shown in the figure, detects another failure.

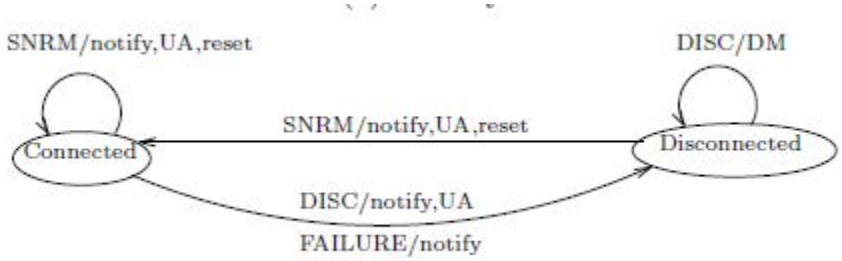
Upon detection of the media failure, the primary discards all information frames in the buffer and reenters *Initialization Mode*. At this point the primary may send either an SNRM or a DISC frame, depending on the instructions received from the higher layer. Figure 2.6(a) demonstrates the situation where the primary sends a DISC frame. Upon receiving the UA frame, it sends SNRM and upon receiving the next UA frame, it enters *Connection Mode*. At this point it accepts new packets from the higher layer. The first such packet is included in a frame with $NS = 0$. The diagram shows a scenario when this frame is lost in the media, but the DLC receives an information acknowledgement frame with $NR = 1$, that is interpreted to acknowledge the lost information frame. Figure 2.6(b) shows that the same problem may result when the primary sends an SNRM frame after the failure detection. Notice that three of the properties required to ensure synchronization (**Follow-up**, **Clear** and **Reset**) are violated by this LI procedure, no matter what action is taken by the higher layers. Thus the HDLC Link Initialization Procedure does not ensure synchronization and the HDLC itself does not ensure data reliability.

[4]Higher Layer is not Layer 3 in ISO, it is part of layer 2 that controls the initialization process

[5]All Unnumbered Frames considered here have the P/F bit set to 1 (see [ISO81] for details).

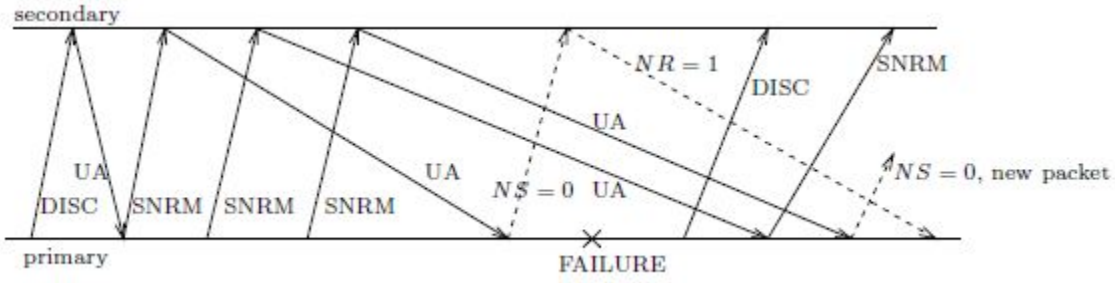


(a) Primary

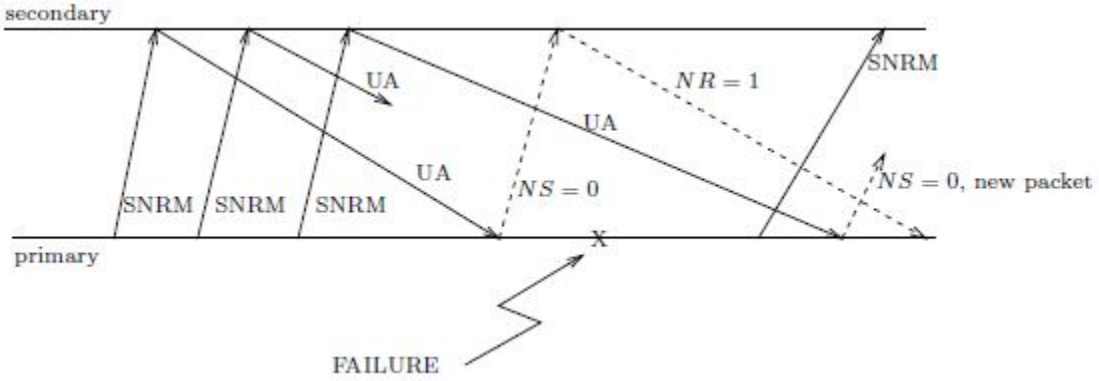


(b) Secondary

Figure 2.5: The HDLC Link Initialization Procedure



(a) With DISC



(b) Without DISC

Figure 2.6: The HDLC LI Procedure does not ensure Synchronization

2.4.2 Link Initialization Procedures that Ensure Synchronization

One way to prevent the situation described in Section 2.4.1 and correct the HDLC Link Initialization Procedure is to use sequence numbers. The LI-Control frames could carry correlated sequence numbers. However, infinitely large sequence numbers would be required to ensure synchronization. In this section we describe several new LI procedures that ensure synchronization without the use of sequence numbers or time-stamps. The first of our procedures, presented in two versions, assumes that one of the stations is predesignated as the primary station and the other as the secondary. The second procedure assumes no such preassignment, but begins by explicitly assigning primary/secondary functions. In the third procedure no preassignment is assumed and no postassignment is performed. In addition to ensuring synchronization, the second version of the primary-secondary procedure and the last two LI procedures also satisfy an additional property that may be important in some environments:

Test: A DLC process in Initialization Mode should not enter Connected State before it observes that the round-trip delay across the system is within a prespecified bound for at least one frame.

More stringent Test requirements, like two round-trips, may be implemented. The purpose of the Test property is to provide insurance that the DLC's will not unnecessarily oscillate between the Connected State and Initialization Mode.

2.4.3 Unbalanced LI Procedures that Ensure Synchronization

This procedure is designed for the situation where one of the DLC's is predesignated as the *primary* and the other as the *secondary*. The finite state diagrams describing the algorithm performed by each station are given in Figure 2.8. When the primary comes up, it enters the Wait-DM state; when the secondary comes up, it enters the Disconnected state. A DLC in Initialization Mode ignores all information frames and all information ACK frames and inhibits the media failure detection mechanism. Similarly, in the Connected state the primary ignores all LI-control frames and the secondary ignores all LI-Control frames other than DISC.

The primary station begins by transmitting DISC control frames at arbitrary time intervals until receiving a DM from the secondary. It then transmits SNRM control frames at arbitrary time intervals until receiving a UA, at which time it transmits a SUCCESS frame, resets its send and receive counter numbers, and enters Connected State. The secondary simply transmits a DM or UA frame whenever it receives a DISC or SNRM respectively. In addition, when the secondary receives a SUCCESS frame it resets its send and receive counter numbers and enters the Connected State.

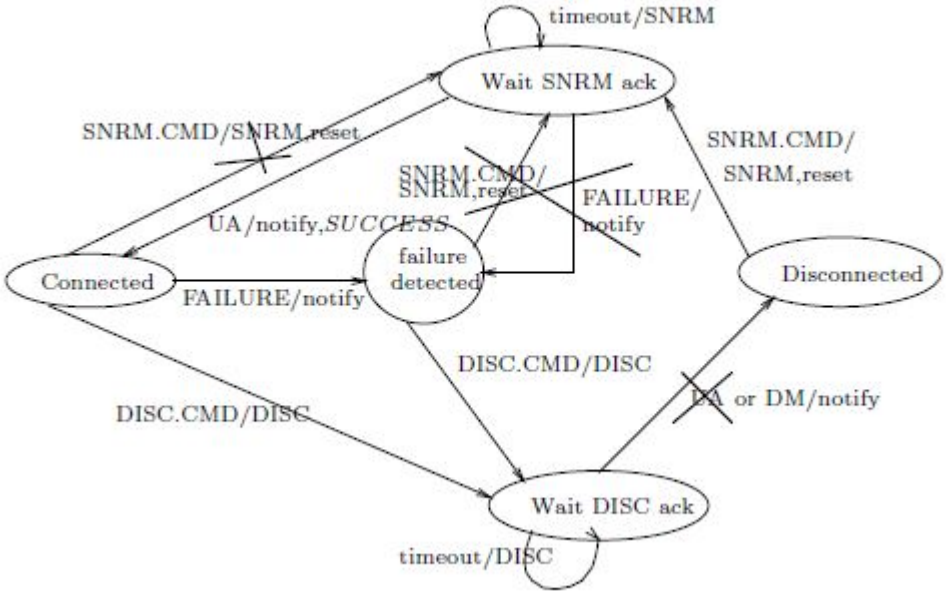
The timeout employed between DISC and SNRM frames is arbitrary from a correctness point of view. The LI procedure ensures synchronization for any value of this timeout. The only reason not to make it too short is performance, since this will unnecessarily clutter the channel with DISC and SNRM frames.

We may point out that the LI procedure of Fig. 2.8 is even simpler than the HDLC LI procedure. We outline here the differences between the two. To obtain Fig. 2.8 from the HDLC LI procedure one has to make the following changes (see Fig. 2.7):

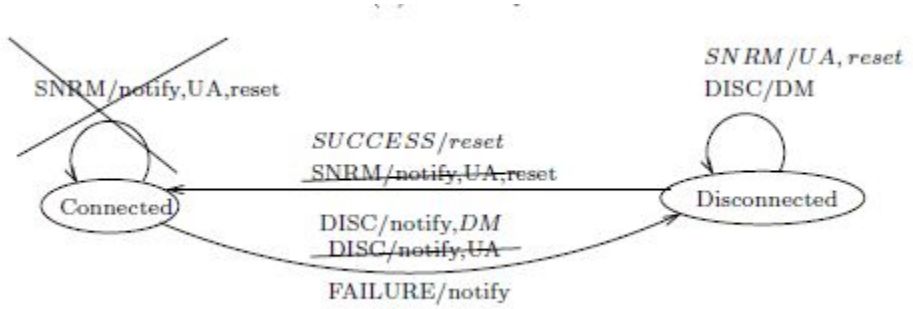
Primary

a) delete the two transitions between "failure detected" and "Wait-SNRM ack" states; in particular, this means that in Wait-SNRM ack, the primary does not react to Failure detected by the failure detection mechanism. In fact, the failure detection mechanism is disabled in Disconnected Mode at both primary and secondary nodes.

b) delete the transition from "Connected" to "Wait-SNRM ack"

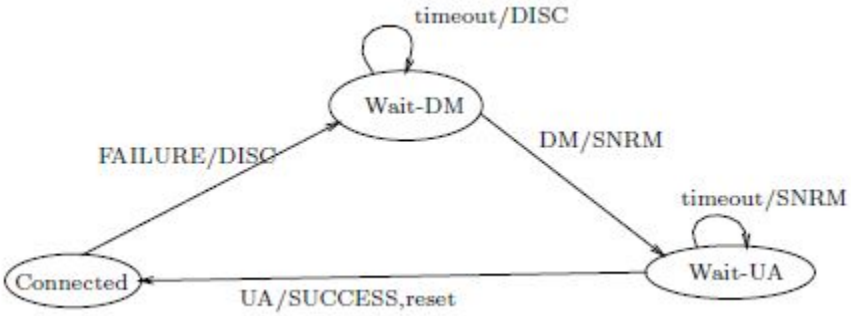


(a) Primary

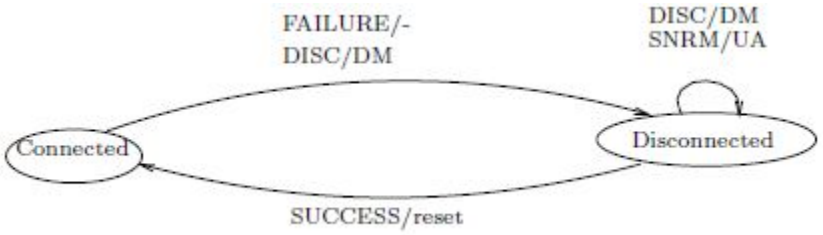


(b) Secondary

Figure 2.7: Corrections to the HDLC LI Procedure



(a) Primary



(b) Secondary

Figure 2.8: An Unbalanced LI that ensures Synchronization

c) in "Wait-DISC ack", do not react to UA frames

d) in the transition from "Wait-SNRM ack" to Connected, send also a SUCCESS frame

e) disregard internal messages like DISC.CMD and SNRM.CMD

Secondary

e) when receiving DISC, always send DM (and never UA).

f) when receiving SNRM in Disconnected state, stay there.

g) SNRM cannot be received in Connected state.

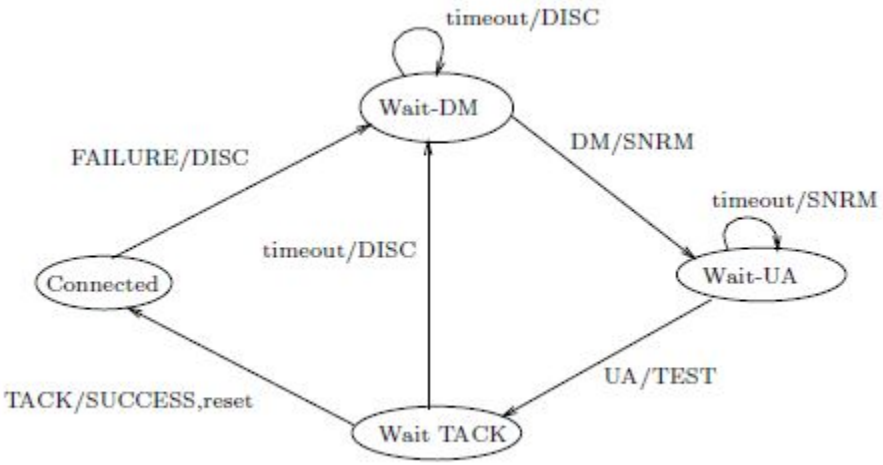
h) when receiving SUCCESS in Disconnected state, transit to Connected state.

In fact the procedure of Fig. 2.8 would have ensured synchronization even without the SUCCESS frame, i.e. without changes d),f),g) and h) above. We prefer the procedure as stated because the primary is the first to go to Connected state.

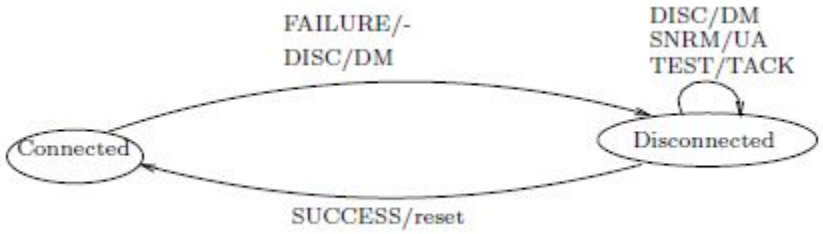
The HDLC LI Procedure fails to ensure synchronization because UA acknowledges both DISC and SNRM frames, the failure detection mechanism is not inhibited in Initialization Mode and it allows operation without use of DISC frames. By correcting these problems, we develop here a Link Initialization Procedure that not only ensures synchronization, but is much simpler than the HDLC LI protocol.

The procedure of Fig. 2.8 suffers from the difficulty that it may bring the DLC's to Connected state even if the media is very bad, and in this case the stations may unnecessarily oscillate between Connected

state and Initialization Mode. In order to avoid this, we introduce in Figure 2.9 a modified version of the Unbalanced LI procedure,where we add a state at the primary, the Wait-Tack state. The purpose of the Wait-Tack state and the associated timeout is to provide a certain degree of insurance that the channel works properly before the DLC's return to Connected state. In the form given in Figure 2.9 we are sure that the *Test* property is satisfied, namely at least one round-trip delay across the communication system did not exceed the bound specified by the TEST timeout. Therefore, the TEST timeout should be set to comply with the required specification. In fact, in various implementations, the Wait-Tack state and the corresponding timeout may be replaced by another "box" if there is a different *Test* requirement.



(a) Primary



(b) Secondary

Figure 2.9: A LI that ensures Synchronization and Test

The remainder of this subsection will be devoted to proving the correctness of the Unbalanced LI Procedure in both versions. We will first show that the procedures, as stated, ensure synchronization and that the second version satisfies the *Test* property, as long as the primary DLC does not fail during execution of the LI procedure. We will then show how to extend the protocols to handle environments in which the primary station can fail during execution of the LI procedure. This extension will require one bit of non-volatile memory at the primary.

Theorem 2.6 *Suppose that the primary station begins operating at a time when the media contains no DISC or DM frames and subsequently does not fail while in Initialization Mode. Then both versions of*

the Unbalanced LI procedure ensure synchronization *and, in addition, the second version satisfies the Test property.*

Proof: Since the secondary station enters Disconnected state after it recovers from a station failure, any failure of the secondary station will have no effect on the operation of the LI procedure other than increased frame delay or loss. Similarly, failures of the physical transmission media are equivalent to frame loss on the media. Thus, we need only consider the case where there are no media or station failures, but there may be arbitrary frame losses and arbitrary long frame delays.

Before continuing, we shall prove the following Lemma, that holds for both versions of the LI procedure:

Lemma 2.7 - *The cleaning property*

Suppose the primary station does not fail in Initialization mode. Suppose also that the primary station enters the Wait-DM state at a time when the system is devoid of DISC or DM frames. Then

a) *at the time t_1 when the primary next exits Wait-DM, only DISC or DM frames may exist in the system and the secondary is in Disconnected state.*

b) *at the time t_2 when the primary next exits the Wait-UA state, only SNRM and UA frames may exist in the system and the secondary is in Disconnected state over the entire interval $[t_1, t_2]$.*

Suppose the assumptions of the Theorem hold. Then

c) *every time the primary enters the Wait-DM state, the system is devoid of DISC and DM frames and a) and b) above hold at every exit of the primary from the Wait-DM and Wait-UA states respectively.*

Proof: Let t_0 be the time when the primary enters the Wait-DM state while the system contains no DISC or DM frames (see Fig. 2.10). The time t_1 defined in a) is the time when the primary receives the first DM after t_0, and therefore exits Wait-DM state. Let t_1' be the time when that DM frame is sent by the secondary and $t_1'' < t_1'$ be the time when the primary sends the DISC frame that causes that DM. By the FIFO property of the media (assumption b) on page 22), frames sent by the primary before t_1'' and by the secondary before t_1' cannot exist in the system at time t_1. Since at time t_0- the system was devoid of DISC and DM, holds $t_1'' \geq t_0$. But between t_0 and t_1 the primary sends only DISC frames and between t_1' and t_1 the secondary sends only DM frames. Therefore only these types of frames may exist in the system at time t_1. In addition, at time $t_1'+$, the secondary is in Disconnected state. Since it cannot receive SUCCESS between t_1' and t_1, it stays in that state until t_1. This completes the proof of part a). Part b) is proved in a similar way.

To prove part c), observe that by the assumption of the Theorem, when the primary comes up, it enters Wait-DM state and at that time the system is devoid of DISC and DM. Consequently a) and b) hold at the times when the primary next exits Wait-DM and Wait-UA respectively. In particular, when Wait-UA is exited, there are no DISC or DM frames in the system. Since such frames are not be generated until the next entrance of the primary into Wait-DM state, at that time the system is devoid of DISC or DM frames. It follows now inductively that a) and b) hold after *every* entrance of the primary into the Wait-DM state, completing the proof of the Lemma. qed

Lemma 2.8 *Suppose that the assumptions of the Theorem hold and let t_3 denote the time when the primary sends some SUCCESS frame (that may or may not arrive at the secondary).*

a) *In the second version (Fig. 2.9), let t_2 be the last time before t_3 when the primary exits Wait-UA state (see Fig. 2.10). Between t_2 and t_3 the primary stays in Initialization Mode. (Note: In the first version holds $t_3 \equiv t_2$).*

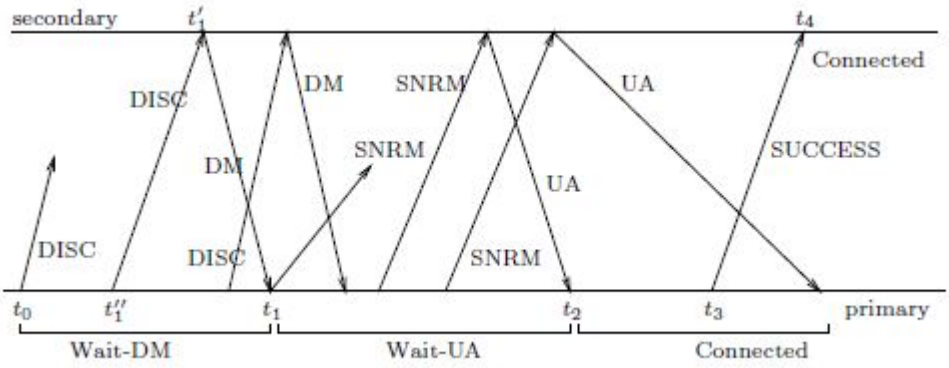


Figure 2.10: Diagram for proof

b) *In both versions, suppose the SUCCESS frame arrives at the secondary, at some time t_4 say. Between t_2 and t_4, the secondary stays in Initialization Mode. Moreover, if the primary is in Connected state at time t_4, then the primary is in Connected state over the entire interval $(t_3, t_4]$.*

Proof: During (t_2, t_3) the primary is in Wait-Tack state hence it is in Initialization Mode. This is part a). In order to prove b), note that from Lemma 2.7, at time t_2 the system contains no other frames except for SNRM and UA and the secondary is in Disconnected state. In addition, the primary sends no SUCCESS frame between t_2 and t_3-, so no such frame can arrive at the secondary between t_2 and t_4. Hence the secondary stays in Disconnected state until t_4. If the primary leaves Connected state between t_3 and t_4, it cannot return to Connected state by t_4. This is because if it does return, there must be a time between t_3 and t_4 when it leaves Wait-DM state. At that time the system may contain only DISC or DM frames, contradicting the fact that the SUCCESS frame is in the system on the entire interval (t_3, t_4). qed

We continue now with the proof of the Theorem. We first prove the Follow-up property. As stated in the definition of the protocol, the primary can enter Initialization Mode only when the failure detection mechanism declares a failure; the secondary can enter Initialization Mode when a failure is detected or when it receives a DISC frame. If the primary enters Initialization Mode when the secondary is in Connected state, it starts sending DISC frames and only those frames. If the media is operational, one of those frames will arrive, forcing the secondary into Initialization Mode (if it is not there already). If not, the failure detection mechanism at the secondary detects a failure, and this forces the secondary into Initialization Mode. If the secondary enters Initialization Mode when the primary is in Connected state, it ignores all information and all information ACK frames. Consequently, the failure detection mechanism at the primary will declare the media as failed, forcing the DLC into Initialization Mode.

The Clear property follows from Lemma 2.8. Suppose first that the primary goes into Initialization Mode and then returns to Connected state at some time t_3 (see Fig. 2.10). At that time it sends a SUCCESS frame. The instant required in the Clear property when the secondary is also in Initialization Mode and the media is devoid of Information frames and information ACK frames can be taken as t_1, the last time before t_3 when the primary exits Wait-DM state. This is because by Lemmas 2.7 and 2.8a), the primary stays in Initialization mode between t_1 and t_3 and the system may contain at t_1 only DISC and DM frames. Now suppose that the secondary goes to Initialization Mode and then returns to Connected state, at some time t_4 when it receives a SUCCESS frame. If t_1 is defined as the last time before t_4 when the primary exits Wait-DM state, then by Lemmas 2.7 and 2.8a), the secondary stays in Initialization Mode between t_1 and t_4 and the media may contain only DISC and DM frames at time t_1. Therefore, time t_1 may again be taken

as the instant required in the Clear property.

To prove the *Reset* property, observe first that according to Lemma 2.8, when the primary enters Connected state, the secondary is still in Initialization Mode. Consequently, the only process that may enter Connected state while the other is already in Connected state is the secondary. It does so when it receives a SUCCESS frame, at time t_4 say (see Fig. 2.10). Let t_3 be the time when that SUCCESS frame was sent. At time t_3 the primary sets $VS = VR = 0$. From Lemma 2.8, between t_3 and t_4 the primary stays in Connected state. Also, during this time, the primary receives no information or information-ACK frames, since the secondary sends no such frames while in Initialization Mode. Consequently, at time t_4+ holds $VS = VR = 0$ at both the primary and the secondary.

We now prove the *Deadlock-free* property. Suppose that both DLC's are in Initialization Mode. In the first version, the DLC's will return to Connected state if some DISC and the corresponding DM will arrive correctly, some SNRM and the corresponding UA will arrive correctly and the subsequent SUCCESS will not be lost or corrupted. Hence the first version satisfies the Deadlock-free property. For the second version there is an additional requirement that the TEST-Tack pair arrive correctly and within the Tack timeout. Hence that version also satisfies the Deadlock-free property.

To prove the *Test* property for the second version, observe that at the time when the primary leaves the Wait-UA state, there can be only SNRM and UA frames in the system. Consequently, in order for the primary to send the SUCCESS frame at t_3, it must be the case that after t_2, the last time before t_3 when the primary left Wait-UA state, it has sent a TEST frame and has received the Tack response within the Tack timeout. Hence the TEST property. qed

Our discussion thus far has been based on the assumption that the primary DLC *does not fail* during execution of the LI procedure. If we drop this assumption, there is the question of what state does the primary station restart in. It is easy to show that no matter which state is selected for restart, Theorem 2.6 no longer holds. Thus the unbalanced LI procedure must be modified so that its operation is unaffected by primary station failures. This can be easily accomplished if the primary station maintains one bit of *non-volatile memory* which is set to 1 every time the Wait-UA state is entered and reset to 0 every time the Wait-UA state is exited. Whenever the primary station fails, it restarts in the Wait-UA state if the value of the bit is 1 and in the Wait-DM state otherwise. This modification will essentially leave the LI procedure unaffected by primary station failures. This is because if the station fails in Wait-DM or Wait-UA state, it will come up in the same state. If it fails in Connected or Wait-Tack state, it will come up in WAIT DM state, and this transition is equivalent to failure detected on the media or TEST timeout expiration respectively. Therefore Theorem 2.6 will once again apply. We mention again that the above is the best one can do because it has been shown in [LMF88] that there exists no LI procedure that ensures synchronization under node failures without the use of non-volatile memory.

2.4.4 A Function-Assigning LI-Procedure

In the Unbalanced LI-Procedure of the previous section it was assumed that the primary/secondary functions are preassigned. This section considers the case where no such preassignment exists, but the LI procedure is required to assign primary/secondary functions to the stations. A procedure of this type is necessary when the stations must use an Unbalanced Operation Mode in Connected State (as in SDLC for example). We note, however, that such an LI procedure can also be used if the stations employ a Balanced Operation Mode in Connected State, by simply disregarding the assignment. The advantage over the Balanced LI Procedure (to be presented in the next section) is that fewer LI-Control frames are used. On the other hand, the Balanced LI Procedure must be used if there is no ordering of the station identities or if there is an explicit

requirement that the entire activity of the LI Procedure be completely symmetric.

The Function-Assignment LI Procedure uses a fairly simple rule for primary/secondary assignment. The station that enters first Initialization Mode becomes the primary, except for the case when each of the two stations enters Initialization Mode before finding out that the other station has also entered this mode. In this latter situation, the station with higher identity number becomes the primary. The finite state diagram for station A is shown in Figure 2.11. Station B has the same diagram except that A and B are interchanged. The idea is that a station (say A) which detects a media failure while in Connected State, enters Wait-DM State and attempts to become the primary by sending DISC frames to the other station. If it receives a DISC frame while in this state, it recognizes that Station B has done the same and if $B > A$, it yields the primary function to the other station by sending DM and entering *disconnected* State. Otherwise, station A will receive DM and declare itself the primary.

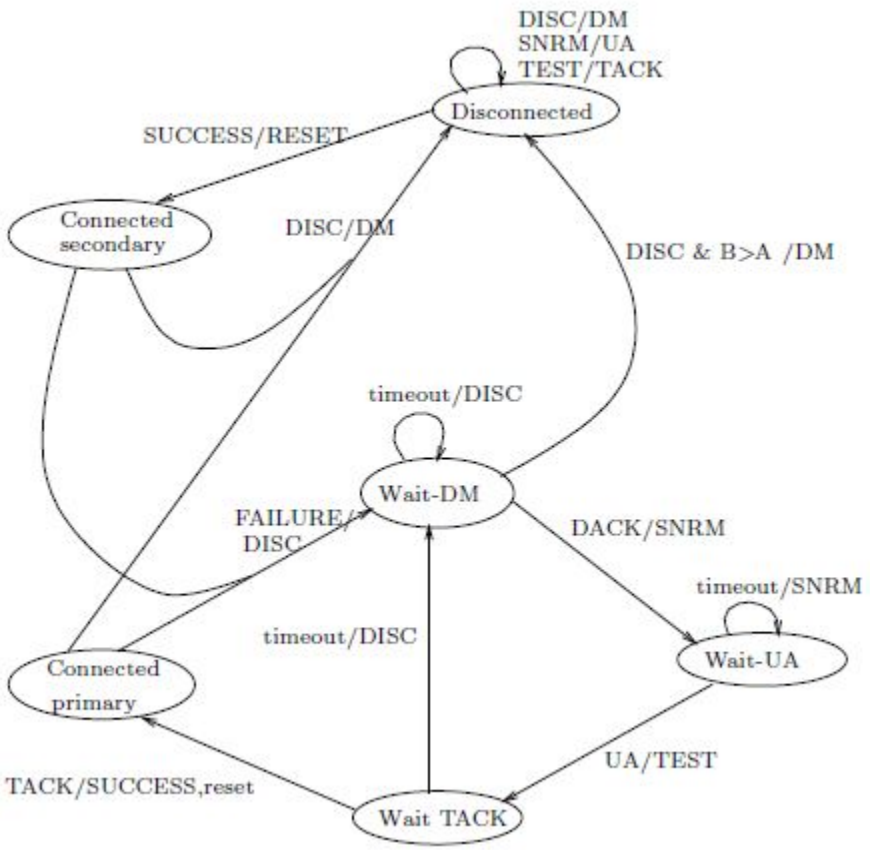


Figure 2.11: A Function-Assigning LI Procedure: algorithm for node A

The correctness of the Function-Assigning LI Procedure follows directly from the proofs for the Unbalanced LI Procedure and several additional observations. Consider first the case where the stations do not fail and, whenever a station begins operating, it enters the Wait-DM state. Then note that:

- A station (say B) can enter *disconnected* state only if it receives a DISC frame and this can be sent only if the other station is in Wait DM state; *disconnected* state can be left only by entering Connected Secondary state.
- A station (say B) can make the transition to Wait-UA state only if it receives a DM frame and this can be

sent only if the other station is in *disconnected* state.

These observations imply that after any station (say A) enters Wait UA state (and until both enter Connected state), the stations operate exactly as in the Unbalanced LI Procedure with A the primary and B the secondary. Therefore all properties of Section 2.4.3 hold here as well.

If the stations may fail while in Initialization Mode, each needs two bits of non-volatile memory to indicate if the failure occurs in *disconnected* state, in Wait-UA state or elsewhere. The station will then restart in *disconnected* state, Wait-UA state or Wait-DM state respectively. The argument why this leaves the LI Procedure unaffected by station failures is the same as in the last paragraph of Section 2.4.3.

2.4.5 A Balanced LI Procedure

The Unbalanced LI Procedure described in Section 2.4.3 can be easily modified to obtain a Balanced LI Procedure for which no primary/secondary assignment is required or performed. The finite state diagram describing the algorithm performed by each DLC is given in Figure 2.12. Notice that this procedure essentially consists of two independent executions (one in each direction across the media) of the unbalanced procedure described in Section 2.4.3. The only difference is the introduction of a Wait-Hold state which guarantees that the two DLC's re-enter the Connected state at about the same time. More specifically, neither station can enter the Connected state until both stations have entered the Wait-Hold state. The correctness of this protocol follows directly from the correctness of the unbalanced protocol.

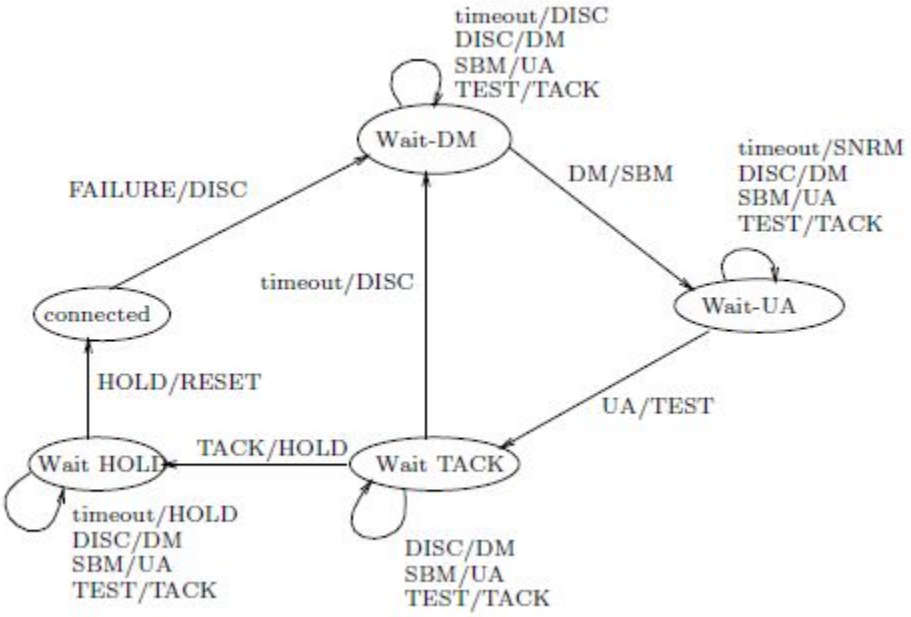


Figure 2.12: A Balanced LI Procedure

Problems

Problem 2.4.1 Consider the HDLC Initialization Procedure with the two transitions between Failure Detected and Wait-SNRM ack deleted. Does this LI procedure ensure synchronization? Prove or give counterexample.

Problem 2.4.2 In the Function Assignment LI procedure there is need to preassign numbers to nodes. Suppose that in order to relax this asymmetry, we try to assign numbers to functions rather than to nodes,i.e. the will be a number for the primary and one for the secondary. If yes, at what time will each node receive the number corresponding to its new function? Explain why the new version works.

Problem 2.4.3 Consider the Unbalanced LI procedure of Fig. 2.8 with the WAIT UA state removed and no frames SNRM or UA. Does it ensure synchronization? Prove or give counterexample.

Problem 2.4.4 On link (A, B) operates a sliding-window DLC protocol with a Link Initialization Procedure that ensures synchronization. Suppose that the source at A sends at time t_1 a packet to B, that is delivered to the sink at B at time τ_1. Also, the source at B sends at time τ_2 a packet, that is delivered to the sink at A at time t_2. Here $\tau_1 < \tau_2$ and $t_1 < t_2$. Show that DLC A is in Connected state during the entire interval $[t_1, t_2]$ if and only if DLC B is in Connected state during the entire interval $\tau_1, \tau_2]$.

Problem 2.4.5 Consider the same situation as in Problem 2.4.4, except that $\tau_2 < \tau_1$. Can it happen that DLC A enters Initialization Mode during $[t_1, t_2]$? Prove or give example.

Problem 2.4.6 On link (A, B) operates a sliding-window DLC protocol with a Link Initialization Procedure that ensures synchronization. Suppose that the source at A sends at times t_2, t_1, where $t_2 < t_1$, two packets to B, that are delivered to the sink at B at times τ_2, τ_1 respectively. Show that $\tau_2 < \tau_1$ and if DLC B is in Connected state during the entire interval $[\tau_2, \tau_1]$, so is DLC A during $[t_2, t_1]$.

Problem 2.4.7 On link (A, B) operates a sliding-window DLC protocol with a Link Initialization Procedure that ensures synchronization. Suppose that the source at A sends at time t_1 a packet to B, that is delivered to the sink at B at time τ_1. At some time $t_2 < t_1$, the source at DLC A sends a packet to B. Suppose that during the entire interval $[t_2, t_1]$, DLC A is in Connected state. Show that the packet sent at t_2 must eventually arrive at B. Let τ_2 be the arrival time. Show also that $\tau_2 < \tau_1$ and that DLC B is in Connected state during the entire interval $[\tau_2, \tau_1]$.

Problem 2.4.8 To increase the chances that the secondary enters Connected state after the primary does, one can change the LI procedure of Sec. 2.4.3 such that the primary sends 3 SUCCESS frames and then goes to Connected. The secondary goes to Connected if it receives at least one SUCCESS. Does this protocol work? Prove or give counterexample.

Problem 2.4.9 Design an LI procedure that satisfies the following TEST property : The time for n round-trips between the two nodes is within a prespecified bound.

Problem 2.4.10 Design a LI procedure that satisfies the following TEST property : When n frames are sent contiguously from one node to the other, they are all being acknowledged within a prespecified bound.

Problem 2.4.11 Consider the following network : Three nodes are connected to a bus. Every frame that is sent by one node, is received by both other nodes. The bus is not reliable, so a frame can be received correctly at one node and received corrupted or not received at all at the other node. A frame can also be corrupted or not received by both receivers.

a) Design a window protocol for the case when a primary node A needs to send the same sequence of frames reliably to B and C.

b) Design an unbalanced LI procedure for this case.

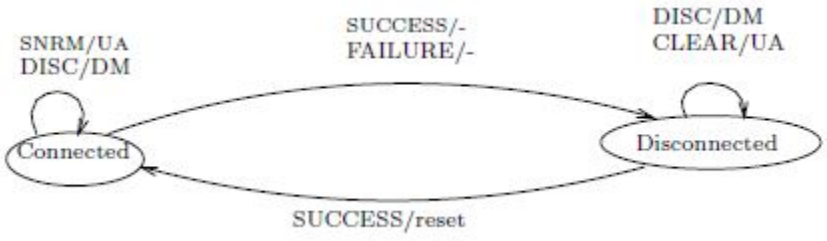


Figure 2.13: Altered diagram for the secondary node

Problem 2.4.12 a) The following change, given in Fig. 2.13 has been done to the secondary node in the unbalanced LI procedure that ensures synchronization (Fig. 2.8b)). Does the new procedure ensure synchronization. If yes, prove. If not, explain which properties are not satisfied and give an example.

b) Repeat (a) for the change in Fig. 2.14:

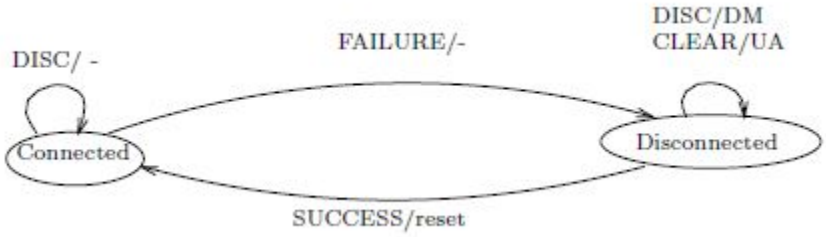


Figure 2.14: Altered diagram for the secondary node

c) Repeat (a) for the change in Fig. 2.15:

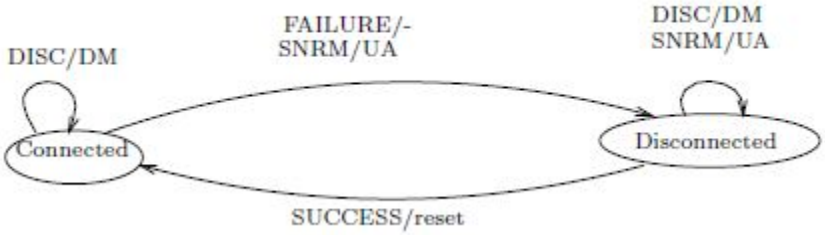


Figure 2.15: Altered diagram for the secondary node

2.5 CONCLUSIONS

ALSO, BASIC DLC PROPERTIES....????

In this chapter we have presented a new class of Link Initialization Procedures that can be used in conjunction with any bit-oriented DLC procedure to achieve data reliability. These LI Procedures were shown to ensure synchronization without the use of sequence numbers or time-stamps. Moreover, they do not rely on special properties of the physical transmission media such as finite life of messages or line flushing mechanisms. The LI Procedures provide in fact the flushing mechanism for the entire communication link.

The LI Procedures were shown to provide insurance that the link stations do not return to the Connected State before the link is capable of reasonable operation. The procedures described in this chapter have been designed to comply with the test requirement of one round-trip transmission within a specified interval. However, they can be easily modified to perform any desired test sequence. The new mechanism can simply be "plugged" into the LI Procedure in place of the Wait TACK State. On the other hand, if there is no Test requirement, the LI Procedures may be simplified by removing the TEST and TACK messages and the Wait TACK State.

In order to ensure synchronizations across station failures, our LI procedures require one or two bits of non-volatile memory.

??????? REWORD, EXPAND | SOURCE (DISTR,NET. PROT) LOSING PACKETS: PERLMAN, ROSEN

Before concluding, we note that DLC procedures are often used to provide the lower layer link protocol that serves various distributed network protocols and applications [Seg83], [SJ86], [HS82]. In this environment the data sources and sinks of Figure 2.1 are the node algorithms that perform the distributed protocols and the packets are the protocol control messages exchanged by the node algorithms. In order to perform correctly, the distributed algorithms require the lower layer link protocols to possess certain properties. These properties are precisely **Follow-up, Deliver, Confirm, FIFO, Crossing** and **Deadlock-Free** as defined in this paper. Consequently, a sliding-window bit-oriented DLC procedure such as HDLC, combined with the LI procedures proposed in this paper, can provide the link-level protocols for distributed network protocols.

Chapter 3

PI-TYPE NETWORK PROTOCOLS

Consider the situation when a UPDATE INTROD number of physically distinct computation units work on a common problem, while their operation is coordinated via communication channels connecting some of these units. Each computation unit has certain processing and memory capability and is preprogrammed to perform its part of the computation, as well as to receive and send control messages over the communication channels. The program residing in each node will be referred to as the *node algorithm* and the ensemble of all algorithms providing the solution to the common problem is named a *distributed protocol.*

The specific protocols considered here will collectively be called **Distributed Network Protocols (DNP)**, to indicate the fact that the common problem that has to be solved is connected with the network topology. Many of the "classical" graph algorithms have their distributed version and, in addition, several new distributed network protocols appear from practical problems. The main application considered so far for DNP's is in data or voice communication networks. In such networks, geographically dispersed devices must transmit information to one another and must somehow coordinate this transmission. The nodes serve as communication processors and/or as switches. In principle, the common goal of these units is to efficiently transmit the required information to achieve certain performance goals, like minimum delay or maximum throughput. With this application in mind, several examples of problems for which DNP's have been proposed or are currently under investigation are routing of information, network resynchronization, shortest path, minimum weight spanning tree, common channel access coordination, information broadcast and others.

The presented protocols have one additional important feature. Since nodes and links may fail and be added asynchronously to the network, the protocols must be able to work under arbitrarily changing network topology. We first consider DNP's for networks with fixed topology and in most cases we extend those protocols to incorporate changing topology.

It is useful to emphasize, at this point, the importance of formal specification and validation of DNP's. The design of DNP's has two main stages: first, the construction of its general idea and then specification of the node algorithms intended to implement it. As with many computer programs, it is normally hard to make sure that the node algorithms indeed perform their intended purpose under all circumstances and the validation proofs attempt to provide this confidence, as well as deeper understanding of the algorithms. The more formal these proofs are, the more confidence one can have in the correctness of the algorithm. Although intuitive presentation and validation of DNP's is important, it is certainly not sufficient and the following chapters try to emphasize this point by introducing formal methods for description and validation of DNP's.

3.1 The Fixed Topology Model

In this section we give the general model and assumptions used in all presented DNP's. Consider a connected network (V, E) where V is a set of nodes and E is a set of links. We assume that the network has fixed topology. We shall use the following assumptions:

a) Each link is bidirectional; the link connecting node i with node j considered in the direction from i to j is denoted (i, j).

b) All messages are *control messages* of the DNP. We observe that those messages are considered as data packets by the DLC's on the links.

c) Associated with each link, there is a Data-Link Control protocol that ensures Data Reliability. Since links and nodes do not fail in this model, Data Reliability means FIFO, Confirm and Delivery.

d) All messages received at a node i are stamped with the identification of the link from which they came and then transferred into a common queue; each node uses one processor for the purpose of the algorithm; the processor extracts the control message at the head of the queue (at that time we say that the node *receives* the message), proceeds to process it and discards the message when processing is completed; actions triggered by receipt of a message are atomic, namely no other operation related to the protocol is performed by the processor while a message is being processed; consequently we may relate all processing that takes place in response to the receipt of a control message to the instant this processing is completed and regard the processing as if it takes zero time.

e) Each node has an identification; before the protocol starts, each node knows the identity of all nodes that are potentially in the network; except when otherwise stated, it knows nothing about the topology of the network and in particular about what nodes actually belong to the network. We denote by $1, 2, ..., |\overline{V}|$ the nodes that are potentially in the network and by $1, 2, ..., |V|$ the nodes actually belonging to the network. We denote by $|E|$ the number of bidirectional links in the network and by $|\overline{E}|$ the number of links potentially in the network.

f) Each node knows its adjacent links, and possibly the identity of its neighbors. The latter can be normally provided by the DLC Link Initialization at the time when the link is brought up. The collection of all neighbors of node i will be denoted by G_i.

g) In some cases, the protocol may be started by only one node and in some others by several nodes asynchronously. This will be stated explictly in the description of each protocol. A node starts the algorithm by receiving a special message *START* from the outside world; a standing assumption is that, once a node has entered the algorithm, it cannot receive *START.*

h) *(don't postpone)* The message delay on a given link is measured from the time when the message is accepted by the DLC until it is delivered by the DLC at the other end to the Network Protocol. The message delays on a given link are assumed to be *strictly positive* and may be time varying, *with the restriction that always a message sent at a later time on a given link arrives at a later time.*
Note that property h), that will be called *don't postpone*, and FIFO are two different properties. The meaning of the *don't postpone* property is that if a given message is postponed, it arrives at a later time than if it were not. The meaning of FIFO is that if two messages are sent on the same link, the one that is sent second arrives at a later time than the one sent first.

Normally, the node or nodes that start the protocol perform a slightly different algorithm than the others, because they receive a *START* message from the higher layer. In order to avoid having to specify

two types of algorithms, one for the special nodes and one for the others, we shall denote throughout this and subsequent chapters the receiving of a message from the higher level by "the node receives a message from *nil*". Sometimes, in the informal description of the protocols, we shall still say that *a node receives START* instead of a node receives a message from *nil* . Moreover, "sending a message to *nil*" will mean that the node *sends no message*. Also, a variable like $e_i(l)$ at node i is undefined if $l = nil$ and the actions involving such a variable in the algorithms are disregarded.

We shall distinguish between the terms *receiving* a message and *accepting* the information in a message. A node receives a message when the algorithm dictates to take the message from the node queue and to process it as defined in d). In some cases, the algorithm at the node dictates that the information in the message be accepted and this means to pass it on to the higher level.

The measures of performance of a given protocol are the amount of communication required, the time it takes to complete and the processing complexity of the algorithms at the node and the required memory. In this and following chapters we shall try to quantify communication and time, and refer to processing and memory requirements only in general terms. Communication complexity C is normally measured in terms of the sum, over the network links, of the number of elementary quantities passed in messages over those links. The elementary quantities are node identities, link weights, capacities, number of nodes or links in certain subsets, etc. The time complexity T of a protocol is the number of units of time required if each communication of an elementary quantity over a link requires one time unit and computation requires negligible time. This is under the proviso that the protocol must continue to work correctly if link delays are arbitrary. [1]

[1] expand discussion of T ????

3.2 The Variable Topology Model

The model introduced in Sec. 3.1 assumes fixed topology. In a general computer network, links and nodes may fail and come up and Distributed Network Protocols must be restructured to take this into account. A topological change of a link is its physical failure or coming up. However, the DLC models and properties that we have developed in Chap. 2 enable us to stow away these physical level events and consider only the events at the DLC levels. Thus, the only topological link-related events that must be considered is the change at a node of a link status from Initialization Mode to Connected state and viceversa. Note that with this definition, two topological events correspond to a link failure, one at each endpoint. Similarly, two or more topological events correspond to each coming up of a link, since each end of the link may enter Connected state and reenter Initialization Mode several times before stabilizing in Connected state. Also, with this model a node failure does not directly indicate a topological event. It does indirectly result in several topological events, since the Follow-up property of the DLC implies that the nodes at the other end of the adjacent links will enter Initialization Mode. In our model a node comes up with all its adjacent links in Initialization Mode. Thus a node coming up does not directly signify a topological event. Topological events will occur upon completion of the Link Initialization procedure on each of its adjacent links. We shall indicate by G_i the collection of all nodes l such that link (i,l) is in Connected state at a node i. Note that this definition does not take into account the current status of link (i,l) at node l. The underlying assumptions that the DLC level must provide to the Network Protocol should also be adapted to the new situation. In particular, the basic assumptions of Sec. 3.1 are still valid here with the following changes:

a'),c') Associated with each link there is a Data-Link protocol that ensures Data reliability, i.e. Follow-up, Deadlock-Free, Crossing, FIFO, Confirm and Delivery.

e') A link is considered to belong to the network, i.e. to be in the set E if both its ends are in Connected state for this link. Therefore $(i,l) \in E$ if and only if both $i \in G_l$ and $l \in G_i$.

Another issue that must be addressed explicitly is the order of actions that a node takes upon coming up, after a failure or at the time it first joins the network. We assume no nonvolatile memory devoted to the state of the node in the Network Protocol, so that we do not distinguish between the events of node recovery and the node first joining the network. As shown in Chap. 2, some nonvolatile memory may be required for the DLC's, but this is of no concern to us here, since we simply assume that the DLC ensures Data Reliability, without being concerned with the way it achieves this. Obviously, if a node fails, some finite time afterwards, the failure detection mechanism at the node at the other end of each adjacent link will be triggered and that node will enter Initialization Mode. Therefore, failure of a node cannot and need not be distinguished from failure of all adjacent operating links. On the other hand, we must explicitly state in the model the order of the actions when a node comes up. Therefore we add to the model of Sec. 3.1 the assumption:

i) When a node comes up, it first performs the actions required by the Network Protocol and then proceeds to perform the Link Initialization Protocol for each of its links.

Note that this order of actions is almost mandatory, since waiting for all Link Initializations to complete before entering the Network Protocol is impossible, because a node does not know a priori what Link Initializations will succeed. In practice, some savings may be obtained by waiting for several links to enter Connected State before entering the Network Protocol, and in this case the node should perform the Network Protocol actions corresponding to the node coming up immediatly followed by the actions corresponding to links coming up.

Although the transient behavior of the network while topological changes take place is important, it is difficult to give precise statements for the desirable behavior. In other words, it is difficult to define reliability of the protocol for its behavior while changes take place. Moreover, if topological changes are too frequent, then no meaningful distributed computation can be done. Consequently, in many situations, the required properties are stated in terms of the behavior after topological and other parameters changes cease. In particular, in all cases where a fixed topology protocol is extended to a changing topology protocol, reliability of the latter means that after topological changes cease, the network converges to the same state as the fixed topology network does in the former. At first glance this requirement seems trivial, because after topological changes cease, the extended protocol works in a fixed topology environment and therefore the node algorithms are identical to the fixed topology protocol, so that obviously it will bring the network to the same final state. However, this is far from correct because neither of the following is trivial in most cases: i) ensuring that the conditions in the network after the last topological event are identical to the initial conditions assumed by the fixed topology protocol, and ii) ensuring that the extended protocol works in a fixed topology network exactly as its fixed topology counterpart. These difficulties will become apparent in the sequel.

While most reliability properties are stated in the context of convergence after topological events cease, one notable exception is the property of route loop-freedom. The latter is often considered not only in steady-state, but also in transient situations, and will be treated in some detail in the oncoming sections.

3.3 Basic Protocols

3.3.1 Propagation of Information (PI)

Suppose that node s receives from the outside world a piece of information that has to be transmitted to all nodes in the network. The simplest procedure to accomplish this, named $PI1$, is "flooding" the network [],[], REFERENCES [Seg83]. As said in Sec. 3.1, we shall say that node s receives a message containing the information to be propagated, called MSG, from nil, at the time when it starts the propagation. Sometimes we shall also say that s receives $START$ instead of saying that it receives MSG from nil. At the time when it receives MSG from nil, node s starts the protocol by transmitting MSG to all its neighbors. Each other node i in the network, when it receives the first MSG, accepts the information and sends a similar message to all its own neighbors. All other MSG's received at i are disregarded. The binary variable m_i indicates receipt of the first MSG at node i by taking on value 0 before the first message was received and 1 afterwards. Another version, that will prove useful later, is named $PI2$. In this version, when it receives the first MSG, from a neighbor l say, a node $i \neq s$ accepts the information, but sends the message only to the other neighbors (and not back to l).

In $PI1$, each node receives exactly one message from each neighbor. Sometimes it is useful to explicitly indicate that a node has indeed received a message from each neighbor. Version $PI3$ is designed to provide this information by requiring every node i to monitor the receipt of messages from neighbors and to reset m_i to 0 upon detecting that it had received a message from each neighbor.

Before specifying the algorithms at the nodes, we discuss the issue of protocol initialization. A simple initial state is to require that at the time when s starts the propagation, i.e. receives $START$, all nodes $i \in V$ have $m_i = 0$ and there are no MSG's on the links in E. However, as with the "correct global initial state" in the treatment of DLC protocols (see Sec. 2.3), not only does such a global initial state contradict the spirit of nonsynchronization in Distributed Protocols, but in fact turns out to be too restrictive for our purposes. Consequently, this restrictive initialization requirement must be relaxed. In $PI1$ and $PI2$, we shall only require that if a MSG reaches a node i, then just before the first MSG arrives at i, holds $m_i = 0$. It is easy to give examples where if this condition does not hold, the information does not reach all nodes in V. We shall do so at the end of this section. Version $PI3$ needs in addition the condition that after the first MSG arrives at i, only MSG's sent in the present protocol arrive at i. The latter condition is necessary only in $PI3$, since in the other versions nodes ignore all received messages except the first one.

We shall now formally specify the algorithms for each node and prove the correctness of the protocols.

Protocol PI1

Messages

$MSG(info)$ - message carrying the information $info$ to be propagated

Variables

G_i - set of neighbors of i
m_i - shows whether node i has already entered the protocol (values 0,1).

Initialization

if i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$

Algorithm for node i

```
A1      receive MSG(info) from l ∈ G_i ∪ {nil}
A2      {   if (m_i = 0) phase1();
        }
B1      phase1()
B2      {   m_i ← 1;
B3          accept(info);
B4          for (k ∈ G_i) send MSG(info) to k;
        }
```

Properties of the protocol

We shall need the following notations: lines in the algorithm are denoted by <•>. The notation <•>$_i$ refers to the event of node i performing line <•> of its algorithm; whenever the corresponding line contains an ***if*** condition, the notation refers only to the cases when the condition holds. The notation $phase1()_i$ refers to the event of node i performing function $phase1()$ in its algorithm. Also, $t(*)$ will denote the time when event $*$ happens.

Theorem 3.1 (PI1) *Suppose that in Protocol PI1, node $s \in V$ receives START. Recall that START is defined as the event when s receives MSG from nil. Then:*

a) All nodes $i \in V$ will accept the information in finite time and exactly once.

b) During the execution of the protocol, exactly one MSG is sent on each link in each direction.

c) The propagation of information is the fastest possible, in the sense that no other protocol can bring the information to any node i faster than PI1.

d) Define a string of messages *as a sequence of messages (of some other protocol), such that each message except the first one is sent by a node i to some neighbor at or after the time when the previous message in the sequence was received by i from some neighbor. Then no string of messages can overtake PI1, i.e. if the originator of the string sends the first message in the string after it has entered PI1, then all messages in the string are received after the respective nodes have entered the PI1.*

Note: Observe that properties *c*) and *d*) are similar, but not identical. Property *c*) says that no node can gain in terms of speed if $PI1$ is *replaced* by another protocol. Property *d*) says that if *both* $PI1$ and another protocol that generates *strings of messages* operate in the network, then no string of the other protocol can overtake $PI1$.

Proof: To prove *a*), suppose the contrary, i.e. that there is at least one node i that never performs $phase1()_i$. Consider the set V' of nodes that do perform $phase1()$ and the set V'' of nodes that never perform $phase1()$. Since $s \in V'$ and $i \in V''$, both sets are nonempty. Since V is connected, there are two neighbors j and k such that $j \in V'$ and $k \in V''$. When j performs $phase1()_j$, it sends MSG to k. The Delivery property of the DLC implies that the MSG will arrive at k. If this is the first MSG that arrives at k, the initialization assumption states that it finds $m_i = 0$, causing k to perform $phase1()_k$, contradiction. If this is not the first MSG that arrives at k, then $phase1()_k$ happened when k had received the first MSG. The fact that each node i cannot accept the information more than once follows from the fact that the parameter m_i becomes 1 exactly once and never changes afterwards.

Property *b*) follows from the fact that each node i sends MSG on all adjacent links at the time when it performs $phase1()_i$ and only then. To prove *c*), suppose that there is a protocol PI' that brings the information earlier to some nodes. Let

t_m, t'_m = time when m accepts the information in $PI1$, PI' respectively
$t(send_m(l))$ = time when m sends MSG to neighbor l in $PI1$

$t(send'_m(l))$ = time when m sends first MSG to neighbor l in PI' (∞ if m does not send any MSG to l in PI')

$t(rcv_m(l))$, $t(rcv'_m(l))$ = time when m receives MSG from neighbor l in $PI1$, PI' respectively

K = set of nodes k for which $t'_k < t_k$

i = node in K with minimum t', i.e. holds $t'_i \leq t'_k \; \forall \, k \in K$.

j = neighbor of i from which i receives the information in PI'.

Clearly $t'_j < t'_i$, hence $j \notin K$ and therefore $t'_j \geq t_j$. In PI' must hold $t(send'_j(i)) \geq t'_j$ and by <A2>, holds $t(send_j(i)) = t_j$, therefore $t(send'_j(i)) \geq t(send_j(i))$. This implies, by the *don't postpone* property (see Sec. 3.1), that $t(rcv'_i(j)) \geq t(rcv_i(j))$. But the definition of j is that $t'_i = t(rcv'_i(j))$. Also holds $t_i \leq t(rcv_i(j))$ hence $t'_i \geq t_i$ contradicting the fact that $i \in K$.

If we replace PI' by the protocol that generates the *string of messages* , the proof of *d*) is identical to the proof of *c*), except that $t(rcv'_j(i)) \geq t(rcv_j(i))$ follows from the FIFO property of the DLC instead of the *don't postpone* property. qed

The communication complexity of $PI1$ is $2 \mid E \mid$. Its time complexity is d, where d is the longest path in the network in terms of number of hops from the node that receives $START$. Hence its worst case time complexity is $(\mid V \mid -1)$.

Protocol PI2

Messages

$MSG(info)$ - message carrying the information $info$ to be propagated

Variables

G_i - set of neighbors of i
m_i - shows whether node i has already entered the protocol (values 0,1).
p_i - neighbor from which the first MSG is received

Initialization

if i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$

Algorithm for node i

```
A1    receive MSG(info) from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0) phase1();
      }
B1    phase1()
B2    {   m_i ← 1;
B3        p_i ← l;
B4        accept(info);
B5        for (k ∈ G_i − {p_i}) send MSG(info) to k;
      }
```

Theorem 3.2 *(PI2) Suppose that in Protocol PI2, a node $s \in V$ receives $START$. Then:*

a) all nodes $i \in V$ will accept the information in finite time and exactly once; after this happens, the links $\{(i, p_i)$, $\forall i \in V\}$ will form a directed spanning tree rooted at s; in addition, for all i holds $t(phase1()_i) > t(phase1()_{p_i})$.

b) During the execution of the protocol, exactly one MSG is sent on each link of the type $\neq (i, p_i)$, in each direction. On links of the type (i, p_i), a MSG is sent only in the direction from p_i to i.

c) The propagation of information is the fastest possible.

d) No string of messages can overtake $PI2$ (in the sense of the definition in Theorem 3.1d)).

Proof: To prove *a*), suppose the contrary, i.e. that there is at least a node i that never performs $phase1()_i$. Consider the set V' of nodes that do perform $phase1()$ and the set V'' of nodes that never perform $phase1()$. Since $s \in V'$ and $i \in V''$, both sets are nonempty. Since V is connected, there are two neighbors j and k such that $j \in V'$ and $k \in V''$. When j performs $phase1()_j$, it cannot be that $p_j = k$, since receipt of a message from k means that k has previously performed $phase1()_k$. Hence at time $t(phase1()_j)$, node j sends MSG to k. The Delivery property of the DLC implies that the MSG will arrive at k. If this is the first MSG that arrives at k, the initialization assumption states that it finds $m_i = 0$, causing k to perform $phase1()_k$, contradiction. If this is not the first MSG that arrives at k, then $phase1()_k$ happened when k had received the first MSG. To complete the proof of *a*), observe that since i enters the protocol (i.e. performs $phase1()_i$), when it receives the first message, from the preferred neighbor p_i, a node i always enters the protocol after its preferred neighbor. Therefore the links $\{(i, p_i)\}$ form a tree and this must be a spanning tree rooted at s.

The proof of *b*) is identical to that of Theorem 3.1b). In order to prove *c*), consider the same notations as in the proof of Theorem 3.1c). If $i \neq p_j$, the proof of contradicting the fact $i \in K$ is identical to the proof in Theorem PI1c). If $i = p_j$, then $t(send_j(i)) = \infty$, so that the same proof does not apply. However, since $t(phase1()_j) > t(phase1()_i)$, holds $t_i < t_j$. By the definitions of i and j, the inequalities $t'_j < t'_i$ and $t'_j \geq t_j$ hold as in Theorem 3.1c). Therefore, $t'_i > t'_j \geq t_j > t_i$, contradicting again the fact that $i \in K$.

If we replace PI' by the protocol that generates the *string of messages* , the proof of *d*) is identical to the proof of *c*), except that $t(rcv'_j(i)) \geq t(rcv_j(i))$ follows from the FIFO property of the DLC instead of the *don't postpone* property. qed

The communication complexity of $PI2$ is $2 \mid E \mid - \mid V \mid$. Its time complexity is d, where d is the longest path in the network in terms of number of hops from the node that receives $START$. Hence its worst case time complexity is $(\mid V \mid -1)$.

Protocol PI3

Messages

$MSG(info)$ - message carrying the information $info$ to be propagated

Variables

G_i - set of neighbors of i
m_i - shows whether node i is in the protocol (values 0,1).
$e_i(l)$ - number of MSG's sent to neighbor l - number of MSG's received from it, for all $l \in G_i$

Initialization

if i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$ and $e_i(l) = 0$ for all $l \in G_i$
- after receiving the first MSG and until m_i returns next to 0, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1      receives MSG(info) from l ∈ G_i ∪ {nil}
A2      {   if (m_i = 0) {
A3              phase1();
            }
A4          e_i(l) ← e_i(l) − 1;
A5          if (e_i(k) = 0 ∀k ∈ G_i) phase2();
        }
B1      phase1()                                        /* similar to PI1 */
B2      {   m_i ← 1;
B3          accept(info);
B4          for (k ∈ G_i){
B5              send MSG(info) to k;
B6              e_i(k) ← e_i(k) + 1;
            }
        }
C1      phase2()
C2      {   m_i ← 0;
        }
```

Note: recall that if MSG is received from nil, the lines containing $e_i(l)$ are disregarded.

Theorem 3.3 *(PI3) Suppose that in Protocol PI3, node $s \in V$ receives $START$. Then:*

a) All nodes $i \in V$ will accept the information in finite time and exactly once.

b) During the execution of the protocol, exactly one MSG is sent on each link in each direction.

c) The propagation of information is the fastest possible.

d) No string of messages can overtake $PI3$.

e) Every node $i \in V$ executes $phase2()_i$ in finite time and after this time it receives no more MSG's.

Proof: Except for the manipulation of $e_i(k)$, protocol $PI3$ is exactly $PI1$. When it enters the protocol due to receipt of MSG from l say, node i sends MSG to all neighbors and sets $e_i(l) \leftarrow 0$ and $e_i(k) \leftarrow 1$ for all other neighbors k. After having received a message from each neighbor, it will have $e_i(k) = 0$ for all neighbors k and hence will perform $phase2()_k$. All other properties follow from Theorem 3.1. qed

The communication and time complexities of $PI3$ are the same as for $PI1$.

To complete this section, we give a simple example where the PI protocols do not work if the initialization requirements do not hold. In Fig. 3.1 suppose that s receives $START$ and starts the protocol at time t_0, but when MSG arrives at b from a, it finds $m_b = 1$. This can happen for example, if at time t_0 all $m_i = 0$, but there is a MSG, containing different information, on the link from c to b and that MSG arrives at b shortly after t_0, setting $m_b \leftarrow 1$. Then a and b will send to each other MSG's containing different information, node b will disregard the information that originated at s and node c will never receive it. Thus $PI1$, $PI2$ and $PI3$ do not work. On the other hand, if the MSG on the link (c, b) arrives at b *after* the MSG with the relevant information arrives at b from a, in $PI1$ or $PI2$ node b disregards the faulty MSG when it arrives and the protocols work. However in $PI3$, the faulty message contradicts the second initialization condition, i.e. that only MSG's sent by the protocol are processed at each node i after it sets $m_i \leftarrow 1$. This faulty message causes b to complete the protocol, i.e. reset $m_b \leftarrow 0$, after which it still receives the MSG sent by c in $PI3$. As a result, node b receives a MSG after executing $phase2()_i$ and reenters the protocol, contradicting Theorem 3.3e) and a).

3.3.2 Propagation of Information with Feedback (PIF)

Sometimes the node s that receives $START$ and propagates information may want to be positively informed when the information has indeed reached all connected nodes. The following protocol can be used for this

Figure 3.1: Counterexample for Initial Conditions

purpose [Seg83]. We start with a $PI2$ protocol, namely: when it receives MSG from nil, node s sends MSG to all neighbors. When it receives the first MSG, from neighbor l say, a node i other than s accepts the information contained in MSG, denotes this neighbor as p_i and sends MSG to all neighbors except to p_i. We refer to p_i as the *preferred neighbor* of i. We continue as follows: node i expects now messages MSG from all neighbors except p_i. When it observes that it had received MSG from all those neighbors, a node i other than s sends MSG to p_i. As shown presently, receipt of MSG from all neighbors at node s can be interpreted as the signal that the information has indeed reached all connected nodes. In this way, the propagation of MSG's occurs in two phases: $phase1()$ from node s into the network according to $PI2$, for purposes of propagation and $phase2()$ from the network back to node s for the purpose of confirmation. The formal description of the protocol follows.

Protocol PIF1

Messages

$MSG(info)$ - message carrying the information $info$ to be propagated

Variables

G_i - set of neighbors of i
m_i - shows if node i has already entered the protocol (values 0,1).
$e_i(l)$ - number of MSG's sent to l - number of MSG's received from l, for all $l \in G_i$
p_i - neighbor from which the first MSG is received

Initialization

if i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$ and $e_i(k) = 0$ for all $k \in G_i$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Note: By definition, a condition on an empty set is always true. For instance, in $<$A5$>$ below, if $G_i - \{p_i\} = \emptyset$, then the condition holds and i should perform $phase2()$.

Algorithm for node i

```
A1    receives MSG(info) from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0) {
A3            phase1();
          }
A4        e_i(l) ← e_i(l) − 1;
A5        if (e_i(k) = 0 ∀k ∈ G_i − {p_i}) phase2();
      }
B1    phase1()                                          /* similar to PI2 */
B2    {   m_i ← 1;
B3        p_i ← l;
B4        accept(info);
B5        for (k ∈ G_i − {p_i}){
B6            send MSG(info) to k;
B7            e_i(k) ← e_i(k) + 1;
          }
      }
C1    phase2()
C2    {   send MSG(info) to p_i
C3        e_i(p_i) ← e_i(p_i) + 1;
      }
```

Note: recall that for a node i that receives MSG from nil, the paprameter p_i becomes nil, the lines containing $e_i(l)$ are disregarded and when eventually node i performs <C2>, it sends MSG to no one.

Theorem 3.4 *(PIF1) Suppose that a node $s \in V$ receives $START$. Then:*

a) all nodes $i \in V$ will perform the event phase1()$_i$ in finite time and exactly once (among other actions, a node accepts the information at the time when it performs phase1()$_i$ and only at that time); after this happens, the links $\{(i, p_i), \forall i \in V\}$ will form a directed spanning tree rooted at s; in addition, for all i holds $t(\text{phase1}()_i) > t(\text{phase1}()_{p_i})$; moreover, the propagation of information is the same as in PI, namely the fastest possible. Note: *some nodes may perform phase2() before all nodes have performed phase1().*

b) for all $k \in G_i$, the variables $e_i(k)$ denotes the number of MSG's sent by i to k minus the number of MSG's received by i from k.

c) all nodes $i \in V$ will perform phase2()$_i$ in finite time and exactly once; moreover $t(\text{phase2}()_i) < t(\text{phase2}()_{p_i})$; node i receives no messages after time $t(\text{phase2}()_i)$; also, at the time when node s performs phase2()$_s$, all nodes in V have completed the algorithm, i.e. have performed phase2(), there are no messages traveling in the network and holds $e_i(k) = 0$ for all $i \in V$ and all $k \in G_i$.

d) exactly one MSG travels on each link in (V, E) in each direction.

e) no string of messages can overtake $PIF1$.

Proof: The propagation of *phase*1() is as in $PI2$, hence *a*) and *e*) follow from Theorem 3.2a) and d). Part *b*) follows directly from the algorithm.

To prove *c*) let k be a leaf of the tree referred to in *a*), i.e. $\nexists$ such that $p_l = k$. Then all neighbors n of k will send MSG to k when they perform *phase*1()$_n$. Node k will receive all these messages, at which time holds from *b*) that $e_k(n) = 0$ for all $n \in G_k - \{p_k\}$, which will enable k to perform *phase*2()$_k$. At that time there are no messages traveling towards k, node k will send MSG to p_k and $e_k(p_k)$ will return to 0. The same will be true for all leaves. Now nodes that are on the last-but-one level in the tree will be able to perform *phase*2() and the procedure will continue downtree all the way to node s. This argument also shows that a node i performs *phase*2()$_i$ before its preferred neighbor does.

To prove *d*), observe that in *phase*1()$_i$, a node i sends MSG to all $k \in G_i - \{p_i\}$ and in *phase*2()$_i$ it sends MSG to p_i. Since it performs each of *phase*1()$_i$ and *phase*2()$_i$ exactly once, *d*) follows. qed

As with Protocol $PI3$, it is useful sometimes to explicitly indicate that node i has already completed the protocol, i.e. has performed $phase2()_i$. This can be done by adding the action $m_i \leftarrow 0$ in $phase2()_i$. We shall refer to this version of PIF as Protocol $PIF2$.

Protocol PIF2

Messages

$MSG(info)$ - message carrying the information $info$ to be propagated

Variables

G_i - set of neighbors of i
m_i - shows if node i is currently in the protocol (values 0,1).
$e_i(l)$ - number of MSG's sent to l - number of MSG's received from l, for all $l \in G_i$
p_i - neighbor from which the first MSG is received

Initialization

if i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$ and $e_i(k) = 0$ for all $k \in G_i$
- after receiving the first MSG and until m_i returns next to 0, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receive MSG(info) from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0) {
A3            phase1();
          }
A4        e_i(l) ← e_i(l) − 1;
A5        if (e_i(k) = 0 ∀k ∈ G_i − {p_i}) phase2();
      }
B1    phase1()                                        /* similar to PI2 */
B2    {   m_i ← 1;
B3        p_i ← l;
B4        accept(info);
B5        for (k ∈ G_i − {p_i}) {
B6            send MSG(info) to k;
B7            e_i(k) ← e_i(k) + 1;
          }
      }
C1    phase2()
C2    {   send MSG(info) to p_i
C3        e_i(p_i) ← e_i(p_i) + 1;
C4        m_i ← 0
      }
```

Note that with the change of $m_i \leftarrow 0$ in $phase2()_i$, there is danger that a node will enter the protocol two or more times. The following Theorem states that this cannot happen.

Theorem 3.5 *(PIF2) Protocol $PIF2$ has the same properties as $PIF1$.*

Proof: We first prove that no node can perform $phase1()$ more than once and that no node can send a MSG on the same link more than once. Note that we cannot deduct this property directly from the properties of Protocol $PI2$ since here the value of m_i returns to 0 at time $t(phase2()_i)$, whereas in $PI2$ it stays 1 forever. Suppose that MSG can be sent on the same link more than once and let i be the first node that sends a second MSG to the same neighbor, at time t say. Note that since s does not receive $START$ twice, holds $i \neq s$. Let t_0 be the time when i enters the protocol, i.e. performs $phase1()_i$, for the first time. Before time t_0, node i sends no MSG and at time t_0 it sends MSG to all $k \in G_i - \{p_i\}$, so $t > t_0$. Let t_1 be the first time at or after t_0 when i completes the algorithm, i.e.performs $phase2()_i$. Since we do not know yet that $phase2()_i$ will ever be performed, we allow $t_1 \leq \infty$. From time t_0 until time t_1, node i sends no MSG and at time t_1 it sends MSG to p_i, so $t > t_1$. In addition, for all $k \in G_i - \{p_i\}$, at time t_0+ holds

$e_i(k) = 1$ and at time t_1- holds $e_i(k) = 0$, so that before or at time t_1-, node i has received at least one MSG from every neighbor. However, in order to send a MSG at time t, node i must receive a MSG from some neighbor l at time $t-$, and since $t > t_1$, this is at least the second MSG received by i from l. Since, by assumption h) in Sec. 3.1, delays are strictly positive, that MSG was sent by l before t. This contradicts the fact that i is the first node to send MSG twice to the same neighbor. This proves that no node can send two MSG's to the same neighbor and therefore that no node can receive two MSG's from the same neighbor. Also, since the only way to perform $phase1()_i$ the second time is to receive a MSG after having reset m_i to 0, and the latter can be done only after having received a MSG from all neighbors, no node i can perform $phase1()_i$ twice.

Now, after having proved that no node can perform $phase1()$ more than once and that no node can send a MSG on the same link more than once, all other properties follow in exactly the same way as in Theorem 3.4. qed

The communication complexity of PIF is $2 \mid E \mid$. Its time complexity from start to termination at s is $2d$, where d is the longest path in the network in terms of the number of hops from s, hence the worst time complexity is $2(\mid V \mid -1)$. The time necessary to propagate information is the same as in PI1 and PI2, namely d.

Problems

Problem 3.3.1 Suppose PIF1(1) is started by s and almost immediately afterwards PIF1(2) is started by s. Prove or give counterexample: no MSG(2) is sent on any link before MSG(1).

Problem 3.3.2 Prove or give counterexample: no string of messages can overtake PIF2.

Problem 3.3.3 Prove Theorems 3.1, 3.2, 3.3 for the case when several nodes receive asynchronously the same message from the higher layer.

Problem 3.3.4 Consider a network with nodes a, b, c, d, s and links and delays (in both directions) as follows:

Link	a,b	a,c	a,s	b,c	b,d	c,d	c,s	d,s
Delay	3	5	3	3	2	2	1	6

Suppose that s receives MSG from nil at $t = 0$ and the nodes perform $PIF1$.

a) Indicate the values of the various variables as a function of time at each node.

b) When does the protocol complete?

c) What is the tree that is formed?

Problem 3.3.5 Consider any two nodes i and j such that i is an ancestor of j in the tree formed in $PIF1$ (i.e. j is in the subtree rooted at i). Show that the MSG's sent by the nodes k on the tree path from j to i when they perform $phase2()_k$ form a *string of messages*.

Problem 3.3.6 Consider a network with nodes a,b,c,d,s. The links and delays on them (in both directions) are given below.

Link	a,b	a,c	a,d	b,s	b,c	b,d	c,d	d,s	a,s
Delay	3	5	8	4	3	3	2	6	3

Suppose s receives START at time t=0, and the nodes perform PIF1.

a) Indicate the values of the various variables as a function of time at each node.

b) When does the protocol complete?

c) What is the tree that is formed?

Problem 3.3.7 Consider a network with fixed topology, fixed delays and a given source node s. If the delays are such that no two mesages from different neighbors can arrive at exactly the same time to a node, all PIF1 runs give the same tree and the same timing of events. This is however not the case if for example delays are integers, messages can arrive at a node from different neighbors at the same time and the order of processing is determined randomly.

a) Does every node perform phase1 at the same time in all PIF runs?
b) Show that there is a node that is a leaf in the PIF tree in all runs.
c) Does every node perform phase2 at the same time in all PIF runs?

Problem 3.3.8 *Source Routing* is a widely used routing method that requires the source to know the entire path to the destination. Consider a network with time-invariant link delays.

a) Show how PIF can be used to provide the source with the fastest path to the destination.
b) Design a PI-type protocol to provide the source with an additional path to the destination that is with high probability as disjoint as possible from the first.

3.4 Repeated Propagations of Information (RPI)

BROADCAST??, TOPO.CHANGES??, SEPARATE CHAPTER?? Suppose that a node s needs to propagate several pieces of information, for short referred to as *packets,* that cannot be included in a single message, either because they are not all available at the same time at s or because they would make together an excessively long message. We require here for each node in V properties similar to FIFO and Delivery in Link Protocols (see Sec. 2.2): packets must be accepted by each node in the same order as sent by the source s, with no gaps or duplicates and every packet is eventually accepted by each node. The simplest way to propagate several packets is to use repeated $PI1$ or $PI2$ protocols. Each PI is assigned a sequence number $r = 0, 1, 2, \ldots$ and the MSG's that belong to a given PI carry its sequence number. Due to the no-overtake property of $PI1$ and $PI2$, consecutive PI's are entered by each node in appropriate order, so packets are accepted at each node in the same order as sent by the source. Consequently there is no need to reorder packets at nodes. Therefore, the sequence numbers are not needed for reordering packets, but only to identify messages containing different packets, to allow nodes to distinguish between new packets and already received ones.

Protocol RPI1

Messages

$MSG(r, P)$ - message with sequence number r carrying information P ($r = 0, 1, 2, \ldots$)

Variables

G_i - set of neighbors of node i
r_i - largest sequence number received by i (values 0,1,2,...)

Initialization

- just before the first packet becomes available, holds $r_s = -1$
- if i receives a MSG, then just before receiving the first MSG, holds $r_i = -1$

Algorithm for node i

```
A1    packet P becomes available
A2    {   deliver MSG(r_s + 1, P) from nil to yourself;
      }
B1    receive MSG(r, P) from l ∈ G_i ∪ {nil}
B2    {   if (r > r_i) phase1(r);
      }
C1    phase1(r)                                        /* similar to PI1 */
C2    {   r_i ← r ;
C3        accept(P);
C4        for (k ∈ G_i) send MSG(r, P) to k;
      }
```

Note: In <C4>, node $i \neq s$ may send $MSG(r, P)$ to all $k \in G_i - \{l\}$, where l is the neighbor MSG was received from, instead of to all neighbors.

Theorem 3.6 *(RPI1) Suppose $s \in V$. Then packets are accepted by each node in V in the same order, without gaps or duplicates, as generated at the source s and all packets are eventually accepted at every node in V.*

Proof: As in Sec. 2.3, let $P(0), P(1), P(2), \ldots$ denote the packets that become available at the source node s. Then a message with sequence number r sent by s contains packet $P(r)$, i.e. has the form $MSG(r, P(r))$. When a node $i \neq s$, sends a message, it copies the incoming one and thus all messages in the network have this form.

Now, temporarily alter Protocol RPI1, so that a sequence of independent $PI1$'s propagate the packets. This will require defining variables $m_i(r), r = 0, 1, 2, \ldots$ and changing line <B2> to **if** $(m_i(r) = 0) phase1(r)$

and line <C2> to $m_i(r) \leftarrow 1$. Now consecutive packets are included in consecutively started PI's, say $PI(r')$ and $PI(r'+1)$. Since the propagation of $MSG(r'+1, P(r'+1))$ from s to any node can be regarded as a *string of messages* that is started after the time when s triggers $PI(r')$, Theorem 3.1d) implies that $MSG(r'+1, P(r'+1))$'s do not overtake $PI(r')$. Therefore, every $MSG(r'+1, P(r'+1))$, and in particular the one that causes acceptance of $P(r'+1)$, is received by every node after the time when the node had entered $PI(r')$, i.e. after it had accepted $P(r')$. Therefore, the condition $r > r_i$ is equivalent to $m_i(r) = 0$, so that the protocol as originally defined has the stated properties. qed

The sequence numbers scheme is the most commonly used method for propagating multiple packets of information mostly because of its conceptual simplicity, its reliability and its obvious extension to changing topology networks [],[]. REFERENCE However, in fixed topology networks, other, not much more complicated, methods can be used. The first fact to realize is that if we use $PI1$'s, sequence numbers need not be carried in MSG's. Variables $e_i(l)$ that hold the difference between the number of messages sent to and messages received from l can do the job. The protocol will be as follows. Source s starts a $PI1$, i.e. sends $MSG(P)$ to all its neighbors, as soon as a new packet P becomes available. We shall prove in Lemma 3.7 that every node receives *on every link* messages exactly in the same order as sent by the source. Also, in $PI1$, a node sends to each neighbor a copy of each new message. Therefore, whenever a node i receives from a neighbor l a message that makes $e_i(l)$ strictly negative, this indicates that node i had just received from l a new packet. This new packet is accepted and a copy of it is sent out to all neighbors of i. It is interesting to point out that in this way, in a fixed topology network, we are able to propagate packets using $PI1$'s without explicitly identifying messages that belong to the different PI's. The difficulty with this protocol, as well as with $RPI1$ is that unbounded variables, r and r_i or $e_i(l)$, are necessary. The specification of the protocol is given below:

Protocol RPI2

Messages

$MSG(P)$ - message carrying information P

Variables

G_i - set of neighbors of i
m_i - shows if node i is in the protocol (values 0,1)
$e_i(l)$ = number of messages sent to l - number of messages received from l, for all $l \in G_i$ (values $0, \pm 1, \pm 2, \dots$)

Initialization

if i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$ and $e_i(k) = 0$ for all $k \in G_i$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1      packet P becomes available
A2      {   deliver MSG(P) from nil to yourself;
        }
B1      receive MSG(P) from l ∈ G_i ∪ {nil}
B2      {   if (m_i = 0) {
B3              m_i ← 1;                                  /* enter protocol */
            }
B4          e_i(l) ← e_i(l) − 1;
B5          if (e_i(l) < 0) phase1();
        }
C1      phase1()                                          /* similar to PI1 */
C2      {   accept(P);
C3          for (k ∈ G_i) {
C4              send MSG(P) to k;
C5              e_i(k) ← e_i(k) + 1;
            }
        }
```

As before, denote by $P(0), P(1), P(2), \ldots$ the packets that become available at the source node s. We want to show that packets are accepted at each node $i \neq s$ exactly in the order $P(0), P(1), P(2), \ldots$.

Lemma 3.7

a) At every node, the sequence of accepted packets is identical to the sequence of packets sent to each neighbor.

b) At all times, $e_i(l)$ equals the number of MSG's sent by i to l so far minus *the number of MSG's received by i from l. Except for the interim value between executions of <B4> and <B5> when $e_i(l)$ takes on value -1, the variables $e_i(l)$ are nonnegative.*

c) Every node i receives on each link the packets exactly in the order $P(0), P(1), \ldots$, without gaps or duplicates.

d) Every node i accepts packets exactly in the order $P(0), P(1), \ldots$, without gaps or duplicates.

Proof: Part *a*) is obvious, since every packet accepted in <C2> is sent to all neighbors in <C4>. Part *b*) is also obvious: all $e_i(l)$ are initialized to 0, every MSG received from l decrements $e_i(l)$, every MSG sent to l increments it, and as soon as some $e_i(l)$ is decremented to -1 in <B4>, it is returned to 0 in <B5>. Parts *c*) and *d*) are proved by a common induction. Suppose both hold at all nodes in V until time $t-$ and let t be the first time when either *c*) or *d*) is contradicted, at node i say. Observe that the packet, $P(K)$ say, that contradicts *c*) or *d*) for the first time, cannot be an out of order packet, it can be only a packet that produces a gap or a duplicate. Now, if arrival of $P(K)$, from neighbor l say, contradicts *c*) at time t, then, by FIFO, packet $P(K)$ had been sent by l with a gap or duplicate in the series $P(0), P(1), \ldots$. By assumption h) in Sec. 3.1, this has occurred at a time earlier than t. However this means by *a*) above that $P(K)$ had been accepted by i with a gap or duplicate, contradicting *d*) at a time before t. This contradicts the assumption that *c*) and *d*) hold at all times at all nodes before t.

Now suppose that *d*) is violated at t, upon receipt of $MSG(P(K))$ by i from l. In view of *b*), message $P(K)$ is accepted at t only if $e_i(l)(t-) = 0$. Since at time $t-$ holds $e_i(l) = 0$, the number of MSG's received from l equals the number of MSG's sent to l. From *a*) follows that at $t-$ the *number* of accepted packets at i equals to the *number* of MSG's sent by i to l and therefore equals to the *number* of $MSG's$ received by i from l. Since by theinduction hypothesis, all packets accepted by i until $t-$ and all packets received by i from l until $t-$ are in order, without gaps or duplicates, the *sequence* of packets accepted by i until $t-$ is identical to the *sequence* of packets received by i from l until $t-$. Thus packet $P(K)$ accepted by i at time t forms a gap or a duplicate not only in the sequence of packets accepted by i, but also in the sequence received by i from l. However, we have shown above that the latter cannot happen at t. This completes the proof of *c*) and *d*) and of the Lemma. qed

Theorem 3.8 *(RPI2) Suppose $s \in V$. In RPI2 packets are accepted by each node in V in the same order as generated at the source s, without gaps or duplicates, and all packets are accepted by each node in V in finite time.*

Proof: The fact that all packets are accepted by each node in the order $P(0), P(1), \ldots$ was established in Lemma 3.7. When a packet is generated at s, the latter sends it to all neighbors. From Lemma 3.7c) and d) follows that when a MSG arrives at i from l that makes $e_i(l)$ strictly negative, this indicates the arrival of a new packet. At that time the packet is accepted and sent to all neighbors. In all other cases MSG's are disregarded, except for updating $e_i(l)$. Hence the propagation of a given packet is exactly a $PI1$, and from Theorem 3.1a) follows that each packet is accepted in finite time at each node. qed

It is interesting to note that the above protocol does not work with $PI2$'s, since in the latter a node does not send a message to the neighbor from which it receives the first message containing a given packet, and this first neighbor may be different from packet to packet[2].

The communication and time complexities of protocol RPI2 per packet are the same as for PI1. The communication required for each packet is one message per link in each direction, i.e. $2 \mid E \mid$ messages networkwise. The time necessary to disseminate the information is the fastest possible. In RPI1, the communication and time complexities are the same, except that each message carries the sequence number in addition to the information packet.

The main problem of protocol RPI2 is that in principle at least, the variables $e_i(l)$ are unbounded. In a network with unknown delays, $e_i(l)$ cannot be bounded. For example, in Fig. 3.2, if link (b, a) is slow, $e_a(b)$ may increase without bound while many new packets arrive on (s, a). Although a large enough field for $e_i(l)$ may solve the problem (64 bits with one message/millisecond would take 500 million years to wrap around), a large amount of work has been devoted to the design of protocols that work with finite fields. It turns out that this can be done in fixed topology networks.

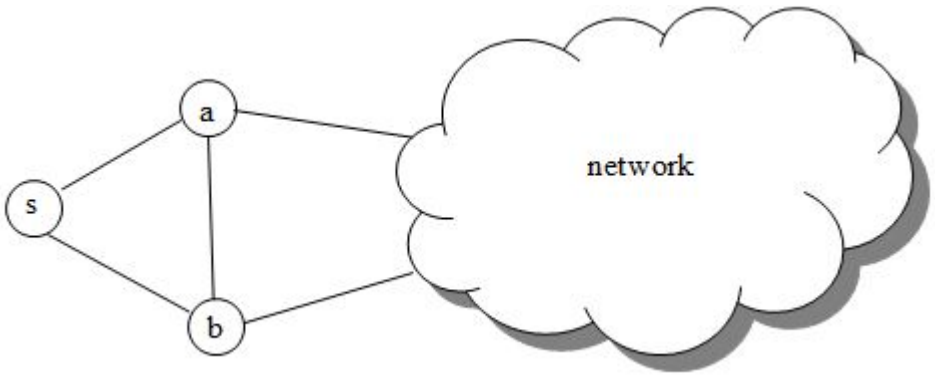


Figure 3.2: Example for RPI2

There are at least three ways to solve the problem of unbounded variables: blocking propagation of $PI1$'s in case of excessively large $e_i(l)$'s, working with $PIF2$'s instead of $PI1$'s and propagating on a tree. In the first scheme, a node i whose largest $e_i(l)$ reaches a certain value, stops accepting messages that would increase this variable even more. Those messages are queued until the value is decreased due to arrival of messages from l. One drawback of this scheme is that messages containing packets are queued at network nodes, thereby occupying network resources.

[2]See Problem 3.4.1.

Another possibility is to perform for each packet a $PIF2$ protocol, whereby each $PIF2$ is started by s after the previous $PIF2$ completes. In $PIF2$, each node resets $m_i \leftarrow 0$ when it completes its part of the protocol. Therefore, when $PIF2$ completes, all nodes $i \in V$ have $m_i = 0$ and there are no MSG's in the network, so that the initialization requirements for the next $PIF2$ are satisfied without need to distinguish between different packets. In this solution, the problem of messages buffered at nodes is solved since messages waiting for the previous PIF to complete will be buffered at the source, outside of the network, but this protocol may be very slow in dissemination of the information, since each packet will have to wait for the previous one to be sent and confirmed. The solution is to run in parallel several independent $PIF2$'s, each containing a different packet. The various PIF's will be distinguished from each other by including a finite field instance number, denoted by r, in each MSG of the PIF. For example, if we use 3 bits for PIF identification, then 8 PIF's can propagate independently in the network. As shown presently, a new packet at the source can be sent in any PIF that is available, without causing information to get out of sequence at any node. Since different PIF's may form different trees, they may complete out of order. For example, PIF(2) may be started after, but completed before, PIF(1) (we use brackets to indicate the instance number of a PIF). In this case, if a new packet becomes available at the source s after PIF(2) is completed, but before PIF(1) is, it can be propagated in a new PIF(2). We shall show that the *no-overtake* property of PI2 (see Theorem 3.2d)) implies that still packets are accepted by each node in V in the same order as generated at the source, so that there is no need to worry about out of sequence information. The reason of using the term *instance number* here, as opposed to the more common term *sequence number,* should be clear by now. The latter does not reflect the meaning of the field r in the present protocol, since consecutive packets do not necessarily carry consecutive numbers.

The third scheme saves a lot of communication and node algorithm complexity. The idea is that dissemination of all but the first packet can take place on the tree formed by the first PIF2. In this case there is no need for instance numbers or for feedback to the source. To implement this, the second packet must wait until the first (and only) PIF is completed, during which nodes tell their preferred neighbors the fact that they have selected them as preferred neighbors. This can be done in the second phase of the PIF, when nodes send MSG's to their preferred neighbors. When the PIF completes, every node knows not only its preferred neighbor (parent in the tree), but also its children in the tree. Therefore from now on, information can be sent only on the tree. No instance numbers, feedback to the source or postponement of propagation are necessary. Simply every node, when it receives a message, accepts the packet and sends a copy of the message to all its sons on the tree. This results in $\mid V \mid -1$ messages per packet, the minimum possible. Since the second packet has to wait and messages are sent on a fixed tree, that is the fastest possible when it is formed, but may deteriorate with time, the time characteristics of this version may be worse than of the previous ones. However, since there is no postponement of propagation other than queueing, either at the source or at the nodes, this version may still work better than the previous ones.

The main outcome here is that in fixed topology networks, repeated propagation of information can be performed without sequence numbers. This will be used in Sec. 5.3 to considerably alleviate the problem of sequence numbers for Topology-and-Congestion-Broadcast in changing topology networks. Again, it is important to distinguish between sequence numbers and with bounded field instance numbers.

We shall specify the protocol with independent $PIF2$'s. The specification and proof of correctness of the others is easy. We shall assume that the packets $P(1), P(2), \ldots$ become available at s and are included in $MSG(r, P)$, where $0 \leq r \leq W - 1$. Here W is a given number, determined by the fixed number of bits allocated to the field r. $PIF(r)$ will denote the $PIF2$ with instance number r.

Protocol RPIF

Messages

$MSG(r, P)$ - message carrying information P and instance number r and also serving as confirmation

Variables

G_i - set of neighbors of node i
$m_i(r)$ - shows if node i is in $PIF(r), r = 0, 1, \ldots, W-1$
$p_i(r)$ - preferred neighbor of node i for $PIF(r)$
$e_i(l)(r)$ = number of $MSG(r)$ sent to l - number of $MSG(r)$ received from l, for all $l \in G_i$

Initialization

if i receives a MSG, then

- just before receiving the first MSG, holds $m_i(r) = 0$ and $e_i(k)(r) = 0$ for all $r = 0, 1, ..., W-1$ and all $k \in G_i$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Note: By definition, a condition on an empty set is always true. For example, in <B5> below, if $G_i - \{p_i(r)\} = \emptyset$, then the condition holds and i should send $MSG(r, P)$ to $p_i(r)$.

Algorithm for node i

```
A1   packet P becomes available
A2   {   while (m_i(r') = 1 ∀r') {} ;
A3       deliver MSG(r, P) from nil to yourself with some r | m_i(r) = 0;
     }
B1   receive MSG(r, P) from l ∈ G_i ∪ {nil}
B2   {   if (m_i(r) = 0){
B3           phase1(r);
         }
B4       e_i(l)(r) ← e_i(l)(r) − 1;
B5       if (e_i(k)(r) = 0 ∀k ∈ G_i − {p_i(r)}) phase2(r);
     }
C1   phase1(r)                                   /* same as PIF1 and PIF2 */
C2   {   m_i(r) ← 1;
C3       p_i(r) ← l;
C4       accept(P);
C5       for (k ∈ G_i − {p_i(r)}){
C6           send MSG(r, P) to k;
C7           e_i(k)(r) ← e_i(k)(r) + 1;
         }
     }
D1   phase2(r)                                   /* same as PIF2 */
D2   {   send MSG(r, P) to p_i(r);
D3       e_i(p_i(r))(r) ← e_i(p_i(r))(r) + 1;
D4       m_i(r) ← 0;
     }
```

Theorem 3.9 *(RPIF)*

a) Packets that become available at s are sent in finite time.

b) If $s \in V$, then packets are accepted by each node in V in the same order as generated at the source s and all packets are eventually accepted at every node in V.

Proof: At the time when a $PIF(r)$ is started, holds $m_s(r) = 0$, namely s has completed the previous $PIF(r)$, and by Theorem 3.5, all nodes $i \in V$ have $m_i(r) = 0$ and $e_i(k)(r) = 0$ for all $k \in G_i$ and there are no messages $MSG(r)$ in (V, E). Consequently, the initial conditions for the new $PIF(r)$ comply with the $PIF2$ initialization requirements (Sec. 3.3.2). Therefore that PIF has the properties given in Theorem 3.5. In particular, each $PIF(r)$ that is started terminates in finite time, thereby making r available for the next packet. Hence *a*).

To prove *b*), observe that from Theorem 3.5, every given packet is disseminated, i.e. performs step $phase1()$ in the algorithm, according to PI2. Now consecutive packets $P(I)$ and $P(I+1)$ are included in consecutively started PIF's, say PIF(r') and PIF(r''). Observe that in general the PIF instance numbers r' and r'' are not consecutive and r' is not necessarily smaller than r''. Since the propagation of $MSG(r'')$ from s to any node can be regarded as a *string of messages* that is started after the time when s triggers PIF(r'), Theorem 3.2d) implies that $MSG(r'')$'s do not overtake $PIF(r')$. Therefore, every message $MSG(r'')$, and in particular the one that causes acceptance of $P(I+1)$, arrives at every node after the time when the node had entered $PIF(r')$, i.e. after it had accepted $P(I)$. Hence *b*). qed

The communication and time complexities of protocol RPIF per packet are the same as for PIF. The communication required for each packet is one message per link in each direction, i.e. $2 \mid E \mid$ messages networkwise, where each message must contain, in addition to the information also $log_2 W$ bits for identification. The time necessary to disseminate the information is the fastest possible if there is no waiting time at the source. If W is designed properly, the hope is that in most cases there will be no or little waiting time at the source.

Protocol $PI3$, where nodes perform a flooding with termination, is similar to $PIF2$, where nodes perform flooding with feedback and termination. A natural question that can be asked is if $PI3$ can be used in a similar way as $PIF2$ to achieve propagation of information without sequence numbers. We give here an example to show that $PI3$ cannot be used for this purpose. For simplicity, we take $W = 1$, so a new $PI3$ can be started by s only when it completes the previous one. Consider the network shown in Fig. 3.3. We start with the situation shown in the inside of the figure. All nodes have completed their part in the previous $PI3$, except for node d that is expecting a late MSG from c, hence $m_d = 1$ and $e_d(c) = 1$. All other m's and e's are 0. Therefore, s is allowed to start a new $PI3$. Clearly, if the MSG sent now by s arrives at d before the old MSG, both initialization conditions of $PI3$ are violated: the first MSG of this $PI3$ finds $m_d = 1$ and after this first MSG is received by d, the latter receives a MSG that was not sent in the present protocol. Indeed, the protocol does not work. When the new MSG, containing the new packet, arrives at d, the latter does not accept it, while m_d remains 1 and $e_d(s)$ becomes -1, and when later the MSG containing the old packet arrives from c, the variable $e_d(c)$ becomes 0. When the MSG containing the new packet arrives from c, it is not accepted again since $m_d = 1$. The final situation is that the system is deadlocked with $m_d = m_c = m_s = 1$, $e_s(d) = e_c(d) = -e_d(s) = -e_d(c) = 1$, as shown on the outside of the figure, and d has not accepted the new packet.

Problems

Problem 3.4.1 RPI2 is a modification of PI1. Let RPI2' be the protocol created when the same modification is applied to PI2. Give an example where RPI2' does not work properly.

Problem 3.4.2 RPI1 is a modification of PI1 that uses sequence numbers. Show that RPI1 with a finite sequence number does not work. Show how it contradicts the initialization condition.

Problem 3.4.3 Give a counterexample similar to the one of Fig. 3.3 for $W = 2$.

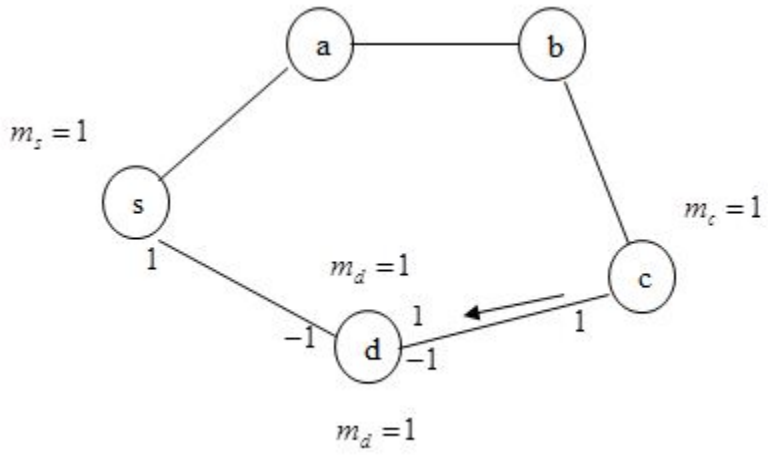


Figure 3.3: Example for PI3

3.5 Multi-Initiator Propagation of Information (MPI) - Reset Protocols

In addition to propagation of information, Protocols PI and PIF of Sec. 3.3.1 are also useful for other purposes, like resetting and cleaning the network. By this we mean that if there are some old messages some other protocol P in the network at the time when a PIF is started, then at the time when the PIF is completed, all nodes in the network are brought to a known state and there are no old messages of P in the network. As seen in the following chapters, those protocols can also be used in order to "wake up" the nodes in a network to start performing some protocol. However, in order to be useful for these purposes, we must allow for the possibility that the PI or PIF is started independently from several sources, as opposed to the versions described in previous sections, where we required a single START. In this section more than one node may receive $START$, still with the condition that only a node i that is not in the protocol, i.e. for which $m_i = 0$, can receive $START$.

In Protocols $PI1$, $PI2$ and $PIF1$ of Sec. 3.3.1, when a node enters the protocol, it sets $m_i \leftarrow 1$ and never changes m_i afterwards. The specification for their multi-initiator counterparts **MPI1, MPI2** and **MPIF1** is identical to the respective one-initiator algorithm, except that the messages MSG do not carry information. Therefore, their specification will not be repeated here. The initialization conditions are also identical. The only difference is that more than one node may receive $START$, provided that this happens before the node has entered the protocol.

For easy reference, we give a short verbal refresh. In $MPI1$, every node $i \in V$ enters the protocol, i.e. performs $phase1()_i$, and sets $m_i \leftarrow 1$ when it receives the first MSG. At that time it sends MSG to all its neighbors. It disregards all subsequent received MSG's. Protocol $MPI2$ is identical to $MPI1$, except that when it enters the protocol, a node i sends MSG's to all neighbors except the one from which it has just received MSG. The initialization condition for $MPI1$ and $MPI2$ is that if a node i receives a MSG, then just before receiving the first MSG, it has $m_i = 0$. Protocol $MPIF1$ is identical to $MPI2$, with the addition that when it realizes that it had received a MSG from each neighbor, a node i sends a MSG to the neighbor from which it had received the first MSG. This is referred to as step $phase2()_i$ in the algorithm, or in words, the time when node i completes its part in the protocol. As in $PIF1$, in order to achieve the properties we seek from $MPIF1$, we require here an additional initialization property: after entering the protocol, i.e. setting $m_i \leftarrow 1$ in step $phase1()_i$, a node i processes only messages of the current instance of the protocol. The properties of the $MPI1$, $MPI2$ and $MPIF1$ protocols are given in the following Theorems. As stated before, in many cases the multi-initiator protocols, as well as the one-initiator protocols which are special cases of the latter, are not intended for actual propagation of information. In these cases, the statements of accept the information in MSG in the specification of the protocol are simply disregarded. As a consequence, we shall avoid referring to acceptance of information in the statement of the protocol properties.

The reason the Multi-Initiator protocols work is that each one-initiator PIF is triggered independently of the others in different parts of the network and propagates undisturbed until it meets another one-initiator PIF. When two (or more) PIF's meet, each receives the MSG's of the other, but does not distinguish them from its own, so the PIF's simply coalesce.

Theorem 3.10 *(MPI1) Suppose that in Protocol $MPI1$, one or more nodes in V receive $START$. Then:*
a) All nodes $i \in V$ will enter the protocol, i.e. perform $m_i \leftarrow 1$, in finite time and exactly once.

b) During the execution of the protocol, exactly one MSG is sent on each link in each direction.

c) The propagation of $m_i \leftarrow 1$ *is the fastest possible.*

d) No string of messages can overtake $MPI1$.

Proof: In the proof of $PI1$, there is no mention of the fact that there is only one initiator. Consequently all properties hold for the multi-initiator protocol. qed

The communication complexity of $MPI1$ is $2 \mid E \mid$. Its time complexity is d, where d is the longest path in the network in terms of number of hops from any node that receives *START*. ????

Theorem 3.11 *(MPI2) Suppose that in Protocol* $MPI2$*, one or more nodes in* V *receive* $START$*. Then:*
a) all nodes $i \in V$ *will enter the protocol, i.e. perform* $phase1()_i$*, in finite time and exactly once; after this happens, the links* $\{(i, p_i),\ \forall i \in V\}$ *form a spanning forest of (disjoint) directed trees rooted at nodes that have received* $START$*; in addition, for all* i *holds* $t(phase1()_i) > t(phase1()_{p_i})$.

b) During the execution of the protocol, exactly one MSG *is sent on each link of the type* $\neq (i, p_i)$ *, in each direction. On links of the type* (i, p_i)*, a* MSG *is sent only in the direction from* p_i *to* i.

c) The propagation of $m_i \leftarrow 1$ *is the fastest possible.*

d) No string of messages can overtake $MPI2$.

Proof: Identical to Theorem 3.2. qed

The communication complexity of $PI2$ is $2 \mid E \mid - \mid V \mid$. Its time complexity is d, where d is the longest path in the network in terms of number of hops from any node that receives *START*. ????

Theorem 3.12 *(MPIF1) Suppose one or more nodes in* V *receive* $START$*. Then:*
a) all nodes $i \in V$ *will enter the protocol, i.e. perform the event* $phase1()_i$*, in finite time and exactly once. After this happens, the links* $\{(i, p_i),\ \forall i \in V\}$ *form a spanning forest of (disjoint) directed trees rooted at nodes that have received* $START$*; in addition, for all* i *holds* $t(phase1()_i) > t(phase1()_{p_i})$*; moreover, the propagation of* $m_i \leftarrow 1$ *is the fastest possible.*

Note: *some nodes may perform* $phase2()$ *before all nodes have performed* $phase1()$.

b) all nodes $i \in V$ *will perform* $phase2()_i$ *in finite time and exactly once; moreover* $t(phase1()_i) \leq t(phase2()_i) < t(phase2()_{p_i})$*; also,* $t(phase2()_i) > t(phase1()_k)$ *for all* $k \in G_i$*; node* i *receives no messages after time* $t(phase2()_i)$.

c) exactly one MSG travels on each link in (V, E) *in each direction.*

d) No string of messages can overtake $MPIF1$.

Note: Observe that although most properties of $PIF1$ carry over to Protocol $MPIF1$, one basic property is missing: although all nodes complete the protocol, i.e. execute $phase2()$, there is no knowledge at any node that the protocol has completed at all nodes in the network; execution of $phase2()$ at a node that has received $START$ signals only completion at the nodes on the tree rooted at this node and entrance into the protocol of the neighbors of these nodes.

Proof: The propagation of $phase1()$ is as in $MPI2$, hence *a*) follows from Theorem 3.11.

To prove *b*) let k be a leaf of a tree referred to in *a*), i.e. $\nexists l$ such that $p_l = k$. Then all neighbors n of k will send MSG to k when they perform $phase1()_n$. Node k will receive all these messages and will be able to perform $phase2()_k$. At that time there are no messages traveling towards k and node k will send MSG to p_k. The same will be true for all leaves. Now nodes that are on the last-but-one level in the tree will be

able to perform $phase2()$ and the procedure will continue downtree all the way to node s. This argument also shows that a node i performs $phase2()_i$ before its preferred neighbor does, but after all its neighbors have entered the protocol, i.e. have performed $phase1()$.

To prove c), observe that in $phase1()_i$, a node i sends MSG to all $k \in G_i - \{p_i\}$ and in $phase2()_i$ it sends MSG to p_i. Since it performs each of $phase1()_i$ and $phase2()_i$ exactly once, c) follows. The proof of d) is identical to the proof in Theorem 3.1. qed

The communication complexity of $MPIF1$ is $2 \mid E \mid$. Its time complexity???

In many applications of the Multi-Initiator Propagation of Information, like network reset after topological changes [AAG87b] and fast queries [CS89], it is necessary that a node that completes the protocol returns to the state it starts from, i.e. resets $m_i \leftarrow 0$. Therefore a node i may be at any time in one of two states: $m_i = 0$ indicates that i is not currently participating in the protocol, while $m_i = 1$ indicates that it is. In particular, as with Repeated PIF's, return to $m_i = 0$ upon completion allows a node to start or to enter a new version of the protocol. As seen presently however, if one is not careful, this may also allow a node to enter several times the same protocol, a potentially waistful phenomenon.

The simplest Multi-Initiator PIF with return to initial state upon completion is the Multi-Initiator version of Protocol $PIF2$, called $MPIF2$. There is one problem that must be solved however in the design of this extension. Suppose a $PIF2$ has been started and is in process of completion. Upon completing its part of the protocol every node resets $m_i \leftarrow 0$, so it is free to start its own $PIF2$ if required. For example, in Fig. 3.4, suppose that node a is the only one that has started a $PIF2$. All nodes except a have completed it and a is only waiting for a MSG from b that is now on the link (b, a) (solid arrow in Fig. 3.4). Since $m_b = 0$, node b can start now a new $PIF2$. Suppose that link (b, a) is slow and the new $PIF2$ reaches d via c, at which time node d sends a MSG to a (dashed arrows in Fig. 3.4). This is an unexpected MSG at a since $e_a(d) = 0$, indicating that a MSG has been already received at a from d. However a cannot disregard this MSG since d expects an answer in the form of a MSG from a, i.e. $e_d(a) = 1$. The simplest solution is for a to send back immediately a MSG to d. This is implemented in Protocol $MPIF2$ below. However. this solution allows a node to enter the same PIF more than once. As defined later, nodes that enter/exit a given PIF are said to enter/exit a *segment*. In our example in Fig. 3.4, node d will exit the segment of b, then a will complete its PIF and later will enter the segment of b, at which time it will send a MSG to d, causing d to enter the same segment twice.

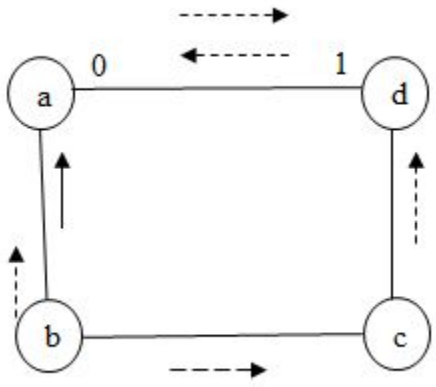


Figure 3.4: Example for MPIF2

Protocol MPIF2

Messages

MSG) - message of the protocol

Variables

G_i - set of neighbors of node i
m_i - shows whether node i is in the protocol (values 0,1)
$e_i(l)$ - number of MSG's sent to l - number of MSG's received from l, for all $l \in G_i$

Initialization

if i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$ and $e_i(l) = 0$ for all $l \in G_i$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Note: By definition, a condition on an empty set is always true. For instance, in <A7> below, if $G_i - \{p_i\} = \emptyset$, then the condition holds and i should send MSG to p_i.

Algorithm for node i

```
A1      receive MSG from l ∈ G_i ∪ {nil}
A2      {   if (m_i = 0){
A3              phase1();
            }
A4          else {
A5              if (e_i(l) = 1) e_i(l) ← e_i(l) − 1;
A6              else send MSG to l;
            }
A7          if (e_i(k) = 0 ∀k ∈ G_i − {p_i}) phase2();
        }
B1      phase1()                                  /* same as PIF1 and PIF2 */
B2      {   m_i ← 1;
B3          p_i ← l;
B4          e_i(l) ← e_i(l) − 1;
B5          for (k ∈ G_i − {p_i}){
B6              send MSG to k;
B7              e_i(k) ← e_i(k) + 1;
            }
        }
C1      phase2()                                  /* same as PIF2 */
C2      {   send MSG to p_i ;
C3          e_i(p_i) ← e_i(p_i) + 1;
C4          m_i ← 0;
        }
```

As with all Multi-Initiator protocols, $MPIF2$ is composed of several one-initiator segments, each segment operating in a similar manner to $PIF2$. In order to state the properties of $MPIF2$, we need to define precisely what we mean by a *segment* of the $MPIF2$. Loosely speaking, a *segment* is the part of the network that enters a given one-originator PIF. More precisely, a *segment* is started when a given node receives $START$ and the messages sent out by that node are said to belong to that segment. In the algorithm, a node sends out a MSG only upon receipt of a MSG. Then we say that the MSG's sent out by the node belong to the same segment as the received MSG. We say that a node *enters a segment* if it enters the protocol, i.e. performs $phase1()$, due to the receipt of a MSG belonging to that segment and *exits the segment* when it next performs $m_i \leftarrow 0$. After it enters a segment and until it exits it, we say that the node is *in the segment*. Note that in general, this allows a node that has entered a given segment to send out MSG's belonging to a different segment, if it receives a MSG of the latter (for example in <A6>). In principle, it is even possible that a node exits a segment due to receipt of a MSG of a different segment. We shall prove in the sequel that the first type of event may occur, but the second cannot.

As seen below, the properties of $MPIF2$ are significantly different from the ones of $MPIF1$. [3]

Theorem 3.13 *(MPIF2)*

a) Suppose one or more nodes in V receive $START$. Then all nodes $i \in V$ will enter the protocol, i.e. perform the event phase$1()_i$, in finite time at least once. The links $\{(i, p_i), \forall i \in V\}$ form at all times a forest of (disjoint) directed trees rooted at nodes that have received $START$; moreover, the propagation of $m_i \leftarrow 1$ is the fastest possible.

b) (Reset and cleaning) Suppose there are a finite number of times when nodes receive $START$. A finite time after the $START$'s stop, all nodes $i \in V$ will have $m_i = 0$ and there are no MSG's in E and this situation does not change. Also, if there is some other protocol P that runs in the network, there are no old messages of that protocol in the network (old messages are defined as messages of P sent by a node before it has entered for the last time MPIF2).

c) In $MPIF2$, a node may enter more than once a given segment. The number of entrances of a given node into a given $PIF2$ is bounded by $\mid V \mid$ ([CS89]).

Protocol MPIF3 solves the problem of multiple entries into the same segment that we found in MPIF2, by adding a third phase to the protocol, that propagates on the tree from the root to the leaves, and allows nodes to return to $m_i = 0$ only upon performing the third phase.

Protocol MPIF3

Messages

$MSG(z)$ - message of the protocol
$z = 1$ if MSG is sent to p_i, $z = 0$ otherwise

Variables

G_i - set of neighbors of i
m_i - shows whether node i is in the protocol (values 0,1).
$e_i(l)$ - number of MSG's sent to l - number of MSG's received from l, for all $l \in G_i$
S_i - set of sons of i

Initialization

if i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$ and $e_i(l) = 0$ for all $l \in G_i$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol[a]

Note: By definition, a condition on an empty set is always true. For instance, in <A10> below, if $G_i - \{p_i\} = \emptyset$, then the condition holds and i should send MSG to p_i.

[a] Not good enough for topo. changes. Partial trees, etc.

[3] Is there need to provide a separate proof of MPIF2, or do properties of this follow from MPIF3???

Algorithm for node i

```
A1    receive MSG(z) from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0){
A3            initialize();
A4            phase1();
          }
A5        else {
A6            if (e_i(l) = 1){
A7                e_i(l) ← e_i(l) − 1;
A8                if (z = 1) S_i ← S_i ∪ {l};
              }
A9            else send MSG(0) to l;
          }
A10       if (e_i(k) = 0 ∀k ∈ G_i − {p_i}) phase2();
      }
B1    receive RELEASE
B2    {   phase3();
      }
C1    phase1()
C2    {   m_i ← 1;
C3        p_i ← l;
C4        e_i(l) ← e_i(l) − 1;
C5        for (k ∈ G_i − {p_i}){
C6            send MSG(0) to k;
C7            e_i(k) ← e_i(k) + 1;
          }
      }
D1    phase2()
D2    {   if (p_i ≠ nil){
D3            send MSG(1) to p_i ;
D4            e_i(p_i) ← e_i(p_i) + 1;
          }
D5        else phase3();
      }
E1    phase3()
E2    {   send RELEASE to all k ∈ S_i;
E3        m_i ← 0;
      }
F1    initialize()
F2        S_i ← ∅;
      }
```

In [AAG87a], [AAG87b] it is argued that MPIF3 works as a regular multi-initiator PIF and hence there is no need to provide a proof for MPIF3 once we know that MPIF1 or PIF2 work. However this is not the case since for example clause <A9>, namely receipt of a MSG from l while $e_i(l) \neq 1$, cannot happen in MPIF1 or PIF2, thus it does not appear there. Moreover, as seen in the proofs, there is need to indicate the exact role of $RELEASE$ messages and show that they cannot arrive too early, resulting in premature return of nodes to $m_i = 0$. Another result that is necessary in the correctness proof, in particular in the proof that a node can enter a given segment at most once, is that no nodes can enter a segment after the segment root leaves it. In PIF2, this result is obvious. The proof of this property here is quite intricate.

Theorem 3.14 *(MPIF3)*

a) Suppose one or more nodes in V receive $START$. Then all nodes $i \in V$ will enter the protocol, i.e. perform the event phase1()$_i$ in finite time at least once. The links $\{(i, p_i), \forall i \in V\}$ such that $m_i = 1$ and there is no $RELEASE$ on link (p_i, i) traveling towards i form at all times a forest of (disjoint) directed trees ; moreover, the propagation of $m_i \leftarrow 1$ is the fastest possible.

b) (Reset and cleaning) Suppose there are a finite number of times when nodes receive $START$. A finite time after the $START$'s stop, all nodes $i \in V$ will have $m_i = 0$ and there are no MSG's in E and this situation does not change. Also, if there is some other protocol P that runs in the network, there are no old messages

of that protocol in the network (old messages are defined as messages of P sent by a node before it has entered for the last time MPIF3).

c) In $MPIF3$, a node can enter a given segment not more than once.

The proof proceeds via a series of Lemmas.

Lemma 3.15 *(Preliminary Properties)*
a) A $RELEASE$ cannot cross paths with a $MSG(1)$ (two messages traveling on the same link in opposite directions are said to cross paths if each is sent before the other is received).

b) If there is a $RELEASE$ message on a link (l, i) (and at the time when such a message is received by i), holds $l = p_i$, $m_i = 1$ and $e_i(k) = 0, \forall k \in G_i$.

c) Denote by $\sigma_i(l)$ the number of MSG messages ever sent by i to l and by $\rho_i(l)$ the number of such messages ever received by i from l. Then

i) *$e_i(l)$ can take values $0, +1$ or -1; if $m_i = 1$, then $e_i(l) = 0$ or 1 for all $l \in G_i - \{p_i\}$ and in addition, $e_i(p_i) = -1$ or 0.*

ii) *$e_i(l) = \sigma_i(l) - \rho_i(l)$.*

iii) *if $m_i = 0$, then $e_i(k) = 0, \forall k \in G_i$.*

Proof: The proof of *a*) and *b*) proceeds by a *common induction*. Suppose *a*), *b*) hold for all $RELEASE$ messages received by any node in V until time $t-$; we show that they cannot be contradicted for $RELEASE$ messages received at t.

Suppose that $RELEASE$ is received by node i at time t from node l and it crosses paths with a $MSG(1)$ (see Fig. 3.5). At the time τ when the $RELEASE$ was sent by l, held $i \in S_l$ and m_l has changed from 1 to 0. At the last time τ_1 before τ when i had entered S_l, node l has received from i a $MSG(1)$. At the time t_3 when the $MSG(1)$ that crosses paths with the $RELEASE$ was sent, holds $m_i = 1$ and $p_i = l$. Let t_2 be the last time before t_3 when $m_i \leftarrow 1$; at that time also $p_i \leftarrow l$. Since during $[t_2, t_3)$, node l sends no messages to i, the $MSG(1)$ received by i at time τ_1 must have been sent before t_2, at time t_1 say. At that time holds $m_i = 1$ and $p_i = l$. However at time t_2, the variable m_i changes from 0 to 1, so that between t_1 and t_2, node i must receive at least one $RELEASE$. Since by the induction assumption on *b*), until time $t-$ nodes receive $RELEASE$ from their preferred neighbors, the first $RELEASE$ received by i after t_1 must be received from l. Since between τ_1 and τ, node l sends no messages to i, that $RELEASE$ was sent before τ_1 and hence crosses paths with the $MSG(1)$ received by l at τ_1. This contradicts the induction assumption on *a*) that states that $RELEASE$ messages received before time t do not cross paths with a $MSG(1)$.

To prove *b*), suppose that $RELEASE$ is received by i from l at time t (see Fig. 3.6). At the time τ when it was sent, held $i \in S_l$. At the last time τ_1 before τ when i has entered S_l, node i has received a $MSG(1)$ from i, sent at time t_1 say. At time t_1+, holds $m_i = 1, p_i = l, e_i(k) = 0, \forall k \in G_i$. The only way for any of these relations not to hold during $[\tau, t)$ is if i receives at least one $RELEASE$ between t_1 and τ. Since by the induction assumption on *b*), all such $RELEASE$'s are received from p_i, the first one is received from l. Since between τ_1 and τ no messages are sent by l to i, such $RELEASE$ was sent before τ_1 and crosses $MSG(1)$, contradicting *a*) before time t. Hence *b*).

From the algorithm, $e_i(k)$ can receive only values 0 or -1 if $k = p_i$ and 0 or 1 for $k \neq p_i$, hence *c*)i). From *b*) follows that $RELEASE$ can be received only when $e_i(k) = 0, \forall k \in G_i$ and from $phase3()_i$, at that time node i sets $m_i \leftarrow 0$. While $m_i = 0$, node i sends no MSG's and upon receipt of the first MSG, it sets $m_i \leftarrow 1$. Therefore all $e_i(k)$ remain 0 while $m_i = 0$, hence *c*)iii). Upon entrance into the protocol, i.e. upon

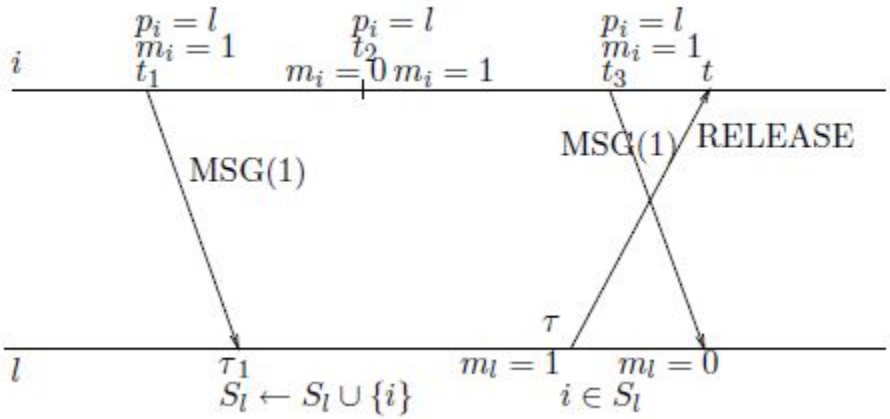


Figure 3.5: Diagram for Lemma

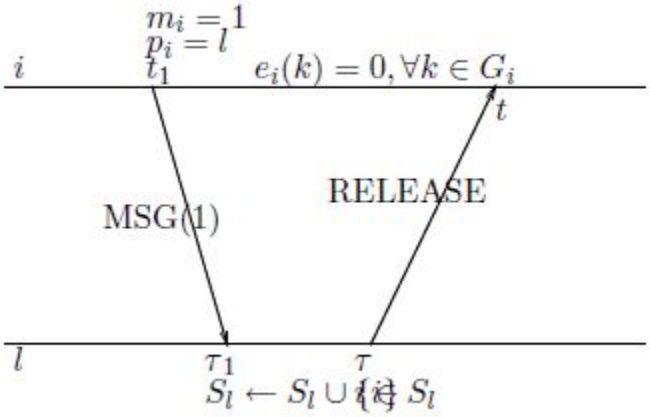


Figure 3.6: Diagram for Lemma

setting $m_i \leftarrow 1$ and while $m_i = 1$, every MSG received from l decrements $e_i(l)$ and every MSG sent to l increments it, therefore the relation $e_i(l) = \sigma_i(l) - \rho_i(l)$ holds throughout. qed

From Lemma 3.15b) and the algorithm we deduct that the events at a node i occur in the following order: enter the protocol, i.e. perform $phase1()_i$, wait until all $e_i(k), \forall k \in G_i - \{p_i\}$ become 0, at which time send $MSG(1)$ to p_i and set $e_i(p_i) \leftarrow 0$, i.e. perform $phase2()_i$, and finally receive $RELEASE$ and exit the protocol, i.e. perform $phase3()_i$. In particular, in view of Lemma 3.15b), $RELEASE$ cannot be received before $phase2()_i$ is performed, since $e_i(p_i) = -1$. We shall say that before the time when $phase2()_i$ is performed, the node is *in the first phase of the protocol* and between the time when it performs $phase2()_i$ and until it exits the protocol, it is *in the second phase of the protocol.*

Lemma 3.16 *(Preliminary Properties 2)*

a) Suppose that at time t' node i receives a message MSG from l when $e_i(l)(t'-) = 0$ and let τ' be the time when that MSG was sent by l (see Fig. 3.7). Then $e_l(i)(\tau'+) = 1$ and l has entered the protocol, i.e. has performed $phase1()_l$ at time τ'. Moreover, no message crosses paths with such a MSG.

b) Suppose that the MSG referred to in a) finds $m_i = 0$ and hence causes i to enter the protocol, i.e. to perform $phase1()_i$, and to set $p_i \leftarrow l$. Then at time t', node l is in the first phase of the protocol and stays there as long as i is in the first phase (and a nonzero period of time afterwards).

c) No message crosses paths with a $MSG(1)$. A $MSG(1)$ that arrives at a node l from a neighbor i, finds $m_l = 1$ and $e_l(i) = 1$ and at that time i is included in S_l.

d) A node i with $m_i = 1$ cannot receive a MSG from its preferred neighbor p_i.

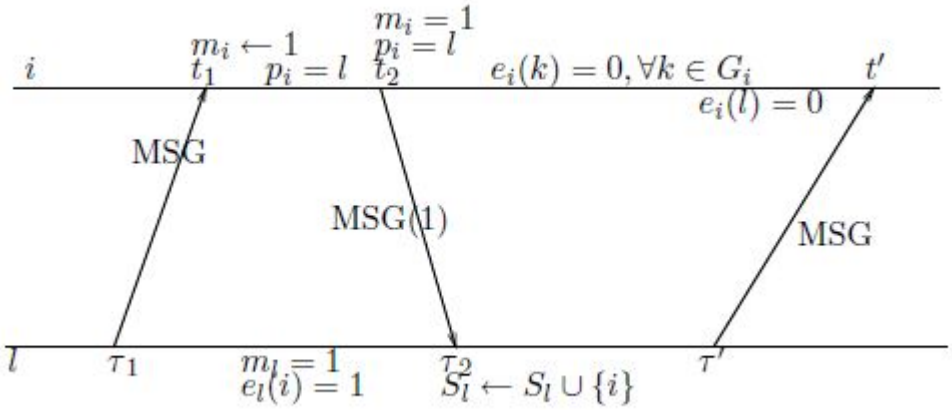


Figure 3.7: Diagram for Lemma

Proof: To prove *a*), observe that, since at time τ' node l sends to i a MSG that is received at t', the FIFO property of the DLC implies that the number $\sigma_l(i)(\tau'-)$ of MSG's sent by l to i before τ' is identical to the number $\rho_i(l)(t'-)$ of MSG's received by i from l before t' (see Fig. 3.7. Hence $\rho_i(l)(t'-) = \sigma_l(i)(\tau'+) - 1$. Moreover, the number of MSG's sent by i to any neighbor until some time t' is always larger than or equal to the number of MSG's received by that neighbor from i until t' or until any earlier time. Thus $\sigma_i(l)(t'-) \geq \rho_l(i)(\tau'+)$. Therefore we have by Lemma 3.15c)

$$0 = e_i(l)(t'-) = \sigma_i(l)(t'-) - \rho_i(l)(t'-) \geq \rho_l(i)(\tau'+) - \sigma_l(i)(\tau'+) + 1 = -e_l(i)(\tau'+) + 1$$

This implies $e_l(i)(\tau'+) \geq 1$, and therefore, from Lemma 3.15c), we have $e_l(i)(\tau'+) = 1$. Moreover, this says that the inequality in the above equation is in fact an equality, so that $\sigma_i(l)(t'-) = \rho_l(i)(\tau'+)$, meaning that all MSG's sent by i to l before t' are received at or before τ'. Thus no MSG crosses paths with the MSG sent by l at τ'. The proof of *a*) is completed by observing that the only case when sending a message results in $e_l(\tau'+) \leftarrow 1$ is in <C5>, so l has performed $phase1()_l$ at time τ'.

In *b*) we assume that the MSG sent at τ' finds not only $e_i(l) = 0$, but $m_i = 0$. From *a*) follows that at the time τ' when the MSG was sent, held $e_l(i)(\tau'+) = 1$. We need to show that l is still in the first phase at time t' and stays in this phase at least as long as i is in the first phase. In order for l to enter the second phase, $e_l(i)$ must become 0, namely l must receive a MSG from i. Since no MSG crosses paths with the MSG sent at τ' and i sends to l no MSG while in the first phase of the protocol, no MSG arrives at l from i from time τ' until after i enters the second phase. Hence l stays during all this time in the first phase,which proves *b*).

To prove *c*), suppose that $MSG(1)$ is sent by i to l, at time t_2 say (see Fig. 3.7). Then at time t_2+ holds $e_i(l) = 0, p_i = l, m_i = 1$. From Lemma 3.15a), no $RELEASE$ can cross paths with the $MSG(1)$. Suppose that a MSG crosses paths with the $MSG(1)$ and let t_3 be the time when the first MSG that crosses paths with this $MSG(1)$ arrives at i. The only way for the situation at i at time t_2+ to change until t_3- is if a $RELEASE$ arrives at i, which cannot happen since from Lemma From Lemma 3.15b), that $RELEASE$ would have to come from l and would cross paths with the $MSG(1)$, contradicting Lemma 3.15a). Hence the MSG received at t_3 by i finds $e_i(l) = 0$ and crosses paths with $MSG(1)$, contradicting part *a*). This proves that no message crosses paths with a $MSG(1)$. To prove the second part of *c*), let τ_2 be the time when $MSG(1)$ is received at l from i (see Fig. 3.7). At time t_2 when $MSG(1)$ is sent, holds $p_i = l$ and let t_1 be the last time before t_2 when p_i was set to l. At t_1, node i receives a MSG from l while $e_i(l) = 0$ and let τ_1 be the time when that MSG was sent. From *a*) and *b*), at τ_1 node l sets $m_l \leftarrow 1$ and $e_l(i) \leftarrow 1$ and that situation does not change until τ_2. This is because a change necessitates receipt of a MSG from i, which cannot happen since i sends no MSG to l between t_1 and t_2 and no MSG can cross the MSG sent by l at τ_1 because of *a*). Therefore at time τ_2- holds $m_l = 1$ and $e_l(i) = 1$ and i is included in S_l in <A8> , hence *c*).

Next we prove *d*). Suppose that node i receives at time t' a MSG from $l = p_i$ while $m_i = 1$, sent by l at time τ'. We distinguish between two cases: $e_i(l)(t'-) = 0$ and $e_i(l)(t'-) = -1$. In the first case (see Fig. 3.7) let t_2 be the last time before t' when node i sets $e_i(l) \leftarrow 0$, i.e. sends $MSG(1)$ to l. During $[t_2, t']$ holds $m_i = 1$. Since $MSG(1)$ crosses paths with no MSG, the $MSG(1)$ must arrive at l before τ', at time τ_2 say. By *c*), at that time holds $m_l = 1$ and i is included in S_l. From *b*), at time $\tau'-$ holds $m_l = 0$, so that between τ_2 and τ', node l receives at least one $RELEASE$. When the first such $RELEASE$ arrives at l it finds $m_l = 1$ and $i \in S_l$, so l sends a $RELEASE$ to i, which arrives at i before t', contradicting the fact that $m_i = 1$ during $[t_2, t']$.

The second case is $e_i(l)(t'-) = -1$. As in the proof of *a*), $\rho_i(l)(t'-) = \sigma_l(i)(\tau'-)$ and $\sigma_i(l)(t'-) \geq \rho_l(i)(\tau'-)$. Therefore,

$$-1 = e_i(l)(t'-) = \sigma_i(l)(t'-) - \rho_i(l)(t'-) \geq \rho_l(i)(\tau'-) - \sigma_l(i)(\tau'-) = -e_l(i)(\tau'-)$$

Therefore, due to the fact that e can take only values 0 or ± 1, holds $e_l(i)(\tau'-) = 1$. But this is a contradiction since in the algorithm, node l never sends a MSG to i if $e_l(i) = 1$. qed

The statements of Theorem MPIF3 are proved now in the following:

Lemma 3.17

a) At any time when $m_i = 1$ and $p_i = l$ hold, either $m_l = 1$ or there is a RELEASE on link (l, i).

b) Links (i, p_i) such that $m_i = 1$ and there is no $RELEASE$ on link (p_i, i) traveling towards i form a forest of disjoint directed trees.

c) From the time when a segment is started and until all nodes that ever enter that segment enter the second phase of the protocol, the nodes that are in a segment *form a tree. The tree is rooted at the node that had started it. No node can enter a given segment after the time when the root exits it.*

d) Every node can enter a given segment at most once.

e) Suppose one or more nodes in V receive $START$. Then every node $i \in V$ will enter the protocol, i.e. perform $phase1()_i$, at least once.

f) Suppose that there are a finite number of times when nodes receive $START$. A finite time after the $START$'s stop, all nodes $i \in V$ will have $m_i = 0$ and there are no MSG's in E and this situation does not change.

Proof: Suppose that at time t holds $m_i = 1$ and $p_i = l$. At t_1, the last time before t when i had set $p_i \leftarrow l$, it had received from l a MSG while m_i was 0 (see Fig. 3.7). At the time τ_1 when that MSG was sent, node l had entered the protocol and had set $m_l \leftarrow 1$ and $e_l(i) \leftarrow 1$ (Lemma 3.16a),b). We shall show that either m_l does not change until time t or there is a $RELEASE$ at time t on the link (l, i) traveling towards i. Since m_l can change only if l receives a $RELEASE$ and the latter can be received only if all $e_l(k)$ are 0 (Lemma 3.15b)), the variable m_l stays 1 at least until l receives a MSG from i that resets $e_l(i)$ to 0. From Lemma 3.16a), that MSG can be sent only after t_1 and hence it is a $MSG(1)$ and from Lemma 3.16c), it also causes inclusion of i into S_l. Therefore, if m_l does change before t, it does so at a time when $i \in S_l$, so l sends $RELEASE$ to i. That $RELEASE$ is still on the link at time t, because otherwise it would have set $m_i \leftarrow 0$ upon its arrival at i, contradicting the fact that $m_i = 1$ on the entire interval $(t_1, t]$. Hence *a*).

To prove *b*), we only have to show that if at time t, a node i enters the protocol, i.e. sets $m_i \leftarrow 1$, and selects l as p_i, it does not close a loop with the property that for all its links (n, p_n) holds $m_n = 1$ and no $RELEASE$ is traveling from p_n to n. But at time $t-$ holds $m_i = 0$, so that by *a*), there is no node j with $m_j = 1$, $p_j = i$ and no $RELEASE$ traveling from i to j, so no such loop can be closed. Hence *b*).

Since when a node i enters a given segment, it sends out MSG's belonging to that segment, all nodes in a segment belong to the same tree, rooted at the node, s say, that initiates the segment by receiving $START$. Since from Lemma 3.16b), while a node i is in the first phase, its preferred neighbor p_i is also in this phase, no node can enter the second phase before all its descendants in the tree do so. Therefore the root s of the tree is the last in the segment to enter the second phase. In fact, the root s exits the segment instead of entering the second phase (see $phase3()$), so that the root can exit the segment only after all nodes in the segment enter the second phase.

The above only shows that the root s does not exit the segment before all nodes already in the tree enter the second phase. It remains to show that no node enters the segment after the root had exited it. Suppose that this is not the case and let node i be the first node to enter the segment, i.e. perform $phase1()_i$, at a time t say, after the time when s had exited the segment. Let l be the node from which i receives a MSG at time t. Node l has entered the segment before the time when s had exited it and on the other hand, node l is at time t still in the first phase (Lemma 3.16b)). This means that s exits the segment while l is still in the first phase, contradicting the statements in the previous paragraph. This proves *c*).

To prove *d*), suppose a node i is the first to enter the same segment for the second time, at time t. This means that prior to t, node i had been in that segment and had exited it. From *c*), this means that the root

of the tree had exited the segment before time t. This contradicts the other part of c), that states that no node can enter a segment after the time when the root had exited it.

The proof of e) proceeds exactly as in Theorem 3.1a). Suppose that there is a node i that never performs $phase1()_i$. Consider the set V' of nodes that do perform $phase1()$ and the set V'' of nodes that never perform $phase1()$. Since nodes that receive $START$ are in V' and $i \in V''$, both sets are nonempty. Since V is connected, there are two neighbors j and k such that $j \in V'$ and $k \in V''$. When j performs $phase1()_j$, it sends MSG to k. The Delivery property of the DLC implies that the MSG will arrive at k. If this is the first MSG that arrives at k, the initialization assumption states that it finds $m_i = 0$, causing k to perform $phase1()_k$, contradiction. If this is not the first MSG that arrives at k, then $phase1()_k$ happened when k had received the first MSG.

To prove f), consider first a leaf i of a tree corresponding to a given segment. When it enters the segment, it sends a MSG to all neighbors k except p_i and sets $e_i(k) \leftarrow 1$. When the MSG arrives at a neighbor k, it finds $m_k = 1$, since otherwise k would select i as its preferred neighbor and i would not be a leaf. Note that k may be in the same segment as i or in a different one and that by Lemma 3.16d), at the time when the MSG arrives at k, the variable $e_k(i)$ cannot be -1. If at that time, $e_k(i) = 0$, then k sends a MSG back to i (see <A9>). If $e_k(i) = 1$, then k had previously sent a MSG to i, that crosses paths with the one sent by i to k. In both cases, when the MSG sent by k is received at i, it sets $e_i(k) \leftarrow 0$. When this happens for all neighbors except p_i, node i enters the second phase and sends $MSG(1)$ to its preferred neighbor. This process continues downtree until all nodes enter the second phase. By Lemma 3.16c), when a node i enters the second phase, its list S_i contains exactly the sons of i in the tree, i.e. the neighbors that had sent $MSG(1)$ to i WHY??. When the segment initiator is supposed to enter the second phase, it exits the segment instead in $phase3()_s$ and sends $RELEASE$ to all neighbors in S_s, i.e. to all sons in the tree. Similarly, when a node i receives $RELEASE$, it exits the segment and sends $RELEASE$ to its sons. Consequently, all nodes in the tree eventually set $m_i \leftarrow 0$ and no more messages of the segment can subsequently be sent or received. qed

Problems

Problem 3.5.1 Show that no loop of *RELEASES* traveling on links can be formed in MPIF3.

Problem 3.5.2 Let MPI3 be the multiple PI created by allowing several nodes to receive a START when executing PI3. Give an example of MPI3 that leads to a deadlock.

Problem 3.5.3 Suppose that there are two copies of four records of information at two nodes i and j in a connected network. Nodes i and j use MPIF3 for propagating these records to all nodes. A message can contain only one record, so after i or j sends a RELEASE message, it starts a new MPIF3 with the next record.

Will all nodes accept all records?
will all nodes acccept the records in order?

3.6 Multi-Initiator Propagation of Information-Topological changes (EMPIF)

Here we are achieving the same reset and cleaning properties as with MPIF, when there are topological changes in the network.

Protocol EMPIF3

Messages

$MSG(z)$ - message of the protocol ($z = 1$ if sent to preferred neighbor, $z = 0$ otherwise)

Variables

G_i - set of neighbors of i, i.e. $l \in G_i$ if (i, l) is in Connected state at i.
m_i - shows whether node i is in the protocol (values 0,1).
$e_i(l)$ - number of MSG's sent to l - number of MSG's received from l, for all $l \in G_i$
S_i - set of sons of i

Initialization

if i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol[a]

Note: By definition, a condition on an empty set is always true. For instance, in $<$B8$>$ below, if $G_i - \{p_i\} = \emptyset$, then the condition holds and i should perform $phase2()$.

[a] Not good enough for topo. changes. Partial trees, etc.

Algorithm for node i

```
A1    node i becomes operational
A2    {   m_i ← 0;
      }
B1    link (i,l) enters Initialization Mode;
B2    {   if (m_i = 0) {                          /* behave as if receives START */
B3            p_i ← nil;
B4            phase1();
          }
B5        else {
B6            if (l ∈ S_i) S_i ← S_i − {l};
B7            if (p_i = l) p_i ← nil;
          }
B8        if (e_i(k) = 0 ∀k ∈ G_i − {p_i}) phase2();
      }
C1    link (i,l) enters Connected State
C2    {   e_i(l) ← 0;
C3        if (m_i = 0) {                          /* behave as if receives START */
C4            p_i ← nil;
C5            phase1();
          }
      }
D1    receives MSG(z) from l ∈ G_i
D2    {   if (m_i = 0) {
D3            p_i ← l;
D4            phase1();
D5            if (e_i(k) = 0 ∀k ∈ G_i − {p_i}) phase2();
          }
D6        else {
D7            if (e_i(l) = 1) {
D8                e_i(l) ← 0;
D9                if (z = 1) S_i ← S_i ∪ {l};
D10               if (e_i(k) = 0 ∀k ∈ G_i − {p_i}) phase2();
              }
D11           else send MSG(0) to l ;
          }
      }
E1    receives RELEASE
E2    {   phase3();
      }
F1    phase1()
F2    {   m_i ← 1;
F3        S_i ← ∅;
F4        e_i(p_i) ← −1;
F5        for (k ∈ G_i − {p_i}) {
F6            send MSG(0) to k;
F7            e_i(k) ← e_i(k) + 1;
          }
      }
G1    phase2()
G2    {   if (p_i ≠ nil) {
G3            send MSG(1) to p_i;
G4            e_i(p_i) ← e_i(p_i) + 1;
          }
G5        else phase3();
      }
H1    phase3()
H2    {   m_i ← 0;
H3        send RELEASE to all k ∈ S_i;
      }
```

In [AAG87b], [AAG87a] it is argued that MPIF3 works as a regular multi-initiator PIF and hence there is no need to provide a proof for MPIF3 once we know that MPIF1 or PIF2 work. However this is not the case since for example there is no clause *phase*3() in those protocols since it cannot happen there. Moreover,

as will be seen in the proofs, there is need to show what exactly is the role of $RELEASE$ messages, that they cannot arrive too early and cause nodes to return to $m_i = 0$ too early, no nodes can enter a segment after the segment root leaves it , etc. RESTATE

The definition of a *segment* needs to be extended in the context of networks with topological changes. We shall need the following definition: we shall say that *there exists a path* from i to s if there exist nodes $i = i_0, i_1, i_2, \ldots, i_n = s$ such that $p_{i_k} = i_{k+1}$ for $k = 0, 1, \ldots, (n-1)$ and $p_s = nil$. A node s is said to start a new segment if it registers an adjacent topological change that causes it to set $p_s \leftarrow nil$ while entering or staying in the protocol. Specifically, node s starts a new segment if:

a) it performs <C4>

b) it performs <B4>

c) it performs <B7>

In the first two cases, an adjacent link fails or recovers while s is not in the protocol, causing it to enter it. In the third case, the link to p_s fails, but s stays in the protocol. In the latter case, we say that any node i that has a path to s enters the segment started by s. A node i is said to enter the segment rooted at a node s if it enters the protocol by receiving a MSG, i.e. in <D4>, and immediatly after the time when it enters the protocol, there exists a path from i to s. For example, in Fig. 3.8, suppose that nodes a, b, c, d, e, f are in the protocol in the first phase, with preferred neighbors indicated by the dashed lines. If link (g, c) fails or a link (g, b) comes up, then g enters the protocol, thereby starting a new segment. MOVE???

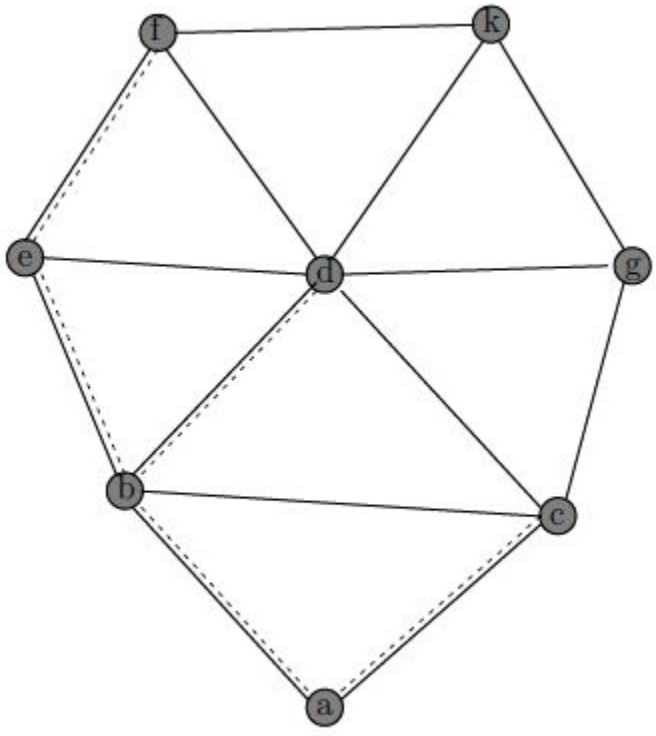


Figure 3.8: Diagram for example

RESTATE

Theorem 3.18 *(EMPIF3)*

a) Suppose one or more nodes in V receive $START$. Then all nodes $i \in V$ will enter the protocol, i.e. perform the event $phase1()_i$ in finite time at least once[4]*. The links $\{(i, p_i), \forall i \in V\}$ such that $m_i = 1$ and there is no $RELEASE$ on link (p_i, i) traveling towards i form at all times a forest of (disjoint) directed trees ; moreover, the propagation of information is the fastest possible.*

b) Suppose there are a finite number of times when nodes receive $START$. A finite time after the $START$'s stop, all nodes $i \in V$ will have $m_i = 0$ and there are no MSG's in E and this situation does not change.

c) In $MPIF3$, a node can enter a given segment not more than once. ??????

The following is an attempt to prove the properties of Protocol EMPIF3. As will be seen, the proof cannot be carried out completely (see Lemma 3.21).

Lemma 3.19 *(Preliminary Properties)*
a) A $RELEASE$ cannot cross paths with a $MSG(1)$ (two messages traveling on the same link in opposite directions are said to cross paths if each is sent before the other is received).

b) Node i can receive $RELEASE$ only from p_i and only when $m_i = 1$ and $e_i(k) = 0, \forall k \in G_i$.

c) Denote by $\sigma_i(l)$ the number of MSG messages sent by i to l since the last time when (i, l) had entered Connected state at i and by $\rho_i(l)$ the number of MSG messages received by i from l since that time. Then

i) *$e_i(l)$ can take values $0, +1$ or -1; if $m_i = 1$, then $e_i(l) = 0$ or 1 for all $l \in G_i - \{p_i\}$ and in addition, $e_i(p_i) = -1$ or 0.*

ii) *if $m_i = 0$, then $e_i(k) = 0, \forall k \in G_i$.*

iii) *if $p_i \neq nil$ and for some $k \in G_i$ holds $e_i(k) = 1$, then $e_i(p_i) = -1$.*

iv) *$e_i(l) = \sigma_i(l) - \rho_i(l)$.*

Proof: The proof of *a*) and *b*) proceeds by a common induction. Suppose *a*), *b*) hold for all $RELEASE$ messages received by any node in V until time $t-$ and we show that they cannot be contradicted for $RELEASE$ messages received at t. Suppose that $RELEASE$ is received by node i at time t from node l and it crosses paths with a $MSG(1)$ (see Fig. 3.9). At the time τ when the $RELEASE$ was sent by l, held $i \in S_l$ and m_l has changed from 1 to 0. At the last time τ_1 before τ when i had entered S_l, node l has received from i a $MSG(1)$. At the time t_3 when the $MSG(1)$ that crosses paths with the $RELEASE$ was sent, holds $m_i = 1$ and $p_i = l$. Let t_2 be the last time before t_3 when $m_i \leftarrow 1$; at that time also $p_i \leftarrow l$. Since during $[t_2, t_3)$, node l sends no messages to i, the $MSG(1)$ received by i at time τ_1 must have been sent before t_2, at time t_1 say. At that time holds $m_i = 1$ and $p_i = l$. However at time t_2-, holds $m_i = 0$. There are two occurrences that can cause m_i to change from 1 at t_1 to 0 at time t_2-: (i) node i receives at least one $RELEASE$ (<E1>), and (ii) node i fails and comes up again (<A1>). The latter case is not possible here, because in view of the *Crossing* property of the DLC on link (i, l), it would require link (i, l) to enter *Initialization* Mode between τ_1 and τ at l (cf. Problem 2.4.4), contradicting the fact that $i \in S_l$ during the entire interval $(\tau_1, \tau]$. To see that case (i) is not possible either, note that by the induction assumption on *b*), until time $t-$ nodes receive $RELEASE$ from their preferred neighbors, the first $RELEASE$ received by i after t_1 must be received from l. Since between τ_1 and τ, node l sends no messages to i, that $RELEASE$ was sent before τ_1 and hence crosses paths with the $MSG(1)$ received by l at time τ_1. This contradicts the induction assumption on *a*) that states that $RELEASE$ messages received before time t do not cross paths with a $MSG(1)$.

[4]This is not proved yet

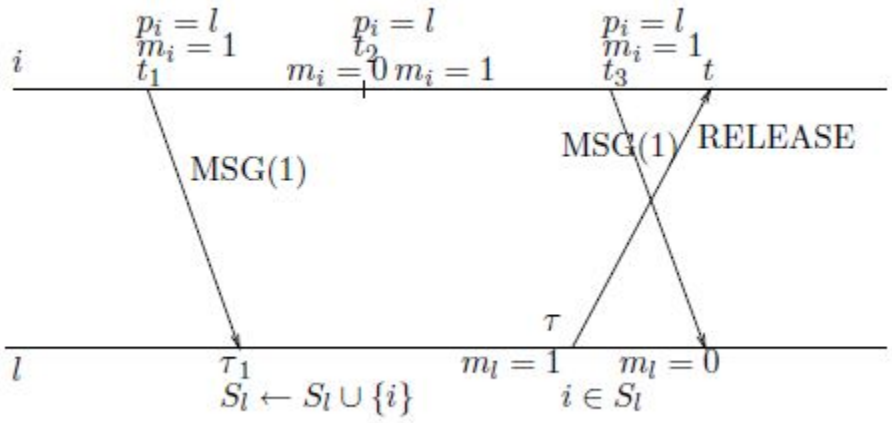


Figure 3.9: Diagram for proof of lemma

To prove *b*), suppose that $RELEASE$ is received by i from l at time t (see Fig. 3.10). At the time τ when it was sent, held $i \in S_l$. At the last time τ_1 before τ when i had entered S_l, node i has received a $MSG(1)$ from i, sent at time t_1 say. At time t_1+, holds $m_i = 1, p_i = l, e_i(k) = 0, \forall k \in G_i$. For any of these relations not to hold at time t, one of the occurrences in the proof of *a*) must occur between t_1 and t. As in *a*), node i cannot fail and recover and (i, l) cannot enter *Initialization* Mode at i, since this would cause i to leave S_l during $(\tau_1, \tau]$. Since by the induction assumption on *b*), all $RELEASE$'s that arrive before t are received from preferred neighbors, the first one is received from l. Since between τ_1 and τ no messages are sent by l to i, such $RELEASE$ was sent before τ_1 and crosses $MSG(1)$, contradicting *a*) before time t. Hence *b*).

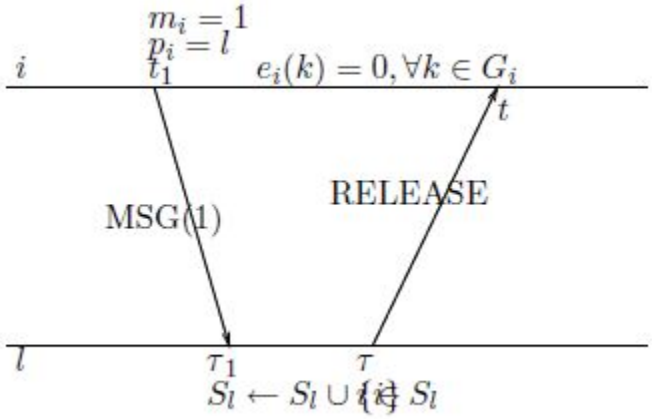


Figure 3.10: Diagram for proof of lemma

From the algorithm, $e_i(k)$ can receive only values 0 or -1 if $k = p_i$ and 0 or 1 for $k \neq p_i$, hence *c*)i). From b) follows that $RELEASE$ can be received only when $e_i(k) = 0, \forall k \in G_i$ and from <H2>, at that time node i sets $m_i \leftarrow 0$. While $m_i = 0$, node i sends no MSG's and upon receipt of the first MSG, it sets $m_i \leftarrow 1$. Therefore all $e_i(k)$ remain 0 while $m_i = 0$, hence *c*)ii). Variables $e_i(k)$ can be set to 1 and p_i can

become $\neq nil$ only when i enters the protocol in $phase1()$ and at that time, if indeed $p_i \neq nil$, node i sets $e_i(p_i) \leftarrow -1$ (see <F4>). Whenever the last $e_i(k)$ switches from 1 to 0, if $p_i \neq nil$, then $e_i(p_i)$ becomes 0 (in <G4>). Hence c)iii). When (i,l) enters *Connected* state at i, node i sets $e_i(l) \leftarrow 0$ (see <C2>). Therefore c)iv) will be proved if we show that as long as (i,l) is in *Connected* state, every MSG received from l decrements $e_i(l)$ and every MSG sent to l increments it. In view of the fact that all $e_i(k)$ are 0 just before any entrance of node i into the protocol (from c)ii) above), this is easily checked for all cases when i enters the protocol. When it enters the second phase with $p_i = nil$, node i sends no MSG's. It can enter the second phase with $p_i \neq nil$ only if it receives a MSG from a neighbor l while $m_i = 0$ (in <F6>), if l is the only neighbor, or while $m_i = 1$ and $e_i(l) = 1$ (in <D10>). In both cases, it sends a $MSG(1)$ to p_i and increments $e_i(p_i)$ from -1 to 0. In the first case, it was -1 because it had been just set to this value, in the second case it was -1 because of c)iii) above. The only other case that needs to be checked is when i receives a MSG from l while $m_i = 1$ (see <D6>). In this case, if $e_i(l) = 1$, then $e_i(l)$ is decremented to 0; otherwise, it is left unchanged and a MSG is returned to l. qed

It is interesting to observe that properties a) and b) above of $RELEASE$ depend only on the fact that $RELEASE$ is sent by a node i to its sons, i.e. to nodes in S_i, and not on the fact that the $RELEASE$ chain in propagated from the root to the leaves of the trees. EXPAND !?????

From Lemma 3.19b) and the algorithm we deduct that the events at a node i occur in the following order: enter the protocol, i.e. perform $phase1()$, wait until all $e_i(k), \forall k \in G_i - \{p_i\}$ become 0, i.e. <B8> holds, at which time send $MSG(1)$ to p_i and set also $e_i(p_i) \leftarrow 0$, and finally receive $RELEASE$ and exit the protocol, i.e. perform $phase3()$. In particular, in view of Lemma 3.19b), $RELEASE$ cannot be received before <B8> is performed, since $e_i(p_i) = -1$. We shall say that before the time when <B8> holds, the node is *in the first phase of the protocol* and afterwards and until it exits the protocol, it is *in the second phase of the protocol.*

Lemma 3.20 *(Preliminary Properties)*

a) Suppose that at time t' node i receives a message MSG from l when $e_i(l)(t'-) = 0$ and let τ' be the time when that MSG was sent by l (see Fig. 3.11). Then $e_l(i)(\tau'+) = 1$ and l has entered the protocol, i.e. has performed $phase1()_l$, at time τ'. Moreover, no message crosses paths with such a MSG.

b) Suppose that the MSG referred to in a) finds $m_i = 0$ and hence causes i to enter the protocol, i.e. to perform $phase1()_i$, and to set $p_i \leftarrow l$. Let t'' be the first time after t' when i enters the second phase. If during (τ', t'') holds $i \in G_l$, then l stays in the first phase during this entire interval and a nonzero period of time afterwards (note that $i \in G_l$ means that node l is operational and (i,l) is in Connected state).

c) No message crosses paths with a $MSG(1)$. Any $MSG(1)$ that arrives at a node i, from a neighbor l say, finds $m_l = 1$ and $e_l(i) = 1$ and at that time i is included in S_l.

d) A node i with $m_i = 1$ cannot receive a MSG from its preferred neighbor p_i.

Proof: To prove a), observe that, since at time τ' node l sends to i a MSG that is received at t', the Crossing property of the DLC implies that during (τ', t'), node i is in Connected state for (i,l). Also, the Crossing and FIFO properties of the DLC imply that the number $\sigma_l(i)(\tau'-)$ of MSG's sent by l to i before τ' is identical to the number $\rho_i(l)(t'-)$ of MSG's received by i from l before t' (see Fig. 3.11). Hence $\rho_i(l)(t'-) = \sigma_l(i)(\tau'+) - 1$. Moreover, the number of MSG's sent by i to any neighbor until time t' is always larger than or equal to the number of MSG's received by that neighbor from i until t' or until any earlier time. Thus $\sigma_i(l)(t'-) \geq \rho_l(i)(\tau'+)$. Therefore we have by Lemma 3.19c),

$$0 = e_i(l)(t'-) = \sigma_i(l)(t'-) - \rho_i(l)(t'-) \geq \rho_l(i)(\tau'+) - \sigma_l(i)(\tau'+) + 1 = -e_l(i)(\tau'+) + 1$$

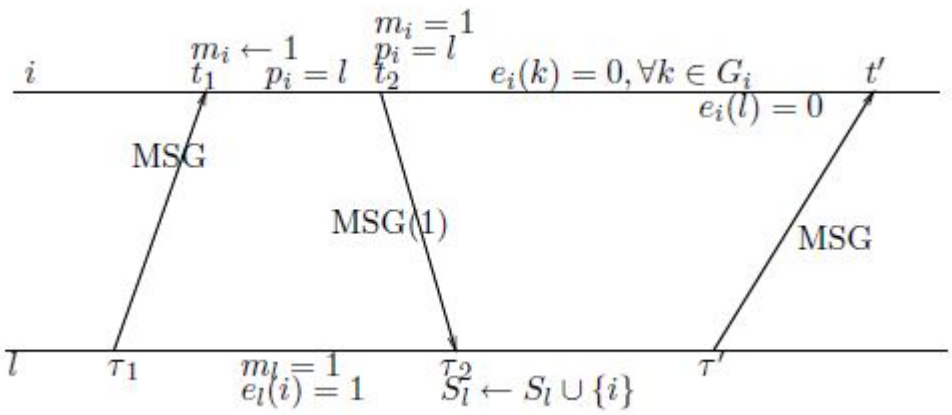


Figure 3.11: Diagram for proof of lemma

This shows $e_l(i)(\tau'+) \geq 1$, and from Lemma 3.19c) holds $e_l(i)(\tau'+) = 1$. Moreover, this says that the inequality in the above equation is in fact an equality, thus $\sigma_i(l)(t'-) = \rho_l(i)(\tau'+)$, meaning that all MSG's sent by i to l before t' are received at or before τ'. Thus no MSG crosses paths with the MSG sent by l at τ'. The proof of a) is completed by observing that the only case when sending of a MSG and $e_l(\tau'+) \leftarrow 1$ occur at the same time, is when i enters the protocol. Thus l has performed $phase1()_l$ at time τ'.

In b) we assume that the MSG sent at τ' finds not only $e_i(l) = 0$, but also $m_i = 0$. From a) follows that at the time τ' when the MSG was sent, held $e_l(i)(\tau'+) = 1$. We also assume that during the entire interval (τ', t''), node l is up and link (i, l) is in *Connected* state at l. Hence the only way for l to leave the first state is by entering the second. In order for l to enter the second phase, $e_l(i)$ must become 0, namely l must receive a MSG from i. Since no MSG crosses paths with the MSG sent at τ' (cf. a)) and i sends to l no MSG while in the first phase of the protocol, no MSG arrives at l from i from time τ' until t'' when i enters the second phase. Hence l stays during all this time in the first phase,which proves b).

To prove c), suppose that $MSG(1)$ is sent by i to l, at time t_2 say (see Fig. 3.12). Then at time t_2+ holds $e_i(l) = 0, p_i = l, m_i = 1$. From Lemma 3.19a), no $RELEASE$ can cross paths with the $MSG(1)$. Suppose that a MSG crosses paths with the $MSG(1)$ and let t' be the time when the first MSG that crosses paths with this $MSG(1)$ arrives at i. Since during (t_2, t'), link (i, l) cannot enter *Initialization* Mode at i because that would contradict the *Crossing* property of the DLC (see Problem 2.4.5) the only way for the situation at i at time t_2+ to change until $t'-$ is if a $RELEASE$ arrives at i. However, this cannot happen since from Lemma 3.19b), that $RELEASE$ would have to come from l and would cross paths with the $MSG(1)$, contradicting Lemma 3.19a). Thus the situation at t_2+ does not change until $t'-$ and in particular the MSG received by i at t' finds $e_i(l)(t'-) = 0$. This means that a MSG that finds $e_i(l) = 0$ crosses paths with the $MSG(1)$, contradicting part a). Therefore, no message crosses paths with a $MSG(1)$. To prove the second part of c), let τ_2 be the time when $MSG(1)$ is received at l from i (see Fig.3.12). At time t_2 when $MSG(1)$ is sent, holds $m_i = 1, p_i = l$ and let t_1 be the last time before t_2 when i had entered the protocol, i.e. m_i was set to 1. At that time $p_i \leftarrow l$ and during $[t_1, t_2]$, link (i, l) is in Connected state at i. At t_1, node i receives a MSG from l while $e_i(l) = 0$ and let τ_1 be the time when that MSG was sent. From a) and b), at τ_1 node l sets $m_l \leftarrow 1$ and $e_l(i) \leftarrow 1$ and that situation does not change until τ_2. This is because a change necessitates either link (i, l) entering *Initialization* Mode or receipt of a MSG from i. The first cannot happen as shown in Problem 2.4.4 and the latter cannot happen since i sends no MSG to l between t_1 and t_2 and no MSG

can cross the MSG sent by l at τ_1 because of a). Therefore at time τ_2- holds $m_l = 1$, $e_l(i) = 1$ and i is included in S_l in <D9>, hence c).

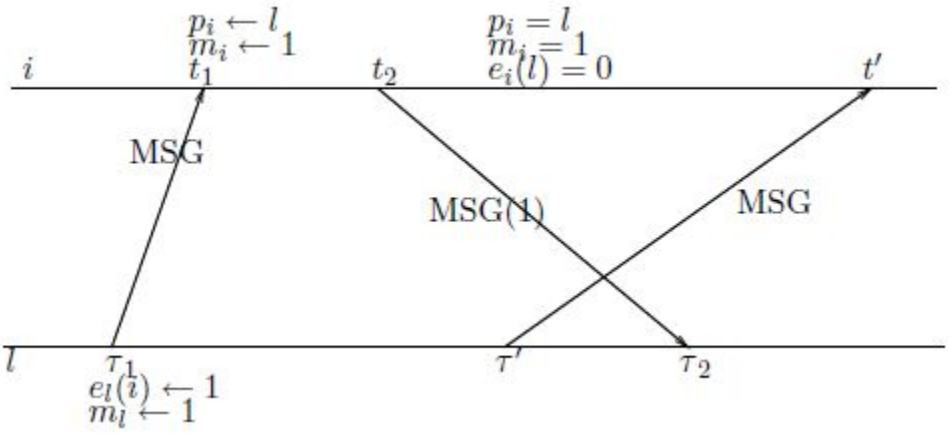


Figure 3.12: Diagram for proof of lemma

Next we prove d). Suppose that node i receives at time t' a MSG from $l = p_i$ while $m_i = 1$, sent by l at time τ'. We distinguish between two cases: $e_i(l)(t'-) = 0$ and $e_i(l)(t'-) = -1$. In the first case, note that $e_i(p_i)$ can be 0 while node i is in the protocol, i.e. if $m_i = 1$, only if i is in the second phase of the protocol, and let t_2 be the time when i had entered that phase. (see Fig. 3.11). Let t_1 be the last time before t' when i has entered the protocol, i.e. has set $m_i \leftarrow 1$ and $p_i \leftarrow l$. At time t_2 node i has sent $MSG(1)$ to l and during $[t_1, t']$, link (l, i) is in Connected state at i and holds $m_i = 1$. Let τ_1 be the time when l has sent the MSG received by i at t_1. From Problem 2.4.6, link (l, i) is in Connected state at l during $[\tau_1, \tau']$. During $[t_1, t']$, node i sends to l only the $MSG(1)$ at t_2 and from a), no message crosses paths with the MSG that arrives at t_1, thus the only message that can arrive at l from i during $[\tau_1, \tau']$ is the $MSG(1)$ sent by i at t_2. Since at τ_1+ holds $e_l(i) = 1$ and from a), at time $\tau'-$ holds $e_l(i) = 0$, some message must arrive on $[\tau_1, \tau']$ at l from i, thus the $MSG(1)$ indeed arrives at l, at time τ_2 say. By c), at that time holds $m_l = 1$ and i is included in S_l. From a), at time $\tau'-$ holds $m_l = 0$, and link (i, l) is in Connected state at l between τ_2 and τ', so that l receives at least one $RELEASE$ during this period. When the first such $RELEASE$ arrives at l, it finds $m_l = 1$ and $i \in S_l$, so l sends a $RELEASE$ to i. From Problem 2.4.7, the latter does not get lost on the link and arrives at i before t', contradicting the fact that $m_i = 1$ during $[t_2, t']$.

The second case is $e_i(l)(t'-) = -1$. As in the proof of a), $\rho_i(l)(t'-) = \sigma_l(i)(\tau'-)$ and $\sigma_i(l)(t'-) \geq \rho_l(i)(\tau'-)$. Therefore,

$$-1 = e_i(l)(t'-) = \sigma_i(l)(t'-) - \rho_i(l)(t'-) \geq \rho_l(i)(\tau'-) - \sigma_l(i)(\tau'-) = -e_l(i)(\tau'-).$$

Due to the fact that e can take only values 0 or ± 1, this implies $e_l(i)(\tau'-) = 1$. But this is a contradiction since in the algorithm, node l never sends a MSG to i at some time τ' if $e_l(i)(\tau'-) = 1$. qed

The statements of Theorem 3.18 are proved now in Lemma 3.21.

Definition: A message is said to be *outstanding* on a link (i, j) towards j if it had been sent by i, but has not arrived yet at j. Note that an outstanding message may never arrive at j, due to the latter entering *Initialization* mode for that link before the message arrives.

Lemma 3.21

a) At any time when $m_i = 1$ and $p_i = l$, if $i \in G_l$, then either $m_l = 1$ or there is a RELEASE outstanding on link (l, i) towards i (the condition $i \in G_l$ says that l is operational and link (i, l) is in Connected state.) If $m_l = 1$, then i and l belong to the same segment.

b) At any time, links $(i, p_i), p_i \neq nil$ such that $m_i = 1$, $i \in G_{p_i}$ and there is no outstanding RELEASE on link (p_i, i) traveling towards i, form a forest of disjoint directed trees. All nodes i in a given tree belong to the same segment.

c) Each tree referred to in b) is rooted at a node i such that $m_i = 1$ and one of the following holds:

(i) $p_i = nil$

(ii) $p_i \neq nil$ *and* $i \notin G_{p_i}$

(iii) $p_i \neq nil$ *and* $i \in G_{p_i}$ *and there is a RELEASE outstanding on* (p_i, i) *towards* i.

The initiator s of a segment is the root of a tree as long as $m_s = 1$. Moreover, nodes in a given tree enter the second phase in order from leaves to root. No node can enter a tree whose root is of the type (iii) above.

d) Every node can enter a given tree $\boxed{\textit{segment???}}$ *at most once.*[5]

e) At the time when a node enters the protocol, it sends a MSG to all neighbors, except to p_i. When this MSG arrives at a neighbor, then either that neighbor enters the protocol, or it is in the protocol already.

f) Suppose that there are a finite number of topological changes. A finite time after those changes stop, all nodes $i \in V$ will have $m_i = 0$ and there are no MSG's in E and this situation does not change (where (V, E) is the final network).

g) An information message sent by a node i after the final $m_i \leftarrow 0$, can be received by a neighbor l only after the final $m_l \leftarrow 1$.

Proof: Suppose that at time t holds $m_i = 1$ and $p_i = l$. At t_1, the last time before t when i had set $p_i \leftarrow l$, it had received from l a MSG while m_i was 0 (see Fig. 3.11). At the time τ_1 when that MSG was sent, node l had entered the protocol and had set $m_l \leftarrow 1$ and $e_l(i) \leftarrow 1$ (Lemma 3.20a),b). Moreover, link (i, l) is in *Connected* state at i during the entire interval $[t_1, t]$, so that the Crossing property of DLC shows that link (l, i) could not have entered *Initialization Mode* at l and returned to *Connected* during $[\tau_1, t]$. Therefore, if at time t holds $i \in G_l$, i.e. (l, i) is in Connected state at l, then it is so during the entire interval $[\tau_1, t]$. We shall show that either m_l, that had value 1 at τ_1+, does not change until time t or there is a $RELEASE$ outstanding at time t on the link (l, i) from l to i. Since m_l can change only if l receives a $RELEASE$ and the later can be received only if all $e_l(k)$ are 0 (Lemma 3.19b)), the variable m_l stays 1 at least until l receives a MSG from i that resets $e_l(i)$ to 0. From Lemma 3.20a), that MSG cannot cross paths with the MSG sent at τ_1, thus it can be sent only after t_1 and hence it is a $MSG(1)$. From Lemma 3.20c), it also causes inclusion of i into S_l. Therefore, if m_l does change before t, it does so at a time when $i \in S_l$, so l sends $RELEASE$ to i. That $RELEASE$ is still outstanding at time t, because if it arrived beforehand, it would have set $m_i \leftarrow 0$, contradicting the fact that $m_i = 1$ on the entire interval $(t_1, t]$. Hence the first part of *a*). If $m_l = 1$ at time t, then l has entered its current segment at τ_1 and has sent to i a MSG of that segment. Thus at time t_1, node i enters the same segment.

To prove the first part of *b*), we only have to show that if at time t, a node i enters the protocol, i.e. sets $m_i \leftarrow 1$, and selects l as p_i, it does not close a loop with the property that for all its links (n, p_n) holds

[5] Example when a node can enter a given segment $\boxed{????}$ twice (link comes up).

$m_n = 1$, no $RELEASE$ is outstanding from p_n to n and $n \in G_{p_n}$. But at time $t-$ holds $m_i = 0$, so that by a), there is no node j with $m_j = 1$, $p_j = i$, $j \in G_i$ and no $RELEASE$ outstanding from i to j, so no such loop can be closed. The second part of b) follows from the second part of a).

A root of a tree referred to in b) is a node i such that $m_i = 1$ and either $p_i = nil$ or (i, p_i) does not satisfy one of the conditions in b). In case (i) holds $p_i = nil$, while in cases (ii) and (iii) one of the conditions in b) are not satisfied. If i is the initiator of a segment, then since $p_i = nil$ as long as $m_i = 1$, the initiator of a segment is the root of the tree as long as $m_i = 1$. Since from Lemma 3.20b), as long as node i is in the first phase and $i \in G_{p_i}$, its preferred neighbor p_i is also in the first phase, no node can enter the second phase before all its sons in the tree do so. Therefore the root of the tree is the last in the tree to enter the second phase. In fact the root exits the protocol instead of entering the second phase (see <G5>), so that the root can exit the protocol only after all nodes in the tree enter the second phase.

The above only shows that the root of a tree does not exit the protocol before all nodes already in the tree enter the second phase. It remains to show that no node enters a tree whose root is of type (iii), namely such that (s, p_s) is in Connected state at both ends and there is a $RELEASE$ outstanding towards s on (p_s, s). Suppose that this is not the case and let node i be the first node to enter such a tree, i.e. perform $phase1()_i$, at a time t say. Let l be the node from which i receives a MSG at time t. Node l has entered the tree before the $RELEASE$ has been sent

the time when s had exited it and on the other hand, node l is at time t still in the first phase (Lemma 3.20b)). This means that s exits the segment while l is still in the first phase, contradicting the statements in the previous paragraph. in the tree is in the first phase.

To prove d), suppose a node i is the first to enter the same segment for the second time, at time t. This means that prior to t, node i had been in that segment and had exited it. From c), this means that the root of the tree had exited the segment before time t. This contradicts the other part of c), that states that no node can enter a segment after the time when the root had exited it.

To prove e), consider first a leaf i of a tree corresponding to a given segment. When it enters the segment, it sends a MSG to all neighbors k except p_i and sets $e_i(k) \leftarrow 1$. When the MSG arrives at a neighbor k, it finds $m_k = 1$, since otherwise k would select i as its preferred neighbor and i would not be a leaf. Note that k may be in the same segment as i or in a different one and that by Lemma 3.20d), at the time when the MSG arrives at k, the variable $e_k(i)$ cannot be -1. If at that time, $e_k(i) = 0$, then k sends a MSG back to i (see $phase3()$). If $e_k(i) = 1$, then k had previously sent a MSG to i, that crosses paths with the one sent by i to k. In both cases, when the MSG sent by k is received at i, it sets $e_i(k) \leftarrow 0$. When this happens for all neighbors except p_i, node i enters the second phase and sends $MSG(1)$ to its preferred neighbor. This process continues downtree until all nodes enter the second phase. By Lemma 3.20c), when a node i enters the second phase, its list S_i contains exactly the sons of i in the tree, i.e. the neighbors that had sent $MSG(1)$ to i. When the segment initiator is supposed to enter the second phase, it exits the segment instead (see $phase4()$) and sends $RELEASE$ to all neighbors in S_s, i.e. to all sons in the tree. Similarly, when a node i receives $RELEASE$, it exits the segment and sends $RELEASE$ to its sons. [6] Consequently, all nodes in the tree eventually set $m_i \leftarrow 0$ and no more messages of the segment can subsequently be sent or received. qed

?????? Partial PIF's

In various situations, it may be important to allow certain nodes to refuse to enter a PIF protocol. This may be the situation if for example, the nodes are temporarily busy in another protocol. The refusal may be temporary, so that receipt of a message at a subsequent time may cause the same node to enter the PIF. Obviously, when nodes are allowed to temporarily refuse entrance in the protocol, one cannot expect the

[6] Show that the above protocol with PI3 instead of $PIF3$ does not work.

information to reach all nodes in the network, but we shall show that in this situation

3.7 Generalized PIF (GPIF)

In this section we generalize the PIF protocol introduced in Section 3.3.2. Many protocols will prove to be special cases of the *Generalized PIF*.

In the *Generalized PIF* it is required that every node i in the network receives at least one message MSG. As in PIF, in the first stage of the *Generalized PIF*, every node i enters the protocol when it receives the first MSG; if l is the neighbor from which that message was received, i sets at that time $p_i \leftarrow l$. In this way a spanning tree rooted at s is defined by the collection $\{(i, p_i), \forall i \in V\}$. However we do not necessarily require that upon entering the protocol, every node i will send messages to all $k \in G_i - \{p_i\}$.

Before introducing the generalization for the second stage, it will be useful to rename the message sent by a node i in $phase2()_i$ of PIF to its preferred neighbor. This message will be called here $ECHO$. Also, for a node i, we introduce the notation S_i as the set of sons of i in the tree constructed in the first phase, i.e. $S_i = \{k : p_k = i\}$. Observe that node i does not know the set S_i.

Now recall that the purpose of PIF was to deliver to the initiator s confirmation that the information has reached all nodes in the network. Observe that in principle, in order to achieve this goal, it is not necessary that nodes wait to receive messages from all $G_i - \{p_i\}$ before performing $phase2()_i$. Receipt of $ECHO$ messages from all sons $k \in S_i$ would suffice. The difficulty in implementing this change is that the set S_i is not known to i. The solution in PIF was to wait for MSG or $ECHO$ from all $k \in G_i - \{p_i\}$. Since the latter set includes S_i, we are sure that when $phase2()_i$ is performed, $ECHO$ messages have indeed been received from all neighbors in S_i. In doing so we have achieved the additional property that when $phase2()_i$ is performed, there are and will be no messages traveling towards i, so that when the Protocol initiator s completes the protocol, i.e. performs $phase2()_s$, the entire network is and will be free of messages. The above properties will also be preserved in the Generalized version.

A *predicate* at node i is a boolean function on the state of node i. Following [CL85] , in a given protocol, we say that a predicate at i is *stable* after a time $t_i \leq \infty$ if it is ***false*** before t_i, becomes ***true*** at time t_i and stays ***true*** forever afterwards. For a neighbor $l \in S_i$, we define $N_i(l)$ as a predicate that is ***true*** after $ECHO$ is received from l and ***false*** beforehand. Obviously, $N_i(l)$ is a stable predicate. A predicate Y_i at i will be said to be *confirming* for a given protocol, if it satisfies the following properties:

a) **stability:** Y_i is stable at i in the protocol and becomes ***true*** at a *finite* time $t_i < \infty$.

b) **detectability:** node i can detect if and when the stable predicate (Y_i ***and*** $N_i(l), \forall l \in S_i$) becomes ***true*** , without the knowledge of S_i.

c) **quiescence:** no message travels on a link (i, k) after the later of the times when in the given protocol (Y_i ***and*** $N_i(l), \forall l \in S_i$) becomes ***true*** and (Y_k ***and*** $N_k(l), \forall l \in S_k$) becomes ***true*** .

For example, the predicate $Y_i =$(node i has received a MSG from all $k \in G_i - S_i - \{p_i\}$) is confirming for PIF. The trivial predicate $Y_i \equiv$ ***true*** is not confirming for any reasonable protocol, since it does not satisfy at least detectability.

Definition: A protocol is said to be a *Generalized PIF* on a network (V, E) if every node $i \in V$ receives at least one message MSG and if the protocol consists of two phases:

i) In the first phase, denoted by $phase1()_i$, every node i enters the protocol when it receives the first message MSG; if l is the neighbor from which this message is received, node i sets $p_i \leftarrow l$.

ii) A node i performs its part of the second phase, denoted by $phase2()_i$, when and if
(Y_i ***and*** $N_i(l), \forall l \in S_i$) becomes ***true*** , where Y_i is a confirming predicate for the protocol. At that time it sends an $ECHO$ message to p_i.

In particular, PIF is a generalized PIF with the confirming predicate Y_i = (node i has received a MSG from all $k \in G_i - S_i - \{p_i\}$).

Protocol GPIF

Messages

$MSG(info)$ - message carrying the information $info$
$ECHO$ -message serving as confirmation

Variables

G_i - set of neighbors of i
m_i - shows if node i has already entered the protocol (values 0,1);
p_i - preferred neighbor, i.e. neighbor from which MSG was received first.
$N_i(l) =$ ***true*** after i has received $ECHO$ from l, $=$ ***false*** otherwise (for all $l \in G_i$)

Initialization

if a node i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receive MSG(info) from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0) {
A3            initalize();
A4            phase1();
          }
      }
B1    receive ECHO from l ∈ G_i
B2    {   N_i(l) ← true ;
      }
C1    (Y_i and N_i(l'), ∀l' ∈ S_i) becomes true
C2    {   phase2();
      }
D1    phase1()
D2    {   m_i ← 1;
D3        p_i ← l;
D4        accept(info);
      }
E1    phase2()
E2    {   send ECHO to p_i
      }
F1    initialize()
F2    {   for (k ∈ G_i − {p_i}) N_i(k) ← false ;
      }
```

Theorem 3.22 *(Generalized PIF) Suppose that in a* Generalized PIF *Protocol node s receives START. Then:*

a) all connected nodes i will perform the event phase$1()_i$ *in finite time and exactly once; after this happens, the links* $\{(i, p_i), \forall i \in V\}$ *will form a directed tree rooted at s; in addition, for all i holds* $t(phase1()_i) > t(phase1()_{p_i})$.

b) node s and all connected nodes i will perform phase$2()_i$ *in finite time and exactly once; moreover* $t(phase1()_i) \leq t(phase2()_i) < t(phase2()_{p_i})$*; also, when node s performs phase*$2()_s$*, all* Y_i*'s hold, all nodes will have completed the algorithm, i.e. performed phase*$2()$ *and there are no messages traveling in the network.*

Proof: In the definition of the *Generalized PIF* it is assumed that every node i enters the protocol. Node i selects the neighbor from which it receives the message that causes i to enter the protocol as p_i. Since the latter can send a message only after it enters itself the protocol, it has previously performed $phase1()_{p_i}$.

This also implies that $\{(i, p_i),\ \forall i \in V\}$ is a tree rooted at s. Hence a). To prove b), let k be a leaf of the tree referred to in a), i.e. $\nexists l$ such that $p_l = k$. Since $S_k = \Phi$, the condition for $phase2()_k$ will become ***true*** when Y_k becomes ***true*** . At that time node k will send $ECHO$ to p_k. The same will be true for all leaves. When those messages arrive, a node i in the last-but-one level in the tree will have $(N_i(l), \forall l \in S_i) =$ ***true*** . Therefore, when its Y_i will become ***true*** ,it will be able to perform $phase2()_i$. The procedure will continue downtree all the way to node s. Since s performs $phase2()$ last in the network, and Y_i has the quiescence property, at time $t(phase2()_s)$ there are no messages in the network, completing the proof. qed

3.7.1 Distributed Snapshots (DS)

In a given protocol P, consider a collection of instances $\{t_i, \forall i \in V\}$ such that every message of P received by a node k from a neighbor i before t_k was sent by i before t_i. The state of node i in P at time t_i is denoted by s_i. The state $s_{(i,k)}$ of the link (i, k) corresponding to this collection of times is defined as the sequence of messages sent by i in P on link (i, k) before t_i and received by k after t_k. A Distributed Snapshot [CL85], [Gaf86] is the collection of node states s_i and link states $s_{(i,k)}$ corresponding to a collection of times t_i with the property mentioned above. Distributed Snapshots are useful in several classes of problems. For example, it can be shown [CL85] that if protocol P is in a deadlock situation during the snapshot, it is in this situation at any time later. Similarly, if the snapshot discovers termination of P. Hence snapshots are useful in discovering stable network states, like deadlock or termination. A $PI1$ protocol can be used to determine a Distributed Snapshot for an arbitrary protocol P as follows [CL85]. The Distributed Snapshot protocol, denoted by DS1, is given below.

Protocol DS1

Algorithm for node i

```
A1    receives MSG from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0) phase1();
A3        else  s_(l,i) ← sequence of messages of P received on (l, i) since t_i;
      }
B1    phase1()
B2    {   m_i ← 1;
B3        p_i ← l;
B4        t_i ← current time;
B5        s_i ← current state of i;
B6        s_(p_i,i) ← empty sequence;
B7        for (k ∈ G_i) send MSG to k;
      }
```

Theorem 3.23 *(DS1) Protocol DS1 generates a Distributed Snapshot.*

Proof: As in Theorem 3.1, let

$t(send_k(i))$ = time when k sends MSG to neighbor i
$t(rcv_k(i))$ = time when k receives MSG from neighbor i.

Then for any two neighbors i and k, holds $t_i = t(send_i(k))$ and $t_k \leq t(rcv_k(i))$. The FIFO property implies that every message of the underlying protocol P received by k from i before $t(rcv_k(i))$ was sent before $t(send_i(k))$ and therefore every message received by k from i before t_k was sent before t_i. Moreover, the state of the link (i, k) is registered in <A3> as the sequence of P messages received by k on (i, k) between t_k and $t(rcv_k(i))$. This is exactly the sequence of P messages sent by i on (i, k) before $t(send_i(k)) = t_i$ and not received by k until t_k. qed

A combination of $PI1$ and $PIF1$ can be used to allow a node s to collect the information of the Distributed Snapshot. This is Protocol $DS2$.

Protocol DS2

Algorithm for node i

```
A1    receives MSG or MSG' from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0) {
A3            initialize();
A4            phase1();
            }
A5          else {
A6            if (received MSG) e_i(l) ← e_i(l) − 1;
A7            else  s_(l,i) ← sequence of messages of P received on (l, i) since t_i;
            }
A8          if (e_i(l') = 0 ∀l' ∈ G_i − {p_i}) phase2();
      }
B1    phase1()
B2    {   m_i ← 1;
B3        p_i ← l;
B4        t_i ← current time;
B5        s_i ← current state of i;
B6        s_(p_i,i) ← empty sequence;
B7        for (k ∈ G_i − {p_i}) {
B8            send MSG to k;
B9            e_i(l) ← e_i(l) + 1;
            }
B10       send MSG' to p_i;
      }
C1    phase2()
C2    {   put (s_i, s_(k,i) ∀k ∈ G_i and all s's received in MSG's ) into MSG;
C3        send MSG to p_i;
      }
D1    initialize()
D2    {   for (k ∈ G_i) e_i(k) ← 0;
      }
```

Except for lines <C1>-<C3>, Protocol $DS2$ is the same as $DS1$ where the message sent by i to p_i in <C3> is renamed MSG'. Therefore $DS2$ generates a Distributed Snapshot. Superimposed on that, the collection of messages MSG defines a PIF, where the MSG's sent to preferred neighbors collect the states of the descendants in the tree. Consequently, when s performs $phase2()_s$ it has the Distributed Snapshot information from the entire network. We have therefore proved:

Theorem 3.24 *(DS2) Protocol DS2 defines a Distributed Snapshot. Also, node s will perform phase2()$_s$ in finite time and at that time it has the information of the Distributed Snapshot states of the entire network.*

3.7.2 The Echo Protocol

The Echo Protocol of [Cha78],[Cha82] accomplishes the same task as PIF, with twice as many messages. We bring it here for completeness and because it will be used in later protocols. The first phase is a $PI2$ protocol. When a node i receives a MSG while $m_i = 1$, it returns on the same link an $ECHO'$ message. When a node i receives $ECHO$ or $ECHO'$ messages from all neighbors, it performs $phase2()_i$ and sends an $ECHO$ message to p_i. Therefore the Echo Protocol is a Generalized PIF, with a $PI2$ as its first phase and with $Y_i = (ECHO'$ received from all $k \in G_i - S_i - \{p_i\})$. In fact it turns out that the actions in response to receipt of $ECHO$ and $ECHO'$ are the same, so we shall distinguish between them only in the explanations and the proofs. In the code, both types will be called $ECHO$.

Protocol ECHO

Messages

$MSG(info)$ - message containing the information $info$ to be distributed (in [Cha78] , this is called an *explorer* message)
$ECHO$ - echo message to MSG, as well as a message serving for confirmation
$START$ - message received from the outside world

Variables

G_i - set of neighbors of i
m_i - shows if node i has already entered the protocol (values 0,1)
p_i - neighbor from which MSG was received first
$e_i(l)$ = number of MSG's sent − number of $ECHO$'s received on link (i,l) ($\forall l \in G_i$)

Initialization

if a node i receives a MSG, then

- just before receiving the first MSG, holds $m_i = 0$.
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Note: By definition, a condition on an empty set is always true. For example, in <B3> below, if $G_i - \{p_i\} = \emptyset$, then the condition holds and i should perform $phase2()$.

Algorithm for node i

```
A1    receives MSG(info) from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0) {
A3            initialize();
A4            phase1();
A5            if (G_i = p_i) phase2();
          }
A6        else  send ECHO to l;
      }
B1    receives ECHO from l ∈ G_i
B2    {   e_i(l) ← e_i(l) − 1;
B3        if (e_i(k) = 0 ∀k ∈ G_i − {p_i}) phase2();
      }
C1    phase1()                                          /* similar to PI2 */
C2    {   m_i ← 1;
C3        p_i ← l;
C4        accept(info);
C5        for (k ∈ G_i − {p_i}){
C6            send MSG(info) to k;
C7            e_i(k) ← e_i(k) + 1;
          }
      }
D1    phase2()
D2    {   send ECHO to p_i;
      }
E1    initialize()
E2    {   for (k ∈ G_i) e_i(k) ← 0;
      }
```

In the proof we shall refer to the $ECHO$ messages sent in <A6> as $ECHO'$ and to the ones sent in <D2> as $ECHO$.

Lemma 3.25 *(ECHO) The Echo Protocol is a Generalized PIF with a PI2 as its first phase and with* $Y_i = (e_i(l') = 0, \forall l' \in G_i - S_i - \{p_i\})$.

Proof: The actions in $phase1()_i$ are identical to those of $PI2$, with the addition that $e_i(l)$ is set to 1 when MSG is sent by i to l. Messages MSG are sent by a node i only in $phase1()_i$ to all $k \in G_i - \{p_i\}$. When a MSG arrives to such a neighbor k, it either finds it with $m_k = 1$, in which case $k \in G_i - S_i - \{p_i\}$ or with $m_k = 0$, in which case $k \in S_i$. In the first case, k sends an $ECHO'$ to i in <A6> and afterwards

sends no messages to i. The receipt of this message at i sets $e_i(k) \leftarrow 0$. Since afterwards no $ECHO$ arrives at i from k and no MSG is sent by i to k, the event $e_i(k) = 0$ is stable and therefore Y_i is stable. After a node $k \in S_i$ sends $ECHO$ to i in $phase2()_k$, it does nothing and that $ECHO$ sets $N_i(k) \leftarrow$ ***true*** when it arrives. Therefore, for $k \in S_i$, after $e_i(k)$ is set to 0, no message arrives to i from k. Since after $t(phase1()_i)$ no message arrives at i from p_i and after the time when $e_i(l) \leftarrow 0$ for any $l \in G_i - S_i - \{p_i\}$, no message arrives at i from l, Y_i satisfies the quiescence property. Obviously it satisfies the detectability property, since $(Y_i$ ***and*** $N_i(l), \forall l \in S_i) \equiv (e_i(l) = 0, \forall l \in G_i - \{p_i\})$. qed

This completes the proof of the Lemma and therefore of the following Theorem.

Theorem 3.26 *(ECHO) Suppose node s receives START. Then:*

a) all connected nodes i will perform the event phase$1()_i$ *in finite time and exactly once; after this happens, the links* $\{(i, p_i)\ \forall i \in V\}$ *will form a directed tree rooted at s; in addition, for all i holds* $t(phase1()_{p_i}) > t(phase1()_i)$*. Moreover, the propagation of information is the fastest possible.*

Note: *some nodes may perform phase*$2()$ *before all nodes have performed phase*$1()$ *)*

b) node s and all connected nodes i will perform phase$2()_i$ *in finite time and exactly once; moreover* $t(phase1()_i) \leq t(phase2()_i) < t(phase2()_{p_i})$ *also, when node s performs phase*$2()_s$ *, all connected nodes will have completed the algorithm, i.e. performed phase*$2()$ *and there are no messages traveling in the network.*

c) on all links $\neq (i, p_i)$*, exactly one MSG and one ECHO travels on each link in each direction; on all links of the type* (i, p_i)*, exactly one MSG travels from* p_i *to i and one ECHO travels in the other direction*[7]*.*

3.7.3 Termination Detection for Diffusing Computations (TDDC)

The problem is the following [DS80]. A node s starts the protocol by sending some messages MSG to some of its neighbors. A node i enters the protocol when it receives the first message MSG. Afterwards it may perform some computations that may require it to send some messages MSG, a finite number of them, to neighbors. Eventually, a node is in a situation when it sends no more messages MSG of the computation. We say then that the node has terminated the computation. A node i that receives a MSG but has no computation to perform and hence sends no messages MSG is considered as if it had terminated the computation. The problem is to superimpose on the above a signalling scheme that will allow the source s to know that all nodes have indeed terminated their computations and will receive no more messages.

The signalling protocol proposed in [DS80] is the following: whenever a node i receives the first MSG, from a neighbor l say, it denotes $p_i \leftarrow l$ and enters the protocol. Subsequently, whenever it receives some MSG, from a neighbor k say, it returns to k a signalling message $ECHO$. It does that whether it is still performing the computation or it had already terminated it. Recall that if it has no computation to perform, node i is considered to have terminated the computation, so that it returns an $ECHO$ even in response to the first MSG. After having terminated the computation, node i waits until it receives an $ECHO$ for every MSG it has sent and then it sends an $ECHO$ message to p_i. The latter is the response to the first received MSG. When the initiator s terminates its computation and receives an $ECHO$ for every MSG sent, it knows that all nodes that have started the computation have terminated it and there are no messages in the network. Observe that this protocol does not guarantee that all nodes that have a computation to perform indeed enter the protocol. The solution to this is given in Problem 3.7.2 and in the second version of TDDC, presented in the next section.

[7] Communication complexity ???
Time complexity ???

Protocol TDDC1

Messages

MSG - message of the computations and of wake-up
$ECHO$ - signalling message

Variables

G_i - set of neighbors of i
m_i - shows if node i has already entered the protocol
p_i - neighbor from which MSG was received first.
$e_i(l)$ - number of MSG's sent $-$ number of $ECHO$'s received on link (i,l) ($\forall l \in G_i$)

Initialization

if a node i receives a MSG, then

- just before it receives the first MSG, holds $m_i = 0$ and $e_i(k) = 0$ for all $k \in G_i$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receive MSG from l ∈ G_i ∪ {nil}
A2    {   if (computation completed) send ECHO to l;
A3        else if (m_i = 0){
A4            phase1();
              }
A5        else send ECHO to l;
      }
B1    computation requires sending message to l
B2    {   send MSG to l;
B3        e_i(l) ← e_i(l) + 1;
      }
C1    receive ECHO from l ∈ G_i;
C2    {   e_i(l) ← e_i(l) − 1;
      }
D1    ( computation completed and e_i(l') = 0 ∀l' ∈ G_i ) becomes true
D2    {   phase2();
      }
E1    phase1()
E2    {   m_i ← 1;
E3        p_i ← l;
      }
F1    phase2()
F2    {   send ECHO to p_i;
      }
```

As said earlier, the $ECHO$ message sent in <A2> or <A5> will be referred to as $ECHO'$. Let V' be the set of nodes that enter the protocol, i.e. perform $phase1()$ and let E' be the set of links of (V,E) that connect those nodes. For a node in V', we denote by G'_i the set of neighbors in (V',E') and, as usual, by G_i the set of neighbors in the original network.

Lemma 3.27 *Protocol TDDC1 is a Generalized PIF over the network* (V',E') *with* $Y_i =$ (*computation completed* ***and*** $e_i(l) = 0, \forall l \in G_i$).

Proof: Node i increments $e_i(l)$ when it sends a MSG to l and decrements it when it receives an $ECHO$ from l. Since node l sends at most one $ECHO$, on the corresponding link, for every received MSG, either immediately (in <A2>) or later (in $phase2()$), and since after the computation is completed at i, node i sends no MSG's, the predicate (computation completed ***and*** $e_i(l) = 0$) is stable for any $l \in G_i$. Hence Y_i is stable.

Now observe that for any $l \in S_i$, if $e_i(l) = 0$ holds, then $N_i(l) =$ ***true*** also holds, hence (Y_i ***and*** $N_i(l), \forall l \in S_i) \equiv Y_i$. Therefore detectability holds for Y_i. Finally, for a link (i,k), if both Y_i and Y_k hold, then there are no messages on the link (i,k), so quiescence holds. qed

This completes the proof of the Lemma and therefore of the following Theorem.

Theorem 3.28 *(TDDC1) Suppose node s receives START. Then node s and all connected nodes i that enter the protocol will perform phase*$2()_i$ *in finite time and exactly once; moreover* $t(phase1()_i) \leq t(phase2()_i) < t(phase2()_{p_i})$; *also, when node s performs phase*$2()_s$*, all nodes that have entered the protocol will have completed the algorithm, i.e. performed phase*$2()$ *and there are no messages traveling in the network.*

3.7.4 Termination Detection for Diffusing Computations - Version 2

Version 1 is not appropriate if there are one or more nodes i that need to perform a computation, but there is no neighbor of i whose computation requires it to send a message to i. This is because such a node i will never enter the protocol. The present version will work for this case. As seen below, the present version is also more economical in terms of messages in most situations.

In order to allow all nodes to enter the protocol, the first phase in this version will be a $PI2$ protocol with wake-up messages, which we call MSG. After entering the protocol, a node starts its computation and sends computation messages, called $COMP$, to neighbors as required. No reply messages to $COMP$ are needed in this version. When it completes its computation, a node i sends a completion message, called $ECHO$, to each of its neighbors, except to p_i. Subsequently, if it receives some (late) message MSG, it responds with an $ECHO$. Finally, when it had received a $ECHO$ message from all neighbors and had completed the computation, it sends a $ECHO$ message to p_i. Receipt of an $ECHO$ message from all neighbors at s indicates completion of the protocol in the entire network.

The present version is more economical in terms of messages if there are many computation messages to be sent, since in the previous version, an $ECHO$ was sent for each of these messages.

Note that the present version is exactly the *Echo* Protocol, except that the transmission of the $ECHO$ messages is postponed until completion of the computation. Hence,

Lemma 3.29 *(TDDC2) The TDDC2 Protocol is a Generalized PIF with* $Y_i =$ *(computation completed* ***and*** $e_i(l') = 0,\ \forall l' \in G_i - S_i - \{p_i\})$ *and with the first phase corresponding to a PI2 protocol.*

3.7.5 Synchronizers

In a synchronous protocol, messages are allowed to be sent only at integer times and the delay of each message is at most one time unit. In an asynchronous network, for each node i, consider a sequence of instances $\{t_i(0), t_i(1), t_i(2), \dots\}$. These sequences of instances define a synchronizer [Awe85a] for a given execution of a protocol P if:

a) messages of P can be sent by i only at times $\{t_i(n), n \geq 0\}$.

b) messages of P sent by i to neighbor k at time $t_i(n)$ arrive at k before $t_k(n+1)$.

c) messages of P sent by i to neighbor k at time $t_i(n)$ arrive at k after $t_k(n)$.

Note that the definition of a synchronizer in [Awe85a] omits requirement c). The latter is however required if the asynchronous protocol is to behave as a synchronous one. The simplest synchronizer, called synchronizer α in [Awe85a] works as follows. The initial times $t_i(0)$ can be set by nodes by a $PI1$ or $PI2$ protocol. Whenever a node i enters the PI, it sets the tick $t_i(0)$. In general, when a node i completes sending to k the messages corresponding to tick $t_i(n)$, it sends to k a message $SYNCH(n)$. The time INCORRECT??,SAFE,ACK, [LT87], [SS91] $t_i(n+1)$ is defined as the time when i has received messages $SYNCH(n)$ from all its neighbors. With this protocol, conditions a) and b) above are satisfied. If

Protocol P is such that it ensures that no P message sent by a node i at tick $t_i(n)$ arrives at k before $t_k(n)$ for all (i,k), then the instances defined above define a synchronizer for P.

Another synchronizer designed in [Awe85a], called synchronizer β, uses asymptotically (in the size of the network) less messages, but longer time. This synchronizer needs an initialization phase, where a leader s is selected in the network and a spanning tree, rooted at s is built. In this protocol it is required that every node i learns that all messages of Protocol P sent to neighbors in round $(n-1)$ have arrived, before being able to participate in the protocol for determining $t_k(n), \forall k$. In order to save messages, we exploit the **Confirm** property of the DLC that states that every considered acknowledged data packet has been delivered to the other side. Recall that messages of Protocol P (as well as any messages of levels higher than the DLC level) are considered as data packets by the DLC. If, when the DLC considers the packet containing a P message acknowledged, it also informs the local synchronizer algorithm, this information can be used by the latter to know that the message has indeed arrived to the other side. Eventually a node i will find out that all P messages sent at tick $t_i(n-1)$ have been delivered. At that time node i is said to be $safe$ with respect to tick $(n-1)$.

The rest now is like a PIF on the tree, except that phase (ii) of the PIF is performed first. When leaves learn that they are $safe$, they send to their father in the tree a $SYNCH(n-1)$ message. In general, when a node learns that it is safe and has received $SYNCH(n-1)$ messages from all sons, it sends a $SYNCH(n-1)$ message to its father. When the leader s receives $SYNCH(n-1)$ from all its sons, it knows that all nodes are $safe$, and triggers broadcast of a $SYNCH(n)$ message over the tree. The time $t_i(n)$ is defined as the time when i receives the $SYNCH(n)$ message. As shown in the Problem 3.7.8, messages sent by i at tick $t_i(n)$ can arrive at a neighbor k between $t_k(n-1)$ and $t_k(n+1)$. Therefore, in order to have a synchronizer, we have to require here also that no P message sent by a node i at tick $t_i(n)$ arrives at k before $t_k(n)$ for all (i,k). The communication load of this protocol is $0(\mid V \mid)$ per tick, plus the communication load of the spanning tree construction and of the leader election, as opposed to the simple synchronizer presented above, whose communication load is $O(\mid E \mid)$ per tick.

Problems

Problem 3.7.1 Give examples of various predicates $\{Y_i\}$ and check whether they are confirming.

Problem 3.7.2 Protocol $TDDC1$ does not ensure that all nodes that have a computation to perform indeed enter the protocol. Indicate a protocol that solves this problem, based on $TDDC1$ and other protocols in this Chapter.

Problem 3.7.3 Write the code for TDDC2.

Problem 3.7.4 Show that with the $SYNCH$ messages sent as in synchronizer α, P messages sent at tick $t_i(n)$ can arrive at a neighbor k only between $t_k(n-1)$ and $t_k(n+1)$.

Problem 3.7.5 Various protocols in this chapter, and synchronizers for them.

Problem 3.7.6 Suggestion: use two types of SYNCH(n) messages, one before P messages of $t_i(n)$ and one after. Does this work???

Problem 3.7.7 Which of the protocols ECHO,PI, PIF, DS, TDDC, can be made more efficient by using the Confirm property of the DLC. Write the corresponding codes and prove their correctness.

Problem 3.7.8 Show that with protocol β , messages sent by i at tick $t_i(n)$ can arrive at a neighbor k between $t_k(n-1)$ and $t_k(n+1)$.

Problem 3.7.9 Analyse precisely communication and time cost of synchronizers α and β (including the initialization cost for β).

Problem 3.7.10 Which of the following will not work properly when the FIFO property of the data-link does not hold : PI, PIF, DS, ECHO, TDDC?

Problem 3.7.11 a)Is the DS protocol a generalized PIF, prove or disprove.
b) Is the α synchronizer a generalized PIF, prove or disprove.

Problem 3.7.12 Is the following protocol a generalized PIF?

Protocol birds

Messages

MSG,
MSG',
START

Variables

m_i **:** contains 1 when in the MSG protocol.
m'_i **:** contains 1 when in the MSG' protocol.
p_i **:** like in PIF.
$N_i(l)$ **:** like in PIF, addressing MSG messages.
$N'_i(l)$ **:** like in PIF, addressing MSG' messages.

Initialization

$m_i = 0$
$m'_i = 0$
p_i = NIL
$(N_i(l), \forall l \in G_i) = 0$
$(N'_i(l), \forall l \in G_i) = 0$

Algorithm for node s

A1 When receiving $START$
A2 $m_s \leftarrow 1$; $m'_s \leftarrow 1$
A3 $\forall l' \in G_s$ ***do***
A4 send MSG to l'
A5 send MSG' to l'
B1 When receiving MSG from neighbor l
B2 $N_s(l) \leftarrow 1$
B3 ***if*** $((N_s(l') = 1) \wedge (N'_s(l') = 1)), \forall l' \in G_s$ ***then***
B4 terminate.
C1 when receiving MSG' from neighbor l
C2 $N'_s(l) \leftarrow 1$
C3 ***if*** $((N_s(l') = 1) \wedge (N'_s(l') = 1)), \forall l' \in G_s$ ***then***
C4 terminate.

Algorithm for node $i \neq s$

D1 When receiving MSG from neighbor l
D2 ***if*** $m_i = 0$ ***then***
D3 $m_i \leftarrow 1$
D4 $p_i \leftarrow l$
D5 $\forall l' \in G_i - \{p_i\}$ ***do***
D6 send MSG to l'
D7 ***else***
D8 $N_i(l) \leftarrow 1$
D9 ***if*** $((N_i(l') = 1), \forall l' \in G_i - \{p_i\}) \wedge ((N'_i(l') = 1), \forall l' \in G_i)$ ***then***
D10 send MSG to p_i
E1 when receiving MSG' from neighbor l
E2 ***if*** $m'_i = 0$ ***then***
E3 $m'_i \leftarrow 1$
E4 $\forall l' \in G_i$ ***do***
E5 send MSG' to l'
E6 $N'_i(l) \leftarrow 1$
E7 ***if*** $((N_i(l') = 1), \forall l' \in G_i - \{p_i\}) \wedge ((N'_i(l') = 1), \forall l' \in G_i)$ ***then***
E8 send MSG to p_i

Problem 3.7.13 The protocol of Problem 3.7.12 has been changed as follows: Two nodes in the network — s and s' receive asynchronously each a START message. The algorithm of all the nodes except s and s' has not been changed.

Protocol birds1

Algorithm for node s

A1 When receiving START
A2 $m_s \leftarrow 1$
A3 $\forall l' \in G_s$ ***do***
A4 send MSG to l'
B1 When receiving MSG from neighbor l
B2 $N_s(l) \leftarrow 1$
B3 ***if*** $((N_s(l') = 1) \wedge (N'_s(l') = 1)), \forall l' \in G_s$ ***then***
B4 terminate.
C1 when receiving MSG' from neighbor l
C2 ***if*** $m'_s = 0$ ***then***
C3 $m'_s \leftarrow 1$
C4 $\forall l' \in G_s$ ***do***
C5 send MSG' to l'
C6 $N'_s(l) \leftarrow 1$
C7 ***if*** $((N_s(l') = 1) \wedge (N'_s(l') = 1)), \forall l' \in G_s$ ***then***
C8 terminate.

Algorithm for node s'

D1 When receiving START
D2 $m'_{s'} \leftarrow 1$
D3 $\forall l' \in G_{s'}$ ***do***
D4 send MSG' to l'
E1 When receiving MSG from neighbor l
E2 ***if*** $m_{s'} = 0$ ***then***
E3 $m_{s'} \leftarrow 1$
E4 $p_{s'} \leftarrow 1$
E5 $\forall l' \in G_{s'} - \{p_{s'}\}$ ***do***
E6 send MSG to l'
E7 ***else***
E8 $N_{s'}(l) \leftarrow 1$
E9 ***if*** $((N_{s'}(l') = 1), \forall l' \in G_{s'} - \{p_{s'}\}) \wedge ((N'_{s'}(l') = 1), \forall l' \in G_{s'})$ ***then***
E10 send MSG to $p_{s'}$
F1 when receiving MSG' from neighbor l
F2 $N'_{s'}(l) \leftarrow 1$
F3 ***if*** $((N_{s'}(l') = 1), \forall l' \in G_{s'} - \{p_{s'}\}) \wedge ((N'_{s'}(l') = 1), \forall l' \in G_{s'})$ ***then***
F4 send MSG to p_i

Is *this* a generalized PIF?

Chapter 4

CONNECTIVITY TEST PROTOCOLS

The purpose of this class of DNP's [Seg83] is to allow each node to learn what nodes are connected to it, i.e. nodes that are in V.

4.1 Protocol CT1

The idea here is to use protocol $PI1$ repeatedly, first to inform all nodes that the protocol is in progress and then for each node to propagate its own identity. Every node (or several nodes) can start the protocol by receiving $START$. A node enters the protocol whenever it receives either $START$ or the first control message from any of its neighbors. The first action taken by a node when entering the protocol is to send a control message containing its own identity to all its neighbors, thereby starting $PI1^i$, i.e. a $PI1$ protocol containing its own identity. In addition, whenever a node i receives the first control message with the identity of some other node j, it marks j as connected and sends a message MSG^j with the identity of j to all neighbors. All further messages with the identity of j are discarded with no action taken.

As in previous sections, a variable with subscript i will indicate that the variable is located at node i. Here and in all subsequent sections, a superscript j in variables, messages,protocol names, etc. will always indicate entities related to some distant node j. For example, $PIF1^j$ will denote a $PIF1$ whose initiator is j whose MSG's are MSG^j and for example, the preferred neighbor of node i in this $PIF1$ will be denoted by p_i^j. The $START$ message received at node i will be denoted by MSG^i received from nil. The latter can be received only if $m_i = 0$. In previous sections we have suppressed the superscript since we considered only one basic protocol at a time, so that no confusion has arisen.

Protocol CT1

Messages

MSG^j - control messages with identity j

Variables

G_i - set of neighbors of node i
m_i - shows whether i has already entered the algorithm (values 0,1)
c_i^j - designates knowledge at i about connectivity to j (values 0,1), for all $j \in \overline{V}$

Initialization

if a node receives at least one MSG,

- just before the time it receives the first one holds $m_i = 0$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receives MSG^j from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0){
A3            m_i ← 1;                                  /* enter protocol */
A4            initialize();
A5            phase1^i();
          }
A6        if (c_i^j = 0) phase1^j();
      }
B1    phase1^j()
B2    {   c_i^j ← 1;
B3        for (k ∈ G_i) send MSG^j to k;
      }
C1    initialize()
C2    {   for (k ∈ V̄) c_i^k ← 0;
      }
```

Theorem 4.1 *(CT1) Suppose that at least one node in V receives $START$. Then for every $i \in V$, the variables c_i^j will become 1 in finite time for all $j \in V$ and will remain 0 forever for all $j \notin V$.*

Proof: The event $m_k \leftarrow 1$ propagates as in $MPI1$ and hence will happen in finite time at all nodes $k \in V$. For a given $j \in V$, after m_j becomes 1, the event $phase1()^j$ propagates again as in $PI1$ and hence will happen in finite time at every node $i \in V$. The fact that c_i^j remains 0 forever for $j \notin V$ is obvious. qed

Theorem 4.2 *With protocol $CT1$, there is no way for node j to know for sure what nodes are disconnected from it or in other words, there is no way for j to know when the algorithm is completed, except for the case when $V \equiv \overline{V}$.*

Proof: Consider first the case of three nodes 1,2,3 with links (1,2) and (2,3). If 1 starts the protocol, it will receive the same sequence of messages whether (2,3) is working or not, except that if it does, it will later receive the identity of 3. Now, after receiving the identity of node 2 and before receiving the identity of 3, there is no way for node 1 to positively know whether it has already completed the protocol or not, i.e. whether new identities are supposed to still arrive. It is easy to see that similar situations may arise for any other topology. qed

Communication cost: The number of bits transmitted on each link in each direction is $\mid V \mid log_2 \mid \overline{V} \mid$. This is because every identity travels exactly once on each link in each direction, there are $\mid V \mid$ identities and it takes $log_2 \mid \overline{V} \mid$ bits to describe an identity. The total number of bits in the network is $2 \mid E \mid\mid V \mid log_2 \mid \overline{V} \mid$, where E is the number of bidirectional links.

The rest of this chapter is devoted to the presentation of several protocols that solve the problem raised in Theorem 4.2, namely allow nodes to positively know that the protocol has indeed been completed. We shall say then that the protocol has the *termination property.* Protocol $CT2$ achieves the property by employing the basic protocol PIF, while the others use a different idea.

Problems

Problem 4.1.1 Let N be a ring consisting of nodes s, a, b, c. The nodes of N perform CT1. Node s receives START at t=0. The delay on each line is 1.
a) Indicate the values of the various variables as a function of time at each node.
b) Repeat the above question for the case when (s,a) and (b,c) do not work.

4.2 Protocol CT2

The protocol is started and entered by nodes in the same way as in CT1, except that when it enters the protocol, every node j triggers a $PIF1^j$ with its identity j instead of a $PI1^j$ as in CT1. It is shown in Theorem 4.3 that at the time it completes its own $PIF1$, a node j has complete knowledge about the identities of nodes in V and those that are not in V. Consequently, the termination property holds for Protocol CT2.

Protocol CT2

Messages

MSG^j - control messages with identity j sent by i

Variables

G_i - set of neighbors of node i
m_i - indicates whether i has entered the protocol (values 0,1)
c_i^j - designates knowledge at i about connectivity to j (values 0,1) for all $j \in \overline{V}$
p_i^j - neighbor from which MSG^j has been received first, for all $j \neq i$.
$e_i^j(l)$ - number of MSG^j sent to l - number of MSG^j received from l, for all $l \in G_i$

Initialization

if a node receives at least one MSG, then

- just before the time it receives the first one, holds $m_i = 0$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receives MSG^j from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0){
A3            m_i ← 1;                                  /* enter protocol */
A4            initialize();
A5            phase1^i();
          }
A6        if (c_i^j = 0) phase1^j();
A7        e_i^j(l) ← e_i^j(l) − 1
A8        if (e_i^j(k) = 0 ∀k ∈ G_i − {p_i^j}) phase2^j();
      }
B1    phase1^j()                                        /* same as PIF1 */
B2    {   c_i^j ← 1;
B3        if (i ≠ j) p_i^j ← l else p_i^j ← nil;
B4        for (k ∈ G_i − {p_i^j}){
B5            send MSG^j to k;
B6            e_i^j(k) ← e_i^j(k) + 1;
          }
      }
C1    phase2^j()                                        /* same as PIF1 */
C2    {   send MSG^j to p_i^j
C3        e_i^j(p_i^j) ← e_i^j(p_i^j) + 1;
      }
D1    initialize()
D2    {   for (j ∈ V̄){
D3            c_i^j ← 0;
D4            for (k ∈ G_i) e_i^j(k) ← 0;
          }
      }
```

In order to analyze the protocol, we shall need the following notations:

$< \bullet >_i^j$ - the event of node i performing line $< \bullet >^j$ of its algorithm regarding node j (i.e. reacting to

receipt of MSG^j) ; whenever the corresponding line contains an ***if*** condition, the notation refers only to the cases when the condition holds.

$phase*()_i^j$ - the event of node i performing the actions corresponding to $phase*()^j$

$t(*)$ - time when event $*$ happens.

The properties of the algorithm are given in the following:

Theorem 4.3 *(CT2) Suppose that at least one node in V receives $START$. Then:*
a) at every node $i \in V$, the variables c_i^j will become 1 in finite time for all $j \in V$ and will remain 0 forever for all $j \notin V$.

b) every $i \in V$ will perform $phase2()_i^i$ in finite time and exactly once, and when this happens, it will have $c_i^j = 1$ for all $j \in V$ and $c_i^j = 0$ for all $j \notin V$. In other words, it will positively know at that time what nodes are connected, resolving the problem raised in Theorem 4.2.

Proof: The event $m_k \leftarrow 1$ propagates as in $MPI1$ and hence will happen in finite time at all nodes $k \in V$. For a given $j \in V$, after m_j becomes 1, the event $phase1()^j$ propagates as in $PI2$ and hence will happen in finite time at every node $i \in V$. The fact that c_i^j remains 0 forever for $j \notin V$ is obvious, completing the proof of *a*).

To prove *b*), observe that for a given node $j \in V$, event $phase2()^j$ propagates in the same way as $phase2()$ in PIF1 and hence $phase2()_j^j$ will happen in finite time and exactly once. It remains to show that $phase2()_j^j$ is indeed the signal indicating that node j knows all $k \in V$, namely to show that $t(phase2()_j^j) > t(phase1()_j^k)$ for all nodes $k, j \in V$. However this follows from the *no-overtake* property of PI2 (Theorem 3.2d)), since for a given k, the event $phase1()_j^k$ propagates according to $PI2^k$, started by k when it received the first MSG and $phase2()_j^j$ can be considered as the end of a *string of messages MSG^j* started by k at some time after it has entered this $PI2$ (cf. Problem 3.3.5).

Communication Cost: Observe that by Theorem 4.3, the communication requirements of $CT2$ are the same as those of $CT1$, namely $\mid V \mid log_2 \mid \overline{V} \mid$ bits per link in each direction. Observe however, that the storage and processing requirements, as well as the required execution time[1]are larger than in $CT1$[2].

Protocols $CT3$-$CT5$ use a different idea for achieving the termination property, $CT3$ is quite wasteful in terms of communication requirements, but it is convenient in order to illustrate the idea and to be used as a basis for developing the more efficient versions $CT4$ and $CT5$. In addition, it can be used for different purposes, like learning the network topology.

Problems

Problem 4.2.1 Show that in CT2, a node can receive messages after it has completed its own PIF, i.e. after it has performed $phase2()_i^i$.

Problem 4.2.2 Augment the CT2 protocol to give nodes a positive indication that no more messages will arrive.

[1]Time complexity??
[2]Roskind

4.3 Protocol CT3

Suppose we use protocol $CT1$, except that for each node j, we propagate in PI^j not only the identity of the node, but also of its neighbors. In other words MSG^j of $CT1$ will now carry the identity of j as well as of all its neighbors, i.e. will have the format $MSG^j(\Lambda)$, where $\Lambda = G_j$, i.e. Λ contains the identities of all neighbors of j. The termination property is achieved using the fact that, if a node k receives a MSG that has originated at j, it will eventually receive MSG's that have originated at all neighbors of j. The termination signal will occur when node k will have heard from all these nodes.

Protocol CT3

Messages

$MSG^j(\Lambda)$ - control messages with identity j and $\Lambda = G_j$

Variables

G_i - set of neighbors of node i
m_i - shows whether i has already entered the algorithm (values 0,1)
c_i^j - designates knowledge at i about connectivity to j (values 0,1,2), for all $j \in \overline{V}$
$= 0$ when i knows nothing about j
$= 1$ while i knows j only as a neighbor of another node
$= 2$ while i knows j directly (i.e. $MSG^j(\Lambda)$ has been received)

Initialization

if a node receives at least one MSG, then

- just before the time it receives the first one holds $m_i = 0$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receives MSG^j(Λ) from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0){
A3            m_i ← 1;                                   /* enter protocol */
A4            initialize();
A5            phase1^i(G_i);
          }
A6        if (c_i^j ≠ 2) phase1^j(Λ);
A7        if (c_i^j = 0 or 2, ∀j ∈ V̄) connectivity known;
      }
B1    phase1^j(Λ)
B2    {   c_i^j ← 2;
B3        for (k ∈ Λ) c_i^k ← max(c_i^k, 1);
B4        for (k ∈ G_i) send MSG^j(Λ) to k;
      }
C1    initialize()
C2    {   for (j' ∈ V̄) c_i^j' ← 0;
      }
```

Theorem 4.4 *(CT3) Suppose that at least one node in V receives $START$. Then:*

a) for every $i \in V$, the variables c_i^j will become 2 in finite time for all $j \in V$ and will remain 0 forever for all $j \notin V$.

b) every $i \in V$ will perform $<A7>_i$ in finite time, and when this happens for the first time, it will have $c_i^j = 2$ for all $j \in V$ and $c_i^j = 0$ for all $j \notin V$. In other words, it will positively know at that time what nodes are connected, resolving the problem raised in Theorem 4.2.

Proof: The event $m_k \leftarrow 0$ propagates as in $MPI1$ and hence will happen in finite time at all nodes $k \in V$. For a given $j \in V$, after m_j becomes 1, the event $c_k^j \leftarrow 2$ propagates again as in $PI1$ and hence will happen

in finite time at every node $i \in V$. The fact that c_i^j remains 0 forever for $j \notin V$ is obvious. Hence a).

For each $j \in V$, propagation of $MSG^j(\Lambda)$ happens as in protocol $PI1$ except that it is triggered by <A6> instead of by $START$ and therefore, every node in V sends exactly one $MSG^j(\Lambda)$ on each of its outgoing links. In order to prove b), consider the situation at the time when all those messages have arrived. Then from <B2> a node i will have $c_i^j = 2$ for all nodes $j \in V$. Since for all $j \notin V$ it has at all times $c_i^j = 0$, <A7> may hold even before the arrival of all messages considered above and it remains to prove that in this case as well, holds $c_i^j = 2$ for all $j \in V$. Suppose $c_i^j = 0$ for some $j \in V$. Let V' denote the subset of V containing nodes k with $c_i^k = 2$ and V'' the subset of V containing nodes k with $c_i^k = 0$. Since $i \in V'$ and $j \in V''$, neither set is empty. Since V is connected, there must exist two neighbors l', l'' such that $l' \in V'$ and $l'' \in V''$. However, since $c_i^{l'} = 2$, holds from <B3> that $c_i^{l''} \geq 1$, contradicting the fact that $l'' \in V''$. qed

Communication Cost: On each link in each direction we send $\mid V \mid (D+1) log_2(\mid \overline{V} \mid)$ bits, where D is the average degree of the nodes (average number of neighbors). Clearly $D = 2 \mid E \mid / \mid V \mid$ and hence the communication cost is $(2 \mid E \mid + \mid V \mid) log_2 \mid \overline{V} \mid$ bits per link in each direction[3].

A similar example to the one given in Sec. 3.3 shows that $CT3$ does not work if the initialization requirements do not hold. Let (V, E) be the network of Fig. 4.1, suppose that s receives $START$ and $m_b = 1$ when the first MSG of the CT3 protocol arrives at b from a. This can happen if for example, at the time when $START$ was given to s, holds $m_i = 0$ for all nodes i, but there is a $MSG^a(s, b, x)$ on the link from c to b, where $x \in \overline{V}$, but is disconnected now from V. Suppose that this MSG arrives at b at about the same time when the MSG^s of CT3 sent by s arrives at a. At that time b will set $m_b = 1, c_b^a = 2, c_b^x = 1$ and will expect, among other messages a $MSG^x(\Lambda)$. The latter will never arrive however. Hence b will never complete the protocol, i.e. will never perform <A7>. If some old $MSG^x(\Lambda)$ is still somewhere in the network, node b may complete the protocol, but with the wrong connectivity information, since it will have $c_b^x = 2$.

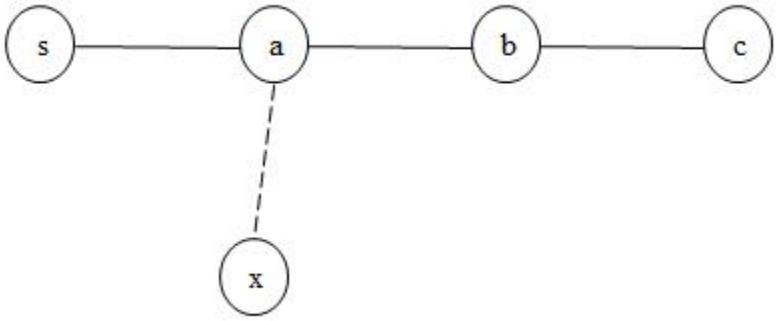


Figure 4.1: Counterexample for Initial Conditions

Although as seen in Chapter 5, protocol $CT3$ is useful for other requirements, for connectivity test it is too waistful in communication and its performance can be considerably improved. One way is to use the position of a variable in a vector to indicate the identity of a node, instead of explicitly mentioning it. This idea was used in a protocol by [Fin79] and we present here an improved version of that protocol.

[3] time, computation,???

4.4 Protocol CT4

Protocol CT4

Messages

C_i $= \{c_i^1, c_i^2, ..., c_i^{|\overline{V}|}\}$, message sent by i
C = message received, we denote its contents by $\{c^1, c^2, ..., c^{|\overline{V}|}\}$

Variables

G_i - set of neighbors of node i
m_i - shows whether i has already entered the algorithm (values 0,1)
c_i^j - designates knowledge at i about connectivity to j (values 0,1,2), for all $j \in \overline{V}$
= 0 when i knows nothing about j
= 1 while i knows j only as a neighbor of another node
= 2 while i knows j directly (i.e. $MSG^j(\Lambda)$ has been received)

Initialization

if a node receives at least one message C, then

- just before the time it receives the first one holds $m_i = 0$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receives C from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0){
A3            m_i ← 1;                                   /* enter protocol */
A4            initialize();
          }
A5        if (∃j ∈ V̄ | c^j = 2 > c_i^j) update();
A6        if (c_i^j = 0 or 2, ∀j ∈ V̄) connectivity known;
      }
B1    update()
B2    {   for (j ∈ V̄) c_i^j ← max(c_i^j, c^j);
B3        for (k ∈ G_i) send C_i to k;
      }
C1    initialize()
C2    {   for (j ∈ V̄) c_i^j ← 0;
C3        c_i^i ← 2;
C4        for (k ∈ G_i) c_i^k ← 1;
      }
```

Note that Finn's protocol [Fin79] requires a node to send messages every time its table is updated, while here messages are sent only when relevant new information is received (see <A5>). In this sense, the present version is more efficient than [Fin79]. The properties of the protocol are summarized in

Theorem 4.5 *(CT4) Suppose at least one node in V receives $START$. Then*
a) no more than $|V|$ messages C traverse each link in each direction

b) every node i will perform <A6> in finite time and when this happens, it will have $c_i^j = 2$ for all connected nodes $j \in V$ and $c_i^j = 0$ for all nodes $j \notin V$.

Proof: The event $m_i \leftarrow 1$ propagates as in $MPI1$. From the algorithm it is clear that c_i^j can only increase and that a message can be sent by i only when some c_i^j is increased from 0 or 1 to 2 and this can happen only once for each j. Hence *a*). Finally *b*) follows in the same way as in Theorem 4.4.

Communication cost: Each message contains $2 \mid \overline{V} \mid$ bits and hence at most $2 \mid \overline{V} \mid\mid V \mid$ bits will travel on each link in each direction.

Protocol CT3 can be improved in another way, resulting in the more efficient protocol CT5[4].

[4]Roskind????

4.5 Protocol CT5

Consider protocol CT3 with the following variation: Upon receiving $MSG^j(\Lambda)$ in <A1>, a node i consults its table containing $\{c_i^k\}$. If $c_i^j = 2$, the MSG is discarded, since such a MSG has been previously received and forwarded to all neighbors; this part is the same as in $CT3$. If $c_i^j < 2$, then $c_i^j \leftarrow 2$ and the MSG is sent to all neighbors, but now, before sending $MSG^j(\Lambda)$, the following pruning operation is performed: For all $k \in \Lambda$, if $c_i^k \geq 1$, then k is deleted from Λ; otherwise k is not deleted from Λ and the variable c_i^k takes value 1. Then $MSG^j(\Lambda)$ is sent to all neighbors.

Node k can indeed be deleted when $c_i^k \geq 1$ because in this case k has been sent before by i to its neighbors, either as a neighbor of some node, in which case, $c_i^k = 1$ or in MSG^k, in which case $c_i^k = 2$. One way or the other, there is no need to send k again. All properties of $CT3$ hold here as well, but the pruning operation assures that the identity of each node k travels no more than twice on each link in each direction, once as a neighbor of some node and once in MSG^k. Hence the communication cost is bounded by $2 \mid V \mid log_2 \mid \overline{V} \mid$ bits per link in each direction.

Problems

Problem 4.5.1 Give an example of a CT5 run, where a node sends $MSG^j(\Phi)$ to a neighbor.

Problem 4.5.2 Suppose that in CT5, nodes send messages MSG only if their list of nodes is nonempty. Will CT5 still work? Explain or give counterexample.

4.6 Extending CT to changing topologies - sequence numbers (ECT)

check Lamport, Time,Clocks,.. 1978 The CT Protocols require specific initial conditions and therefore their extension to handle topological changes must include reinitialization after every such change. This can be implemented by restarting a new cycle of the protocol after every topological event. In order to distinguish between messages and node states belonging to different cycles, we employ global sequence numbers [Seg83], [Fin79], [Gal76]. The cycles of the protocol will be labeled with increasing numbers, every node remembers the highest cycle number known to it so far and each of the cycles corresponds now to the original (nonextended) protocol. When a node wants to trigger a new cycle due to an adjacent topological event, it resets its variables, increments the cycle number and acts as if it has received START for a new cycle with this number. Here resetting variables means to adjust the appropriate variables to their required initial value as stated in each of the protocols. The number of the new cycle will be carried by all messages belonging to this cycle. A node disregards and discards messages whose cycle number is lower than the highest cycle number known to the node so far. A node that receives a message with cycle number larger than the highest known to it, resets its own variables, increases its registered maximal cycle number accordingly and acts as if it enters the protocol now (i.e. the corresponding cycle of the extended protocol). In this way the cycle with higher number will cover the lower-number cycles, in the sense that when a higher cycle reaches any node, the node will forget the previous knowledge and will participate only in the most recent cycle. Observe that several nodes may start the same new cycle independently because of multiple topological events, but the protocol allows this situation to happen, considering it in the same way as if several nodes receive START in the nonextended protocol.

Connectivity Test protocols require that following a topological change, all nodes discard their previous knowledge and restart the protocol from scratch. We shall see in the following sections that other protocols can work more smoothly, whereby only the information affected by the topological change is renewed, whereas the rest of the network adapts smoothly to the new situation.

As an example, we shall write exactly the extended CT3 protocol.

Protocol ECT3

Messages

$MSG^j(R, \Lambda)$ - control messages with identity j and Λ = list G_j of neighbors of j

Variables

G_i - set of neighbors of node i
c_i^j - designates knowledge at i about connectivity to j (values 0,1,2), for all $j \in \overline{V}$
= 0 when i knows nothing about j
= 1 while i knows j only as a neighbor of another node
= 2 while i knows j directly (i.e. $MSG^j(\Lambda)$ has been received)
R_i - highest sequence number known to i (values: 0,1, ...);

Algorithm for node i

A1 node i becomes operational
A2 { $R_i \leftarrow 0$;
}
B1 link (i,l) enters *Connected* state or *Initialization* Mode
B2 { update G_i;
B3 $R_i \leftarrow R_i + 1$; /* enter protocol, replaces $m_i \leftarrow 1$ */
B4 $initialize()$;
B5 $phase1^i(G_i)$;
}
C1 receives $MSG^j(R,\Lambda)$ from $l \in G_i$
C2 { ***if*** $(R \geq R_i)\{$
C3 ***if*** $(R > R_i)\{$
C4 $R_i \leftarrow R$; /* enter protocol, replaces $m_i \leftarrow 1$*/
C5 $initialize()$;
C6 $phase1^i(G_i)$;
}
C7 ***if*** $(c_i^j \neq 2)$ $phase1^j(\Lambda)$;
C8 ***if*** $(c_i^j = 0 \text{ or } 2, \forall j \in \overline{V})$ connectivity known;
}
}
D1 $phase1^j(\Lambda)$
D2 { $c_i^j \leftarrow 2$;
D3 ***for*** $(k \in \Lambda)$ $c_i^k \leftarrow max(c_i^k, 1)$;
D4 ***for*** $(k \in G_i)$ send $MSG^j(R_i,\Lambda)$ to k;
}
E1 $initialize()$
E2 { ***for*** $(j \in \overline{V})$ $c_i^j \leftarrow 0$;
}

Note that <B3> and <C4> here correspond to <A3> in CT3. Clearly, similar extended protocols can be given for the other protocols. Their properties are similar to the ones of ECT3, as summarized in:

Theorem 4.6 *(ECT3) Consider an arbitrary* finite *sequence of topological events with arbitrary timing and location and let (E,V) denote a connected subnetwork in the final topology within each at least one node has entered the protocol. Then there is a finite time after the sequence is completed after which no messages travel in (V,E) and all nodes $i \in V$ will have the same cycle number R_i, with $c_i^k = 2$ for all $k \in V$ and with $c_i^k = 0$ for all $k \notin V$.*

Proof: Consider the topology of the network after all topological changes cease. Consider in this topology a given connected subnetwork (V,E). From $<$B3$>$, each topological event adjacent to a node $i \in V$ increments the cycle counter R_i at node i. Let $\{i_n\}$ be the collection of nodes in V that register change of status of an adjacent link, and let $\{t_n\}$ be the corresponding collection of times when the status change is registered. Since there is a finite number of topological events, the collections $\{i_n\}$, $\{t_n\}$ are finite. Let $R = \max\{R_{i_n}(t_n+)\}$ over all n. Then R is the highest cycle number ever known in network (V,E) and the cycle with number R is started by (one or more) nodes $i \in \{i_n\} \in V$ that increment their R_i to R as a result of a topological event. These nodes can be considered as if they receive START in the CT3 protocol and, indeed, the network covered by the cycle with number R registers no more topological events, since no counter number R_i is ever increased to $(R+1)$. Also, the initial conditions of CT3 hold for the R cycle as follows. A node with $R_i < R$ is considered as having $m_i = 0$, a node i with $R_i = R$ is considered as having $m_i = 1$. WHAT ABOUT PROTOCOLS WHERE m_i RETURNS TO 0??? Since R_i is nondecreasing, the first $MSG(R)$ that arrives at a node i finds $R_i < R$, namely $m_i = 0$. Also, after <C4>, a node disregards all messages with sequence number less than R, so that the condition that nodes receive only messages of the present protocol is also satisfied.

Moreover, from the Follow-up property of DLC follows that in the final topology, $l \in G_i$ if and only if $i \in G_l$, so that the assumption of bi-directionality (Assumption a) in Sec. 3.1) holds in the final topology. Consequently, the evolution of the cycle with sequence number R is the same as in protocol CT3 on (V, E) and therefore Theorem 4.4 holds here, completing the proof.

Here we can see for the first time the reason for requiring asynchronous Initial Conditions in the Fixed Topology algorithms as opposed to synchronous ones: there is a time t_0 when all $m_i = 0$ and there are no messages on the links. One can attempt to find such a time t_0, for example the time when the first message with $R_i = R$ is received by any node in (V, E). However, there is no guarantee that at that time there are no messages on the links. Some links may even have messages with $R_i = R$.

Problems

Problem 4.6.1 Consider each of the 4 properties of *ensuring synchronization* of LI Procedures (see Sec. 2.4). For each property, check whether ECT3 still works in case that property doesn't hold.

Problem 4.6.2 Does ECT3 work if we change line <A2> to $R_i \leftarrow 50$?

Chapter 5

TOPOLOGY and PARAMETER BROADCAST

Link state routing protocols [MRR80] ,[Per83], [Ros80], [Hui95], like OSPF in the Internet, are based on the principle that every node contains a map of the entire network topology, as well as of various fixed and varying parameters of the links and nodes. These parameters may include link speeds, error rates, congestion, etc. In order to make this information available at each node, it is necessary to broadcast it in the network.

5.1 Broadcasting topology and parameters (TPB)

One way to proceed is to use Protocol CT3, where a node j includes in MSG^j not only the list of neighbors G_i, but also the parameters of interest about itself and the adjacent links. For brevity, we shall denote the collection of these parameters at node j by Δ_j^j. The other change compared with $CT3$ is that a node i keeps not only identities of nodes known to it, but the entire information received in MSG^j. Consequently, when a node completes the $CT3$, it has the entire topological and parameter information of the network. The protocol is as follows:

Protocol TPB1

Messages

$MSG^j(\Lambda, \Delta)$ - control messages with identity j, containing $\Lambda = G_j$ and $\Delta = \Delta_j^j$

Variables

G_i - set of neighbors of node i
m_i - shows whether i has already entered the algorithm (values 0,1)
c_i^j - designates knowledge at i about connectivity to j (values 0,1,2), for all $j \in \overline{V}$

$= \mathbf{0}$ when i knows nothing about j
$= \mathbf{1}$ while i knows j only as a neighbor of another node
$= \mathbf{2}$ while i knows j directly (i.e. $MSG^j(\Lambda, \Delta)$ has been received)

Δ_i^i - the local parameters at i
Λ_i^j - list that will contain the identities of neighbors of $j \in \overline{V}$
Δ_i^j - will contain the parameters of $j \in \overline{V}$ as known by i

Initialization

if a node receives at least one MSG, then

- just before the time it receives the first one holds $m_i = 0$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receive MSG^j(Λ,Δ) from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0){
A3            m_i ← 1;                                   /* enter protocol */
A4            initialize();
A5            phase1^i(G_i, Δ_i^i);
          }
A6        if (c_i^j ≠ 2) phase1^j(Λ,Δ);
A7        if (c_i^j = 0 or 2, ∀j ∈ V̄) topology and parameters known;
      }
B1    phase1^j(Λ,Δ)
B2    {   c_i^j ← 2;
B3        Λ_i^j ← Λ;
B4        Δ_i^j ← Δ;
B5        for (k ∈ Λ) c_i^k ← max(c_i^k, 1);
B6        for (k ∈ G_i) send MSG^j(Λ,Δ) to k;
      }
C1    initialize()
C2    {   for (j' ∈ V̄) c_i^j' ← 0;
      }
```

Theorem 5.1 *(TPB1) Suppose that at least one node in V receives $START$. Then:*

a) For every $i \in V$, the variables c_i^j will become 2 in finite time for all $j \in V$ and will remain 0 forever for all $j \notin V$.

b) If $c_i^j = 2$, then $\Lambda_i^j = G_j$ and $\Delta_i^j = \Delta_j^j$, in other words, node i knows the topology and parameters at and adjacent to j. Every $i \in V$ will perform $<A7>_i$ in finite time and when this happens for the first time, it will have $c_i^j = 2$ for all $j \in V$ and $c_i^j = 0$ for all $j \notin V$.

Proof: The event $m_k \leftarrow 1$ propagates as in $MPI1$ and hence will happen in finite time at all nodes $k \in V$. For a given $j \in V$, after m_j becomes 1, the event $c_k^j \leftarrow 2$ propagates again as in $PI1$ and hence will happen in finite time at every node $i \in V$. The fact that c_i^j remains 0 forever for $j \notin V$ is obvious. Hence *a*).

For each j, propagation of $MSG^j(\Lambda, \Delta)$ happens as in protocol $PI1$ except that it is triggered by <A6> instead of by $START$. The message carries $\Lambda = G_j$ and $\Delta = \Delta_j^j$. When a node i receives for the first

time a message $MSG^j(\Lambda, \Delta)$, it copies those lists into its local topological database Λ_i^j, Δ_i^j. The rest of b) is identical to Theorem 4.4b). qed

Communication Cost: If we count each set of parameters as D_i elementary entities, where D_i is the number of links adjacent to i, then we send on each link in each direction $\mid V \mid (2D+1)$ elementary entities. Here D is the average degree of the nodes (average number of neighbors). Clearly $D = 2 \mid E \mid / \mid V \mid$ and hence the communication cost over the entire network is $2 \mid E \mid (2 \mid E \mid + \mid V \mid)$ elementary quantities[1].

5.2 Fixed Topology, changing parameters

In many cases, parameters at various nodes change, while the network topology remains fixed. These changes must be broadcast to all nodes in the network. Since the topology is known to every node when it completes the $TPB1$ protocol, there is no need to repeat the protocol. All that is needed is to have every node broadcast the new parameters when they change. Any of the protocols introduced in Sec. 3.4 for repeated propagation of information can be used. The most commonly used protocol is RPI1, the repeated $PI1$ protocol with increasing sequence numbers. Each node in the network runs a separate RPI1 Protocol, with its own sequence numbers. The protocols for different nodes are completely independent, and we shall describe the protocol for a given node s. As long as the topology remains fixed, this protocol achieves the goal of correctly broadcasting the information. In Sec. 5.4, we shall deal with the difficulties encoutered by this protocol when topology may change.

Protocol TPB2

Messages

$MSG(r, \Delta)$ - message with sequence number r carrying the local topology and the local parameters at s $(r = 0, 1, 2, \ldots)$

Variables

G_i - set of neighbors of node i
r_i - largest sequence number received by i (values $0, 1, 2, \ldots$)
Δ_s - the local parameters at s
Δ_i - will contain the parameters at s as known by i

Initialization

* just before the first message is sent by s, holds $r_s = -1$
* if i receives a MSG, then

- just before receiving the first MSG, holds $r_i = -1$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1      when Δ_s changes
A2      {   deliver MSG(r_s + 1, Δ_s) from nil to yourself;
        }
B1      receive MSG(r, Δ) from l ∈ G_i ∪ {nil}
B2      {   if (r > r_i) phase1(r);
        }
C1      phase1(r)                                   /* similar to PI1 */
C2      {   r_i ← r ;
C3          Δ_i ← Δ;
C4          for (k ∈ G_i) send MSG(r, Δ) to k;
        }
```

Note: In <C4>, a node i may send $MSG(r, \Delta)$ to all $k \in G_i - \{l\}$.

[1] time, computation,???

Theorem 5.2 *(TPB2) Suppose that parameter changes at s stop. If $s \in V$, then a finite time afterwards, every node $i \in V$ will have $\Delta_i = \Delta_s$ and this information will never change afterwards.*

Proof: Protocol $TPB2$ is exactly $RPI1$, therefore all information sent out by s is accepted by each node in V in order and in finite time. Hence, the last information is accepted last, causing Δ_i to contain the list of neighbors and the parameters of s respectively. qed

Another protocol that can be used for the same purpose is $RPIF$ combined with $CT2$. Its advantages over $RPI1$ are that it uses bounded sequence numbers and a node $i \in V$ has positive acknowledgement when it knows the topology of V. Although there is no positive acknowledgement about knowledge of the parameters at all nodes, topology is more critical in most cases, since routing through a congested area is not as bad as routing into a nonexistent link. The main disadvantage of $RPIF$ is that it is somewhat more complicated than $RPI1$. It is interesting to note though that the speed of information dissemination is identical for both protocols. The protocol is essentially a $CT2$ protocol with repeated PIF's and MSG^j carrying the topology and parameters adjacent to j. The PIF started by node j with instance number r, will be denoted by $PIF^j(r)$. Here $0 \leq r \leq W-1$, where W is determined by the number of bits allocated to the instance number. Since a $CT2$ protocol is performed here, the specification cannot be provided separately for each node[2].

Protocol TPB3

Messages

$MSG^j(r, \Lambda, \Delta)$ - message of $PIF^j(r)$

Variables

G_i - set of neighbors of node i
m_i - shows if node i is in the protocol (values 0,1)
$m_i^j(r)$ - shows if node i is in $PIF^j(r), r = 0, 1, \ldots, W-1$
$p_i^j(r)$ - preferred neighbor of node i for $PIF^j(r)$
$e_i^j(l)(r)$ = number of $MSG^j(r)$ sent to l - number of $MSG^j(r)$ received from l, for all $l \in G_i$
c_i^j - designates knowledge at i about connectivity to j (values 0,1), for all $j \in \overline{V}$
Δ_i^i - the local parameters at i
Λ_i^j - list that will contain the identities of neighbors of $j \in \overline{V}$
Δ_i^j - will contain the parameters of $j \in \overline{V}$ as known by i

Initialization

if a node receives at least one MSG, then

- just before the time it receives the first one holds $m_i = 0$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

[2]Is it possible to design a protocol with one of the MPI or MPIF to ensure that routing tables are calculated at times that will allow no loops in the routing tables. In other words can one know when topology and parameter info. is consistent???

Algorithm for node i

```
A1   parameters Δ_i^i change
A2   {  while (m_i^i(r') = 1 ∀r') {};                          /* same is in RPIF1 */
A3      deliver MSG^i(r, G_i, Δ_i^i) from nil to yourself with some r | m_i^i(r) = 0;
     }
B1   receive MSG^j(r, Λ, Δ) from l ∈ G_i ∪ {nil}
B2   {  if (m_i = 0){
B3         m_i ← 1 ;                                           /* enter protocol */
B4         initialize();
B5         phase1^i(0, G_i, Δ_i^i);
        }
B6      if (c_i^j = 0) update^j();
B7      if (m_i^j(r) = 0) phase1^j(r, Λ, Δ);
B8      e_i^j(l)(r) ← e_i^j(l)(r) − 1;
B9      if (e_i^j(k)(r) = 0 ∀k ∈ G_i − {p_i^j(r)}) phase2^j(r);
     }
C1   update^j()
C2   {  c_i^j ← 1;
C3      Λ_i^j ← Λ;
     }
D1   phase1^j(r, Λ^j, Δ^j)                                    /* same as in CT2 */
D2   {  m_i^j(r) ← 1;
D3      if (i ≠ j) p_i^j(r) ← l else p_i^j(r) ← nil;
D4      Δ_i^j ← Δ;
D5      for (k ∈ G_i − {p_i^j(r)}){
D6         send MSG^j(r, Λ^j, Δ^j) to k;
D7         e_i^j(k)(r) ← e_i^j(k)(r) + 1;
        }
     }
E1   phase2^j(r)                                              /* same as in CT2 */
E2      send MSG^j(r, Λ, Δ) to p_i^j(r);
E3      e_i^j(p_i^j(r))(r) ← e_i^j(p_i^j(r))(r) + 1;
E4      m_i^j(r) ← 0;
     }
F1   initialize()
F2   {  for (j' ∈ V̄) {
F3         c_i^j' ← 0;
F4         for (all r') {
F5            m_i^k(r') ← 0;
F6            for (k ∈ G_i) e_i^j'(k)(r') ← 0;
           }
        }
     }
```

Note that broadcast of the adjacent topology G_i is required only in the first PIF started at node i. Nodes disregard this information in all subsequent PIF's. Therefore the protocol may be altered to save communication by using two types of MSG's, one type containing both topology Λ and parameters Δ and the other type containing parameters only. The first type will be used by each node when starting its first PIF and then all subsequent PIF's will use MSG's of the second type.

Theorem 5.3 *(TPB3) Suppose that at least one node in V receives $START$. Then:*

a) for every $i \in V$, the variables c_i^j will become 1 in finite time for all $j \in V$ and will remain 0 forever for all $j \notin V$.

b) if $c_i^j = 1$, then $\Lambda_i^j = G_j$, namely node i knows the topology adjacent to j; every $i \in V$ will perform $phase2()_i^i(r)$ for some r in finite time, and after the first time when it does that, holds $c_i^j = 1$ for all $j \in V$ and $c_i^j = 0$ for all $j \notin V$.

c) suppose parameter changes at a node $j \in V$ stop; a finite time afterwards every node $i \in V$ will have $\Delta_i^j = \Delta_j^j$, i.e. every node in the network will know the parameters at j and this information will never change afterwards.

Proof: Part *a*) follows from the fact that nodes update the variables c_i^j exactly as in Protocol $CT2$. To prove *b*), observe that a node i sets $c_i^j \leftarrow 1$ at the same time when it sets $\Lambda_i^j \leftarrow G_j$. The rest follows from Theorem 4.3.

Now *c*) follows from the fact that after entering the protocol, every node acts as the source s in the RPIF Protocol, where the packets to be propagated are the local parameters. Since Theorem 3.9b) says that packets are accepted by each node in V in the same order as generated and every packet is eventually accepted by each node, the last generated packet containing the final value of the parameters is accepted last by each node. qed

Communication Cost:

????

In Protocols TPB1 and TPB2, every node $i \in V$ must start propagation of adjacent topology and parameters in the form of PI or PIF when it enters the protocol. It is natural to inquire whether it is possible to indicate an initialization situation such that certain, hopefully most, nodes will be absolved from doing so. The reason is that presumably, this is not the first time the protocol is run in the network and most nodes have already sent out their local information. If this information has not changed since the last time when it was sent out, why is there need to send it out again. However it turns out that no initialization assumption is sufficient, short of assuming that all nodes in V have the information. This is because if some nodes have some incorrect old information about j and now are receiving the correct one, they have no means of distinguishing the old ?????

have been required to start a new broadcast. The knowledge in the network could have been possible maybe if previous to the entrance in the TPB protocol, node j has propagated the local information, after which the information has not changed. However, maybe information from some node may already be available at all nodes in V, maybe from previous One may ask if this is necessary if, say from previous propagations, one can make sure that all nodes in V have .

????

Problems

Problem 5.2.1 TPB1 is based on CT3. Can CT5 be used in the same way for implementing TPB? If not, explain why, if yes, write the code of the corresponding protocol.

Problem 5.2.2 Repeat the above question for CT4.

5.3 Topology and Parameter Broadcast - Topological Changes (ETPB)

Similarly to the protocol of Sec. 4.6, one can define a Protocol for the topological changes version of the Topology and Parameter Broadcast Protocol, that will be called the Extended Topology and Parameter Broadcast (ETPB3) protocol. This protocol uses global sequence numbers similar to the ones of Sec. 4.6[3].

Protocol ETPB3

Messages

$MSG^j(R, r, \Lambda, \Delta)$ - message of $PIF^j(r)$ with global sequence number R

Variables

G_i - set of neighbors of i, i.e. $k \in G_i$ if (i, k) is in Connected state at i
R_i - highest sequence number known to i (values: 0,1, ...)
m_i - shows if node i is in the protocol (values 0,1)
$m_i^j(r)$ - shows if node i is in $PIF^j(r), r = 0, 1, \ldots, W-1$
$p_i^j(r)$ - preferred neighbor of node i for $PIF^j(r)$
$e_i^j(l)(r)$ = number of $MSG^j(r)$ sent to l - number of $MSG^j(r)$ received from l
c_i^j - designates knowledge at i about connectivity to j (values 0,1), for all $j \in \overline{V}$
Δ_i^i - the local parameters at i
Λ_i^j - list that will contain the identities of neighbors of $j \in \overline{V}$
Δ_i^j - will contain the parameters of $j \in \overline{V}$ as known by i

[3]Is it possible to design a protocol with one of the MPI or MPIF to ensure that routing tables are calculated at times that will allow no loops in the routing tables. In other words can one know when topology and parameter info. is consistent???

Algorithm for node i

A1 Node i becomes operational
A2 { $R_i \leftarrow 0$;
}
B1 Link (i,l) enters *Connected* state or *Initialization* Mode
B2 { $R_i \leftarrow R_i + 1$; /* enter protocol, replaces $m_i \leftarrow 1$ */
B3 $initialize()$;
B4 $phase1^i(0, G_i, \Delta_i^i)$;
}
C1 parameters Δ_i^i change
C2 { ***while*** $(m_i^i(r') = 1\ \forall r')$ {};
C3 deliver $MSG^i(R_i, r, G_i, \Delta_i^i)$ from nil to yourself with some $r \mid m_i^i(r) = 0$;
}
D1 receives $MSG^j(R, r, \Lambda, \Delta)$ from $l \in G_i$
D2 { ***if*** $(R \geq R_i)$\{
D3 ***if*** $(R > R_i)$\{
D4 $R_i \leftarrow R$; /* enter protocol, replaces $m_i \leftarrow 1$ */
D5 $initialize()$;
D6 $phase1^i(0, G_i, \Delta_i^i)$;
}
D7 ***if*** $(c_i^j = 0)$ $update^j()$;
D8 ***if*** $(m_i^j(r) = 0)$ $phase1^j(r, \Lambda, \Delta)$;
D9 $e_i^j(l)(r) \leftarrow e_i^j(l)(r) - 1$;
D10 ***if*** $(e_i^j(k)(r) = 0\ \forall k \in G_i - \{p_i^j(r)\})$ $phase2^j(r)$;
}
E1 $update^j()$ /* same as in TPB3 */
E2 { $c_i^j \leftarrow 1$;
E3 $\Lambda_i^j \leftarrow \Lambda$;
}
F1 $phase1^j(r, \Lambda^j, \Delta^j)$ /* same as in TPB3 */
F2 { $m_i^j(r) \leftarrow 1$;
F3 ***if*** $(i \neq j)$ $p_i^j(r) \leftarrow l$ ***else*** $p_i^j(r) \leftarrow nil$;
F4 $\Delta_i^j \leftarrow \Delta^j$;
F5 ***for*** $(k \in G_i - \{p_i^j(r)\})$\{
F6 send $MSG^j(R_i, r, \Lambda^j, \Delta^j)$ to k;
F7 $e_i^j(k)(r) \leftarrow e_i^j(k)(r) + 1$;
}
}
G1 $phase2^j(r)$ /* same as in TPB3 */
G2 { send $MSG^j(R_i, r, \Lambda, \Delta)$ to $p_i^j(r)$;
G3 $e_i^j(p_i^j(r))(r) \leftarrow e_i^j(p_i^j(r))(r) + 1$;
G4 $m_i^j(r) \leftarrow 0$;
}
H1 initialize() /* same as in TPB3 */
H2 { ***for*** $(j' \in \overline{V})$\{
H3 $c_i^{j'} \leftarrow 0$;
H4 ***for*** (all r')\{
H5 $m_i^k(r') \leftarrow 0$;
H6 ***for*** $(k \in G_i)$ $e_i^{j'}(k)(r') \leftarrow 0$;
}
}
}

Theorem 5.4 *(ETPB3) Consider an arbitrary* finite *sequence of topological events with arbitrary timing and location and let (V, E) denote a connected subnetwork in the final topology. Then there is a finite time after the sequence is completed after which:*

a) for every $i \in V$, the variables c_i^j are 1 for all $j \in V$ and will remain 0 forever for all $j \notin V$.

b) every $i \in V$ will perform <D10>$_i$ in finite time and after the first time when it does that holds $c_i^j = 1$ and $\Lambda_i^j = G_j$ for all $j \in V$ and $c_i^j = 0$ for all $j \notin V$.

c) suppose parameter changes at a node $j \in V$ stop; a finite time afterwards every node $i \in V$ will have $\Delta_i^j = \Delta_j^j$, i.e. every node in the network will know the parameters at j and this information will never change afterwards.

Proof: Consider the topology of the network after all topological changes cease. Consider in this topology a given connected subnetwork (V, E). From <B2>, each topological event adjacent to a node $i \in V$ increments the cycle counter R_i at the node i adjacent to the change. Let $\{i_n\}$ be the collection of nodes in V that register change of status of an adjacent link, and let $\{t_n\}$ be the corresponding collection of times when the status change is registered. Since there is a finite number of topological events, the collections $\{i_n\}$, $\{t_n\}$ are finite. Let $R = \max\{R_{i_n}(t_n+)\}$ over all n. Then R is the highest cycle number ever known by nodes in V and the cycle with number R is started by (one or more) nodes $i \in \{i_n\} \in V$ that increment their R_i to R as a result of a topological event. These nodes can be considered as if they receive START in the TPB3 protocol and, indeed, the network (V, E) covered by the cycle with number R registers no more topological events, since no counter number R_i is ever increased to $(R + 1)$. Moreover, from the Follow-up property of DLC follows that in the final topology, $l \in G_i$ if and only if $i \in G_l$, so that the assumption of bidirectionality (Assumption a) in Sec. 3.1) holds in the final topology. Moreover, the initialization conditions for protocol TPB3 hold WHY???. Consequently, the evolution of the cycle with sequence number R is the same as in protocol TPB3 and therefore Theorem 5.3 holds here, completing the proof. qed

DELETE !!! In Protocol ETPB3, every node $i \in V$ must start propagation of adjacent topology and parameters in a PIF when it enters a new cycle of the protocol. The question is whether nodes for which the local information has not changed since the last update was sent out, can be absolved from doing so. The intuitive reasoning is that if the local information has not changed, then why is it necessary to send it out again. The problem with this is however that the previous

The reason is that presumably, this is not the first time the protocol is run in the network and most nodes have already sent out their local information. If this information has not changed since the last time it was sent out, why is there need to send it out again. However it turns out that no initialization assumption is sufficient, short of assuming that all nodes in V have the information. This is because if some nodes have some incorrect old information about j and now are receiving the correct one, they have no means of distinguishing the old ?????

have been required to start a new broadcast. The knowledge in the network could have been possible maybe if previous to the entrance in the TPB protocol, node j has propagated the local information, after which the information has not changed. However, maybe information from some node may already be available at all nodes in V, maybe from previous One may ask if this is necessary if, say from previous propagations, one can make sure that all nodes in V have

5.4 Topology and Parameter Broadcast with node-associated sequence numbers - Topological Changes

The protocol $ETPB3$ of Sec. 5.3 uses global sequence numbers R, as well as node-associated instance numbers r. The main advantage of this method, is that, as seen in Theorem 5.4, it works under any sequence of topological changes, including network separations and node crashes without non-volatile memory. However the communication price to be paid to achieve this is too high: every time there is a topological change in the

network, all nodes re-broadcast their local information, even if the latter has not changed. As a result, the more commonly used method is to employ node-associated sequence numbers only, namely to use the RPI1 protocol as described in Sec. 5.2, except that both topology and parameter information are broadcasted. The simplistic protocol is identical to protocol TPB2 of Sec. 5.2, except that it is started by the source node s whenever adjacent topology G_s changes as well as when adjacent parameters Δ_s change and the $PI1$ protocol broadcasts both topology and parameter information. To recapitulate, the protocol consists of each node incrementing its sequence number and starting a new $PI1$ protocol with the new sequence number, whenever the local topology or parameters change. As in TPB2, the protocol evolves independently from source to source and thus the superscript s is suppressed in the pseudo-code.

Protocol ETPB2

Messages

$MSG(r, \Lambda, \Delta)$ - message with sequence number r carrying the local topology and the local parameters at s ($r = 0, 1, 2, \ldots$)

Variables

G_i - set of neighbors of node i
r_i - largest sequence number received by i (values $0, 1, 2, \ldots$)
Δ_s - the local parameters at s
Λ_i - list that will contain the identities of neighbors of s
Δ_i - will contain the parameters at s as known by i

Initialization

* just before the first message is sent by s, holds $r_s = -1$
* if i receives a MSG, then

- just before receiving the first MSG, holds $r_i = -1$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1      node i becomes operational
A2      {   r_i ← 0;
        }
B1      when G_s or Δ_s changes
B2      {   deliver MSG(r_s + 1, G_s, Δ_s) from nil to yourself;
        }
C1      receive MSG(r, Λ, Δ) from l ∈ G_i ∪ {nil}
C2      {   if (r > r_i) phase1(r);
        }
D1      phase1(r)                                        /* similar to PI1 */
D2      {   r_i ← r ;
D3          Λ_i ← Λ;
D4          Δ_i ← Δ;
D5          for (k ∈ G_i) send MSG(r, Λ, Δ) to k;
        }
```

Note: In <D5>, a node i may send $MSG(r, \Lambda, \Delta)$ to all $k \in G_i - \{l\}$.

However, in a changing topology network, this simplistic protocol does not operate correctly. For example, if a node fails, when it recovers, it will set its sequence number to 0. Its updates will be disregarded by other nodes whose stored information about this node appears with a higher sequence number because of previous updates. Only when the sequence number reaches the value that was last used before the failure will the updates be registered.

Incorrect operation may also occur due to network disconnections and reconnections. Suppose the network is split into two non-connected parts V' and V''. Updates initiated by nodes in V' do not reach nodes in V''. Then, if a link connecting the two parts comes up, there is no trigger for updating nodes in V" regarding those

updates. The solution in existing networks, like the Internet, that uses link-state protocols in OSPF [Hui95] is to employ periodic updates. Nodes start broadcast of topology and parameter values on a periodic basis, even if there are no adjacent changes. This solves the disconnection problem, but does not solve the problem of sequence number reset to 0 after a node failure. One solution for the latter problem is to use a timer associated with each table entry and to delete entries for which updates have not been received for a long time [Hui95],[BG92]. Moreover, messages carry an *age* field, and messages that are too old are discarded by nodes.

5.5 SPTA - Topology Broadcast without sequence numbers - Topological Changes

The following protocol *Shortest Path Topology Algorithm (SPTA)* [BG92] allows broadcast of topology without sequence numbers. The main idea is that a node believes the status of a distant link as received from the neighbor that is on the shortest path from the node to that link.

Protocol SPTA

Messages

$MSG(C)$ - message of the protocol (C is list of nodes whose status has changed)

Variables

T_i - main topology of node i.
$T_i(j)$ - port topology of i for neighbor node j
$S_i^{(m,n)}(j)$ - status of link (m,n) according to port topology $T_i(j)$ (values $up, down$)
$S_i^{(m,n)}$ - status of link (m,n) according to main topology T_i (values $up, down$)
P_i - a temporary group of nodes, that holds at the k-th iteration all nodes whose distance from i is no more than k links
$P_{i,old}$ - the temporary group P_i from last iteration
$M_{labeled}$ - the group of nodes that are labeled in each iteration.
G_i - set of neighbors of i, i.e. $l \in G_i$ if (i,l) is in Connected state at i
$Label_i(j)$ - label of node j at node i
C_i - set of nodes whose status has changed

Initialization

not clear

-

Algorithm for node i

A1 node i becomes operational
A2 { do nothing;
}
B1 link (i,l) enters *Initialization Mode*;
B2 { $S_i^{(i,l)} \leftarrow down$; $S_i^{(i,l)}(l) \leftarrow down$;$C_i \leftarrow \{((i,l), down)\}$;
B3 **for** $(k \in G_i)$ send $MSG(C_i)$ to k;
B4 *update*();
}
C1 link (i,l) enters *Connected* State
C2 { $S_i^{(i,l)} \leftarrow up$; $S_i^{(i,l)}(l) \leftarrow up$;
C3 **for** $(k \in G_i)$ send $MSG(\{(i,l), up\})$ to k;
C4 send $MSG(T_i)$ to l;
C5 *update*();
}
D1 receives $MSG(C)$ from $l \in G_i$
D2 { enter information in C into $T_i(l)$;
D3 $S_i^{(l,i)} \leftarrow up$; $S_i^{(l,i)}(l) \leftarrow up$;
D4 *update*();
}
E1 *update*()
E2 { $P_{i,old} \leftarrow i$; $P_i \leftarrow i$; $C_i \leftarrow \emptyset$;
E3 **for** $(j \in G_i)$ {
E4 $P_i \leftarrow P_i \cup \{j\}$;
E5 $Label_i(j) = j$;
}
E6 **while** $(M_{labeled} \neq \emptyset)$ {
E7 $M_{labeled} \leftarrow \emptyset$;
E8 **for** $(m \in P_i - P_{i,old})$ {
E9 **for** $((m,n) \in T_i)$ **and** $(n \notin P_{i,old}))$ {
E10 $j \leftarrow Label_i(m)$;
E11 **if** $(S_i^{(m,n)} \neq S_i^{(m,n)}(j))$ $C_i \leftarrow C_i \cup ((m,n), S_i(m,n))$;
E12 $S_i^{(m,n)} \leftarrow S_i^{(m,n)}(j)$;
E13 **if** $(S_i^{(m,n)} = up)$ **and** $(n \notin P_i))$ {
E14 $Label_i(n) \leftarrow j$;
E15 $M_{labeled} \leftarrow M_{labeled} \cup \{n\}$;
}
}
}
E16 $P_{i,old} \leftarrow P_i$;
E17 $P_i \leftarrow P_i \cup M_{labeled}$;
}
}

Chapter 6

DISTRIBUTED DEPTH-FIRST-SEARCH PROTOCOLS

Here we study Distributed Depth-First-Search Protocols. The basic protocol appears in [Che83].

Protocol DFS1

Messages

MSG - message trying to find new tree nodes
$REPLY$ - reply to MSG

Variables

G_i - set of neighbors of i
m_i - shows if node i has already entered the protocol
p_i - parent of i, i.e.neighbor from which MSG was received first.
$v_i(l) = 1$ if node i knows that l has been visited; $= 0$ otherwise ($\forall l \in G_i$)

Initialization

if a node i receives a MSG, then

- just before it receives the first MSG, holds $m_i = 0$ and $c_i(k) = 0$ for all $k \in G_i$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receive MSG from l ∈ G_i ∪ {nil}
A2    {   if (m_i = 0){
A3            phase1();
A4            continue();
          }
A5        else {
A6            v_i(l) ← 1;
A7            send REPLY to l;
          }
      }
B1    receive REPLY from l
B2    {   v_i(l) ← 1;
B3        continue();
      }
C1    phase1()
C2    {   m_i ← 1;
C3        p_i ← l; v_i(p_i) ← 1 ;
      }
D1    phase2()
D2    {   send REPLY to p_i;
      }
E1    continue()
E2    {   if (v_i(k) = 1 ∀k ∈ G_i − {p_i}) phase2();
E3        else {
E4            select any m ∈ G_i − {p_i} with v_i(m) = 0;
E5            send MSG to m ;
          }
      }
```

The properties of the DFS1 protocol are given in Theorem 6.2. Note that although the main properties are similar to PI/PIF type protocols, the steps of the proof are somewhat different.

First note that at any given time, exactly one message travels in the network. This is because whenever a node receives a message, it sends a message.

Lemma 6.1

a) If a node i sends a MSG to l, then from l it can receive only $REPLY$.

b) After a node i sends a MSG to l, the next $REPLY$ it receives is from l.

c) No node can send two messages on the same link.

Proof: If i sends MSG to l, when the MSG arrives, then l sets $v_l(i) = 1$, and no MSG's are sent on links with $v = 1$, hence *a*).

We prove *b*) and *c*) by a common induction. Let i be the first node that either receives a $REPLY$ on a link (i, k) on which it hasn't last sent a MSG or that sends a second message on some link (i, j), at time t say.

Suppose that the first kind of event happens at time t. If the message sent by i to k found $m_k = 1$, then k would have sent $REPLY$ to i and the next event at i would have been to receive $REPLY$ from k. Therefore $m_k = 0$ when it receives the MSG and therefore i is the parent of k. Let m be the neighbor from which i receives $REPLY$ at time $t-$. $REPLY$ can be sent by m either in <A7> or <D2>. In both cases, m must have previously received a MSG from i and, by the induction hypothesis, i must have received a $REPLY$ from m. This means that the $REPLY$ received at time $t-$ is a second message sent by m to i, contradicting the second hypothesis of the common induction.

Suppose now that at time t, node i sends a second message on the same link (i, j) say. First we argue that this message cannot be MSG. If the first message that i has sent to j was $REPLY$, then $v_i(j)$ was set to 1, either at the time $REPLY$ was sent, in <A6>, or beforehand, in <C3>. Therefore the second message

cannot be a MSG, since such messages are sent only on links with $v = 0$. If on the other hand, the first message that was sent by i to j was MSG, then by b) a $REPLY$ was received from j before $t-$, which has set $v_i(j) \leftarrow 1$ and again the second message could not be a MSG.

Therefore the message sent by i at t is a $REPLY$. This cannot happen as a result of i receiving at time $t-$ a MSG from j, since when j has received the first message from i, it has set $v_j(i) = 1$ and nodes do not send MSG's on links with $v = 1$. Therefore j is the parent of i. When the first $REPLY$ was sent to j, all $v_i(l), l \in G_i - \{p_i\}$ were 1, namely i has received a message on each of these links. Therefore the message at time t is sent upon i receiving a second message on some link (i, m), meaning that m has sent a second message on some link before time t, contradiction. qed

Lemma 6.1 implies that the protocol terminates in finite time. It can terminate only at s, since only the parent of s is nil. It remains to show that it covers the entire network and it produces a (spanning) tree.

Theorem 6.2 *(DFS1) Suppose that a node $s \in V$ receives $START$. Recall that this is defined as the event when s receives MSG from nil. Then:*

a) all nodes $i \in V$ will perform the event phase1()$_i$ in finite time and exactly once ; after this happens, the links $\{(i, p_i), \forall i \in V\}$ will form a directed spanning tree rooted at s;

b) all nodes $i \in V$ will perform phase2()$_i$ in finite time and exactly once; moreover $t(\text{phase2}()_i) < t(\text{phase2}()_{p_i})$; node i receives no messages after time $t(\text{phase2}()_i)$; also, at the time when node s performs phase2()$_s$, all nodes in V have completed the algorithm, i.e. have performed phase2() and there are no messages traveling in the network

c) exactly one message MSG or $REPLY$ travels on each link in (V, E) in each direction.

Proof: Since this protocol simulates Tremaux's algorithm for DFS [Eve79], all properties follow from the properties of DFS. qed

The above protocol has $2 \mid E \mid$ message and time complexities. The reason is that the links are explored serially. An improved protocol was proposed by B. Awerbuch [Awe85b]. When a node enters the protocol, it first informs its neighbors that it has been visited. Upon receiving acknowledgment to these messages, it continues the DFS. In this way, only the tree links are explored serially, leading to $O(\mid V \mid)$ time complexity, while the message complexity is only doubled to $4 \mid E \mid$.

Protocol DFS2

Messages

MSG - message trying to find new tree nodes
$REPLY$ - reply to MSG, or when delivered to itself, indicates that all ACK's have been received
$VISITED$ - informs neighbor that the sending node has been visited
ACK - ack to $VISITED$

Variables

G_i - set of neighbors of i
p_i - neighbor from which MSG is received
$e_i(k)$ - number of $VISITED$ sent - number of ACK's received on link (i, k) ($\forall k \in G_i$)
$v_i(k)$ = 1 when i knows that neighbor k has been visited, = 0 beforehand ($\forall k \in G_i$)

Initialization

if a node i receives MSG, then

- just before it receives the MSG, holds $e_i(k) = v_i(k) = 0$ for all $k \in G_i$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1      receives MSG from l ∈ G_i ∪ {nil}
A2      {   phase1();
A3          if (G_i = {p_i}) phase2();
        }
B1      receives REPLY from l
B2      {   v_i(l) ← 1;
B3          continue();
        }
C1      receives VISITED from l
C2      {   v_i(l) ← 1;
C3          send ACK to l;
        }
D1      receives ACK from l
D2      {   e_i(l) ← e_i(l) − 1
D3          if (e_i(k) = 0 ∀k ∈ G_i − {p_i}) continue();
        }
E1      phase1()
E2      {   p_i ← l;
E3          for (k ∈ G_i − {p_i}){
E4              send VISITED to k;
E5              e_i(k) ← e_i(k) + 1;
            }
        }
F1      phase2()
F2      {   send REPLY to p_i;
        }
G1      continue()
G2      {   if (v_i(k) = 1 ∀k ∈ G_i − {p_i}) phase2();
G3          else {
G4              select any m ∈ G_i − {p_i} with v_i(m) = 0;
G5              send MSG to m ;
            }
        }
```

The time complexity of DFS2 is $4 \mid V \mid -2$, the message complexity is $4 \mid E \mid$.

The $VISITED$ messages informing neighbors that a node enters the protocol are distributed in DFS2 in parallel to all neighbors. However, this is perfomed in series with the rest of the protocol. A node waits to receive ACK's to the $VISITED$ messages before proceeding to advance the protocol. The protocol can be further improved as follows. Instead of sending $VISITED$, waiting for ACK's and then sending MSG, a node i will send MSG to one of its neighbors, l say, and, at the same time, $VISITED$ to all other non-parent neighbors. From each of the neighbors except l and p_i, node i may receive $VISITED$ or nothing. The response from l can be either $REPLY$, meaning that the parent of l is i, or $VISITED$, meaning that l has been visited before the time when l receives the MSG from i. Note that in fact node i needs not distinguish between the two cases, since the protocol does not require nodes to know their sons. Therefore the two types of messages can be collapsed into one, which we call $VISITED$. When this message is received, node i will send MSG to one of the neighbors from which it has received nothing, if any. If there are no such nodes, it will send $VISITED$ to its parent p_i. Note that with this protocol, there is no need for ACK messages.

Protocol DFS3

Messages

MSG - message trying to find new tree nodes
$VISITED$ - informs neighbor that the sending node has been visited

Variables

G_i - set of neighbors of i
m_i - shows if node i is already in the protocol
p_i - neighbor from which MSG is first received
c_i - neighbor of i currently being investigated
$v_i(k) = 1$ when i knows that neighbor k has been visited, $= 0$ beforehand ($\forall k \in G_i$)

Initialization

if a node i receives a MSG message, then

- just before it receives the first MSG, holds $m_i = 0$ and $v_i(k) = 0$ for all $k \in G_i$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1      receives MSG from l ∈ G_i ∪ {nil}
A2      {   if (m_i = 0) {
A3              phase1();
            }
        }
B1      receives VISITED from l
B2      {   v_i(l) ← 1;
B3          if (l = c_i) {                          /* replaces REPLY */
B4              if (current() = nil) phase2();
B5              else {
B6                  c_i ← current();
B7                  send MSG to c_i;
                }
            }
        }
C1      phase1()
C2      {   m_i ← 1;
C3          p_i ← l;
C4          if (current() = nil) phase2();
C5          else {
C6              c_i ← current();
C7              send MSG to c_i
C8              for (k ∈ G_i − {p_i} − {c_i}) send VISITED to k;
            }
        }
D1      phase2()
D2      {   send VISITED to p_i;
        }
E1      function current()
E2      {   if (v_i(k) = 1 ∀k ∈ G_i − {p_i}) current() ← nil;
E3          else {
E4              select any m ∈ G_i − {p_i} with v_i(m) = 0;
E5              current() ← m;
            }
        }
```

If all link propagation delays are the same, the time complexity of DFS3 is $2 \mid V \mid$ and the message complexity is $2 \mid E \mid$, the minimum possible. We don't know the complexities for varying propagation delays. I think it can be shown that the performance in better than DFS2.

Problems

Problem 6.0.1 Why isn't one type of messages sufficient in DFS1?

Chapter 7

MINIMUM-WEIGHT SPANNING TREE PROTOCOLS

In this chapter we investigate the Minimum Spanning Tree protocol of Gallager, Humblet and Spira [GHS83].

Protocol MST

Messages

$Connect(Z)$	-
$Initiate(Z, F, S)$	-
$Test(Z, S)$	-
$Accept$	-
$Report(w)$	-
$ChangeRoot$	-

Variables

			GSH notation
s_i	-	state of node i	SN
- $Sleeping$ = initial state			
- $Find$ = participating in search for minimum outgoing edge			
- $Found$ = at other times			
$s_i(l)$	-	state of link (i, l) as seen by i	$SE(m)$
- $Basic$ = unknown yet			
- $Branch$ = edge is branch in MST			
- $Rejected$= edge is non-branch connecting two tree nodes			
F_i	-	fragment identity	
Z_i	-	fragment level	LN
$BestEdge_i$	-	edge pointing towards minimum-weight outgoing edge	
$BestWt_i$	-	weight of minimum-weight outgoing edge	
$TestEdge_i$	-	adjacent edge currently being tested	
p_i	-	branch pointing towards core	$in - branch$
$FindCount_i$	-	Number of *Report* messages to be received	
$w_i(l)$	-	weight of link (i, l)	$w(j)$

Initialization

In the beginning :
$s_i = Sleeping$

Algorithm for node i

```
A1   Node i wakes up
A2   {   wakeup()
     }
B1   wakeup()
B2   {   ∀ adjacent l, set s_i(l) ← Basic;
B3       BestEdge_i ← adjacent edge of minimum weight;
B4       BestWt_i ← w(BestEdge_i) ; Z_i ← 0 ; s_i ← Found ;
B5       FindCount_i ← 0 ; p_i ← nil;                                              @
B6       ChangeRoot() ;                                                            @
     }
C1   receives Connect(Z) on edge j
C2   {   if  (s_i = Sleeping) wakeup() ;
C3       if  (Z < Z_i) {                                  Z ≤ Z_i, see item 3) below
C4           s_i(j) ← Branch;
C5           send Initiate(Z_i, F_i, s_i) on edge j;
C6           if  (s_i = Find)FindCount_i ← FindCount_i + 1;
         }
C7       elseif  (s_i(j) = Basic) place message on end of queue ;wait for until i completes building the tree and ch
C8           else  {
C9               send Initiate(Z_i + 1, w(j), Find) on edge j;
             }
     }
D1   receives Initiate(Z, F, s) on edge j                                          Z > Z_i
D2   {   Z_i ← Z ; F_i ← F; s_i ← s;  p_i ← j ;
D3       BestEdge_i ← nil; BestWt_i ← ∞ ;
D4       ∀m ≠ j such that s_i(m) = Branch {
D5           send Initiate(Z, F, s) on edge m ;
D6           if  (s = Find) FindCount_i ← FindCount_i + 1 ;
         }
D7       if  (s = Find)  test() ;
     }
E1   test()
E2   {   TestEdge_i ← minimum-weight adjacent edge in state Basic ;
E3       send Test(Z_i, F_i) on TestEdge_i ;
E4       report() ;
     }
F1   receives Test(Z, F) on edge j
F2   {   if  (s_i = Sleeping) wakeup() ;
F3       if  (Z > Z_i) place message on end of queue ;          wait to get to higher level
F4       elseif  (F ≠ F_i) send Accept on edge j ;                  what if Z < Z_i ??
F5           else  {                                                    same fragment
F6               if  (s_i(j) = Basic)  s_i(j) ← Rejected;
F7               if  (TestEdge_i ≠ j) send Reject on edge j ;
F8               else  test() ;                                              check this
             }
     }
```

Figure 7.1: The MST protocol

```
G1      receives Accept on edge j
G2      {   TestEdge_i ← nil;
G3          if (w(j) < BestWt_i) {BestEdge_i ← j; BestWt_i ← w(j)}
G4          report();
        }
H1      receives Reject on edge j
H2      {   if (s_i(j) = Basic) s_i(j) ← Rejected;
H3          test();
        }
I1      report()
I2      {   if (FindCount_i = 0 and TestEdge_i = nil) {
I3              s_i ← Found ;
I4              send Report(BestWt_i) on p_i ;
            }
        }
J1      receives Report(w) on edge j          do we need to distinguish p_i = nil vs. p_i ≠ nil ???
J2      {   if (j ≠ p_i){                                                          my fragment
J3          FindCount_i ← FindCount_i − 1;
J4          if (w < BestWt_i) {BestEdge_i ← j; BestWt_i ← w};
J5          report();
            }
J6          elseif (s_i = Find) place message on end of queue;
                                                          at core, wait to get to Found state
J7              elseif (w > BestWt_i)ChangeRoot();
J8                  elseif (w = BestWt_i = ∞) halt;
        }
K1      ChangeRoot()
K2      {   if (s_i(BestEdge_i) = Branch) {
K3          send ChangeRoot on BestEdge_i;                                  towards new core
            }
K4          else {                                                              at new core
K5          send Connect(Z_i) on BestEdge_i ;
K6          s_i(BestEdge_i) ← Branch ;
        }
L1      receives ChangeRoot
L2      {   ChangeRoot();
        }
```

* critical changes

@ cosmetic changes

Initial properties:

1. For every i and adjacent l , $s_i(l)$ is initialized to *Basic* <B2> and can change at most once, either to *Rejected* <F6>,<H2> or to *Branch* <C4>,<K6>.(the latter is not easy to prove, one must prove that the state was not *Rejected* beforehand).

Description of the Protocol

1. *Sleeping* node awakens <B1>:
   (a) minimum-weight adjacent edge is marked as *Branch* <B6>
   (b) message *Connect* is sent over it <B6>
   (c) node goes to state *Found* <B4>
2. Determining minimum-weight outgoing edge from a level Z fragment:

(a) Core nodes start broadcasting $Initiate(Z, F, s)$ messages <C8>

(b) $Initiate(Z, F, Find)$ messages are sent outward on the fragment branches <D5> and to lower level fragments <C5> (in the paper it says layer $(L-1)$, but it's not clear that indeed this is the case in <C3>). For every *Initiate* message sent, a node expects a *Report* message back. The variable $FindCount_i$ keeps track of the number of *Initiate*'s sent minus the number of *Report*'s received.

(c) A node finds its minimum-weight outgoing edge via $Test/Reject/Accept$ messages (see *6*) below).

(d) Nodes collaborate via *Report* messages converging to the core node <I4> to find the minimum-weight fragment outgoing edge. $BestEdge$ is saved and nodes go into $Found$ state.

3. Receipt of *Connect*

(a) A *sleeping* node i can receive *Connect* only if the sending node k has sent it upon its own *wakeup*, otherwise k has previously sent $Test$ to i, which would have woken i up.

(b) Upon entering <C3> holds $Z \leq Z_i$. This is because of the following:

- If $Z_i = 0$, then $Z = 0$ as explained above.
- If $Z_i > 0$, then Z cannot be larger than Z_i because the sending node k has previously sent $Test$ to i and that $Test$ would have been placed on the end of queue in <F3> until Z_i would reach the level of Z.

4. Changing the core

(a) The two core nodes exchange *Report* <I4>. When a core node receives *Report*, it waits, if necessary, to get into state $Found$ <J6>. The core node with the lower $BestWt$, starts <J7> propagation of *ChangeRoot* towards minimum-weight edge <K3>.

(b) When the *ChangeRoot* message reaches the node adjacent to the minimum-weight outgoing edge, the (i, p_i) edges form a rooted tree, rooted at this node.

(c) When the *ChangeRoot* message reaches the node adjacent to the minimum-weight outgoing edge, the node with the minimum-weight outgoing edge sends a $Connect(Z)$ message over this edge <K5>.

(d) If two fragments at level Z have the same minimum-weight outgoing edge, each sends $Connect(Z)$ over the edge, which causes the edge to become a $Z+1$-level core <C9>

5. Connecting a low-level fragment to a high-level one <C3>

(a) Suppose node i in fragment F_i with level Z_i sends a *Connect* message to node i' in fragment F' with level $Z_{i'}$, where $Z_{i'} > Z_i$. When it receives the *Connect* message, node i' responds with an *Initiate* message <C5>.

(b) If i' is in state $Find$, the low-level fragment joins the search for the minimum-weight outgoing edge < C6>. If i' is in state $Found$, we can deduce that an outgoing edge from node n' has a lower weight than the minimum-weight outgoing edge from F_i. Thus there is no need for fragment F_i to join the search. (??? Is it possible that edge (i, i') is the minimum-weight outgoing edge from F_i ? Is there a problem in this case ? Maybe it works, but it is not the normal operation as described in the paper.)

6. Finding minimum-weight outgoing edge at a node (*test*() procedure, <E1>)

(a) starts when node i receives *Initiate* in state $Find$, <D7>, at which time node i sends $Test(Z_i, F_i)$ on its minimum-weight adjacent edge in state $Basic$

(b) At the receiving node, if the node's level Z_i is less than the received level Z, the node delays making any response <F3>.

(c) Otherwise, if the receiving node is not in the same fragment as the sending node, *Accept* is sent back to the sending node <F4>.

(d) Otherwise, the link is put in *Rejected* state and, if $Test$ was not previously sent on this link by the receiving node, the latter sends *Reject* back. (Note: Probably the $Test$ and *Reject* messages can be combined in one, in <F7>, the node can send $Test(Z_i, F_i)$ and the procedure for *Reject* can be deleted.

Chapter 8

MINIMUM-HOP-PATH PROTOCOLS

In previous chapters, we have presented protocols for propagation of information in networks. In particular, we developed in Chap. 5 protocols like Topology and Parameter Broadcast, which require nodes to distibute to all network nodes their local topology and adjacent parameters. The result is that all nodes maintain a map of the entire network, allowing them to participate in routing protocols of the link state type, like the Internet OSPF [Hui95]. Another method for routing in communication networks, including the Internet, is Distance Vector [Hui95]. With this method, nodes do not maintain the entire network topology. They keep only a table of the estimated next-hop and estimated shortest distance to each other destination in the network. By exchanging these table with the neighbors, nodes update their own tables. In the present and the following chapters, we shall discuss such protocols. The present chapter assumes unity-weights on all links, so that we are looking for minimum-hop protocols, while Chapter 9 deals with protocols for networks with variable link weights.

8.1 Protocol MH1

The problem considered next is to obtain the paths with smallest number of links (hops) from each node to each other node. As before, at the beginning of the algorithm a node knows only its own identity and the adjacent links. When the algorithm is completed at a node i, we want the node to know its distance d_i^k in terms of number of links to all other nodes to which it is connected and a preferred neighbor p_i^k through which it has the minimum-hop path to k. Observe that we do not require nodes to know the entire minimum-hop path.

If the travel time of control messages were identical on all links, then we could have accomplished the minimum-hop-path by using protocol $PI1$ (see Theorem 3.1c)). However, as stated before, such an assumption is not practical, and the problem is to design a Distributed Network Protocol where nodes will receive the first message with a given identity from the neighbor providing the shortest path, even if link delays are arbitrary. Such a protocol has been proposed by Gallager [Gal76], [Gal82].

A node enters the algorithm in the same way as in the CT protocols, namely when receiving $START$ or the first control message, at which time it knows its own identity and the identity of all its neighbors, i.e. all nodes that are at distance 0 and 1 from itself. At that time it sends the identity of its neighbors to all neighbors. After having received the identity of the neighbors of all its neighbors, node i knows all nodes

that are at distance 2 from it. Node i keeps the information, sends it to all neighbors and then waits to receive the lists of all nodes that are at distance 2 from each of its neighbors. The union of these lists minus the set of nodes already known to i, i.e. those that are at distance 0,1 or 2 from it, is exactly the set of nodes that is at distance 3 from i. This information is kept again at i and also distributed to neighbors, and the procedure is repeated. If at some level, the union of the lists received from all neighbors contains no nodes that are unknown to i, then node i has completed the algorithm. It sends to all neighbors a message saying that it has no new node identities to send and stops. Any further message it may receive is disregarded.

Protocol MH1

Messages

$MSG(LIST_i)$ - message sent by node i
$START$ - $MSG(\emptyset)$ from nil

Variables

d_i^k - distance from i to k; set initially to $\mid \overline{V} \mid$ for all k (values $0, 1, \ldots \mid \overline{V} \mid$)
p_i^k - preferred neighbor of i for k, for all k
Z_i - state of node i showing distance covered by the protocol up to now (values $0, 1, \ldots, \mid \overline{V} \mid -1$)
m_i - shows if node i is currently participating in the protocol (values $0, 1$)
$N_i(l)$ - level of last message received on link (i, l) (values $0, \ldots, \mid \overline{V} \mid -1$), for $l \in G_i$

Initialization

- just before node i enters the protocol, it has $Z_i = 0$.
- after entering the protocol, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receive MSG(LIST) from l ∈ G_i ∪ {nil}
A2    {   if (Z_i = 0){
A3            initialize();
A4            level();
          }
A5        if (m_i = 1){
A6            update();
A7            if (Z_i ≤ N_i(l') ∀l' ∈ G_i) level();
          }
      }
B1    initialize()
B2    {   for (k ∈ V̄){
B3            d_i^k ← | V̄ |;
B4            p_i^k ← nil;
          }
B5        for (l' ∈ G_i) N_i(l') ← 0;
B6        m_i ← 1;
B7        d_i^i ← 0;
B8        for (k ∈ G_i){
B9            d_i^k ← 1;
B10           p_i^k ← k;
          }
      }
C1    update()
C2    {   N_i(l) ← N_i(l) + 1;
C3        for (k ∈ LIST){
C4            if (d_i^k > N_i(l) + 1){
C5                d_i^k ← N_i(l) + 1;
C6                p_i^k ← l;
              }
          }
      }
D1    level()
D2    {   Z_i ← Z_i + 1;
D3        LIST_i ← {k | d_i^k = Z_i};
D4        for (k ∈ G_i) send MSG(LIST_i) to k;
D5        if (LIST_i = ∅) m_i ← 0;
      }
```

Note: Observe that the variable p_i^k is not needed by the algorithm, and only designates the neighbor corresponding to the minimum hop path to k.

At first glance it seems that we could use the concept of synchronizers of [Awe85a] to prove the properties of Protocol $MH1$. The time when $Z_i \leftarrow n, n = 1, 2, \ldots$ could be taken as the synchronizer time $t_i(n)$. The MSG sent by i to all neighbors at that time could be taken as $SYNCH_n$ in synchronizer α given in [Awe85a]. However, as seen in Lemma 8.2, these sequences of instances do not satisfy a basic property required in [Awe85a], that a MSG sent by i at $t_i(n)$ can arrive at a neighbor k before $t_k(n)$. Therefore there is no synchronizer for this protocol and hence the proof must be carried out directly.

The following definition and theorem summarize the main properties of the protocol.

Definition: The number of links on the minimum-hop path from i to k is called the *hop-distance* from i to k.

Theorem 8.1 *(MH1) Suppose START is delivered to a node (or several nodes) in V. Then every node $i \in V$:*

a) will enter the protocol (i.e. perform <A2>) in finite time;

b) will complete the protocol (i.e. perform <D5>) in finite time, with d_i^k, p_i^k corresponding to the minimum-hop path from i to k for all nodes $k \in V$ and with $d_i^k =\mid \overline{V} \mid$, $p_i^k = nil$ for all nodes $k \notin V$.

Proof: The proof is given in the following two lemmas. The first indicates several preliminary properties of protocol MH1 connected to message exchanges and variable updates, while in the second we use Lemma 8.2 to validate the basic properties of the protocol.

Lemma 8.2 *Suppose START is delivered to a node (or several nodes) in V. Then for any $i \in V$ holds:*
a) i will enter the protocol in finite time;

b) messages are sent by node i if and only if Z_i is incremented at the same time; if MSG is sent by i while $Z_i \leftarrow Z$, receipt of the MSG at neighbor l will cause $N_l(i) \leftarrow Z$;

c) Z_i and $N_i(m)$ for each $m \in G_i$ change only by increments of +1;

d) for each $m \in G_i$, holds $N_i(m) = Z_i$ or $Z_i \pm 1$ and there is at least one m for which $N_i(m) = Z_i - 1$ (note: this implies $Z_i = min_m N_i(m) + 1$);

e) no message can arrive on links (i, m) for which $N_i(m) = Z_i + 1$;

f) if Z_i is incremented at time t, then for all $m \in G_i$ holds $N_i(m)(t+) = Z_i(t+)$ or $Z_i(t+) - 1$.

Proof: Part *a*) holds since propagation of <A2> happens as */phase*1/ in PI1. Assertion *b*) holds since Z_i is incremented whenever MSG is sent (<D2>, <D4>), $N_i(l)$ is incremented whenever MSG is received from l(<C2>) and both are initialized to 0. In addition, *c*) follows from <D2>. Property *d*) is true immediately after the time when node i enters the algorithm, at which time either $Z_i = 1$ and $min_m N_i(m) = 0$, or $Z_i = 2$ and $min_m N_i(m) = 1$, the latter if i has only one neighbor and enters the algorithm by receiving MSG from it. Suppose now that the property is true at node i up to time $t-$ and we want to show that it will hold at time $t+$ as well. The variables $N_i(\bullet)$ or Z_i can change at time t only if a MSG is received, from neighbor l say. Let $Z_i(t-) = Z$. We have several cases:

i) $N_i(l)(t-) = Z - 1$ and $\exists m \neq l$ with $N_i(m)(t-) = Z - 1$; then $N_i(l)(t+) = Z_i(t+) = Z$ and all other $N_i(\bullet)$ do not change, hence d) continues to hold at time $t+$.

ii) $N_i(l)(t-) = Z - 1$ and $\nexists m \neq l$ with $N_i(m)(t-) = Z - 1$; then $N_i(l)(t+) = Z$ and $Z_i(t+) = Z + 1$, since <A7> holds at t, and d) continues to hold at $t+$.

iii) $N_i(l)(t-) = Z$, in which case $N_i(l)(t+) = Z + 1$ and $Z_i(t+) = Z$, hence d) continues to hold at time $t+$.

iv) We claim that $N_i(l)(t-)$ cannot be $Z+1$. Suppose $N_i(l)(t-) = Z+1$. Then $N_i(l)(t+) = Z+2$, and from b) follows that at time $t_1 < t$, node l has sent $MSG(LIST_l)$ while $Z_l = Z + 2$. From <A7>, <D2>,<D4> we have $Z_l(t_1-) = Z + 1$ and $N_l(i)(t_1+) \geq Z + 1$. This means that $\exists t_2 < t_1$ when i has sent $MSG(LIST)$ to l, while $Z_i(t_2+) = Z + 1$. But the latter and $Z_i(t-) = Z$ contradicts the monotonicity of Z_i (see *c*)).

This completes the proof of *d*). Observe now that *e*) is exactly case iv) in *d*). Finally, observe that scanning cases i)-iv) of *d*), we see that Z_i is incremented only in case ii) and *f*) clearly holds in this case, completing the proof of the lemma. qed

Lemma 8.3 *Recall the definition of the term* hop-distance *just before Theorem 8.1. Under the same conditions as in Lemma 8.2 holds:*
a) If a node i has nodes at hop-distance r, then it sets $Z_i \leftarrow r$ in finite time and then sends $MSG(LIST_i)$, where $LIST_i$ contains exactly all nodes k that are at hop-distance r; for all those nodes holds $d_i^k = r$ and p_i^k = first link on minimum-hop path to k and these d_i^k, p_i^k are final.

b) Let S_i be the largest hop-distance from node i in the network, i.e. node i does have nodes at hop-distance S_i, but has no nodes at hop-distance (S_i+1); then node i will set $Z_i \leftarrow (S_i+1)$ in finite time, at which time it sends $MSG(LIST_i)$ with $LIST_i = \emptyset$ and performs <D5>; node i will not increase Z_i any further.

Proof: Setting of $Z_i \leftarrow 1$ while sending $MSG(LIST_i)$ with $LIST_i = \{G_i\}$ propagates as in PI1 and hence will happen at all nodes in finite time. Now suppose *a*) holds for *all* nodes that have nodes at hop-distance $(r-1)$. Consider a node i that has nodes at hop-distance r. Then itself and all its neighbors m have nodes at hop-distance $(r-1)$ and by the induction hypothesis, they set $Z_m \leftarrow (r-1)$ and send $MSG(LIST_m)$. When such a message arrives at i, it sets $N_i(m) \leftarrow (r-1)$ and after all such messages arrive, <A7> will hold with $Z_i = (r-1)$. This causes $Z_i \leftarrow r$. At this time we have from Lemma 8.2f), $N_i(m) = r$ or $(r-1)$ for all m.

Now suppose k is at hop-distance r from i. Then there is a *neighbor* m of i such that k is at hop-distance $(r-1)$ from m and there is no neighbor m of i such that k is at hop-distance strictly less than $(r-1)$ from m. By the induction hypothesis, k was sent by m in $MSG(LIST_m)$ while $Z_i \leftarrow (r-1)$ and hence was received at i while $N_i(m) \leftarrow (r-1)$, but was sent by no neighbor m' while $Z_{m'} \leftarrow Z < (r-1)$. Hence at the time $Z_i \leftarrow r$ we have $d_i^k = r$, and therefore k is sent in $MSG(LIST_i)$. From <C5>,<C6> it is clear that this d_i^k and the corresponding p_i^k are final and correct. A similar argument shows that nodes at hop-distance other than r cannot be included in the $LIST_i$ considered above. This completes the proof of *a*).

First consider a node i s.t. $S_i = min\{S_j\}$ where the min is over all nodes in the network. All its neighbors m have nodes at distance S_i and by *a*) they send $MSG(LIST_m)$ while $Z_m \leftarrow S_i$. When all these messages arrive to i, Z_i will become S_i+1, but since i has no nodes at hop-distance S_i+1, holds $LIST_i = \emptyset$ and hence i performs <D5>. Now suppose by induction that *b*) holds for all nodes i for which $S_i \leq S-1$. Consider a node j with $S_j = S$. Node j has a node k at hop-distance S and k is included in $LIST_j$ when j sends $MSG(LIST_j)$ while $Z_j \leftarrow S$. We need to show that Z_j will eventually take on value $(S+1)$. First we show that for all neighbors m of j, Z_m will become S. For an arbitrary neighbor m of j, node k is at hop-distance $(S-1)$, S or $(S+1)$ from m and hence $S_m \geq S-1$. If $S_m \geq S$, then *a*) implies that Z_m will become S in finite time. If $S_m = S-1$, then Z_m will become S in finite time from the induction hypothesis. Hence from Lemma 8.2b), $N_j(m)$ will become S in finite time for all neighbors m of j and hence Z_j will become $(S+1)$. Since j has no nodes at hop-distance $(S+1)$, <D5> will hold and this completes the proof of the lemma. qed

Now, Lemma 8.2a) and Lemma 8.3a),b) are exactly Theorem 8.1 and this completes the proof of the Theorem.

Communication cost: From the proof of Theorem 8.1 follows that the identity of every node travels exactly once on each link, and hence we need $\mid V \mid log_2 \mid \overline{V} \mid$ bits on each link in each direction, for a total of $2 \mid E \mid\mid V \mid log_2 \mid \overline{V} \mid$ bits.

Time complexity = $O(\mid V \mid^2)$

Computation ????

Problems

Problem 8.1.1 * Wrong Initial Conditions

Problem 8.1.2 Consider a network with nodes a, b, c, d, s and links and delays (in both directions) as follows:

Link	a,b	a,c	a,s	b,c	b,d	c,d	c,s	d,s
Delay	3	5	3	3	2	2	1	6

The nodes in this network perform MH1. Node s receives START at $t = 0$.

1. Give the sequence of messages received at s, and the sequence of messages sent by s.

2. When does the protocol complete?

3. Give the tables at each node when the protocol completes?

Problem 8.1.3 Suppose that after an execution of the MH1 protocol is completed at all nodes, a new link goes up between nodes i and j (the network was connected before this event).

a) Explain and prove how can i and j exchange only one message to update their tables (namely, to update $p_i^k, d_i^k, p_j^k, d_j^k$ for all $k \in V$.

b) Under what condition on the new and old tables of i and j are the tables of all other nodes correct in spite of the link addition?

Problem 8.1.4 a) Give an upper bound on the number of messages a node can receive after performing <D5>. What can be said on their contents? Are the LIST's of those messages necessarily empty?

b) Prove or give counterexample: Every node receives at least one MSG after performing <D5>.

8.2 Extending MH1 to changing topologies

CHANGE SECTION NAME | check Lamport, Time,Clocks,.. 1978 | As with CT protocols, MH1 requires specified initial conditions and therefore its extension to handle topological changes must include reinitialization after every such change. This is implemented as in Sec. 4.6 by restarting a new cycle of the protocol after every topological event. The cycles of the protocol will be labeled with increasing numbers, every node remembers the highest cycle number known to it so far and each of the cycles corresponds now to the original (nonextended) protocol. When a node wants to trigger a new cycle due to an adjacent topological event, it resets its variables, increments the cycle number and acts as if it has received START for a new cycle with this number. Here resetting variables means to adjust the appropriate variables to their required initial value as stated in the corresponding assumption in each of the protocols (e.g. in $MH1$, $p_i^k \leftarrow nil$, $d_i^k \leftarrow \mid \overline{V} \mid$ for all k and $Z_i \leftarrow -1$, $N_i(m) \leftarrow -1$ for all $m \in G_i$). The number of the new cycle will be carried by all messages belonging to this cycle and now, any node receiving a message with cycle numbers lower than the one known to it so far discards this message. A node receiving a message with higher cycle number than the highest known to it, resets its own variables, increases its registered maximal cycle number accordingly and acts as if it enters the protocol now (i.e. the corresponding cycle of the extended protocol). In this way the cycle with higher number will cover the lower-number cycles, in the sense that when a higher cycle reaches any node, the node will forget the previous knowledge and will participate only in the most recent cycle. Observe that several nodes may start the same new cycle independently because of multiple topological events, but the protocol allows this situation to happen, considering it in the same way as if several nodes receive START in the nonextended protocol.

???? There is a question, whether it is indeed necessary for all nodes to forget their entire previous knowledge, as opposed to protocols where only the information affected by the topological change is discarded, while the rest of the network adapts smoothly to the new situation. For the PU protocol, such a protocol appears in [xxx], [yyy], [zzz], but for the others this is still an open question. ?????

As an example, we shall write exactly the extended MH1 protocol.

Protocol EMH1

Messages

$MSG(R, LIST)$ message

Variables

G_i - set of neighbors of i, i.e. $k \in G_i$ if (i,k) is in Connected state at i
d_i^k - distance from i to k
p_i^k - preferred neighbor from i to k for all k
Z_i - state of node i showing distance covered by the protocol up to now (values $0, 1, \ldots, \mid \overline{V} \mid -1$)
m_i - shows if node i is currently participating in the protocol (values 0,1)
$N_i(l)$ - level of last message received on link (i,l) (values $0, \ldots, \mid \overline{V} \mid -1$), for $l \in G_i$
R_i - highest sequence number known to i (values: 0,1, ...);

Algorithm for node i

A1 node i becomes operational
A2 { $R_i \leftarrow 0$;
}
B1 link (i,l) enters *Connected* state or *Initialization* Mode
B2 { $R_i \leftarrow R_i + 1$; /* enter protocol */
B3 $Z_i \leftarrow 0$;
B4 $initialize()$;
B5 $level()$;
}
C1 receives $MSG^j(R,\Lambda)$ from $l \in G_i$
C2 { ***if*** $(R \geq R_i)\{$
C3 ***if*** $(R > R_i)\{$
C4 $R_i \leftarrow R$; /* enter protocol */
C5 $Z_i \leftarrow 0$;
C6 $initialize()$;
C7 $level$;
}
C8 ***if*** $(m_i = 1)\{$
C9 $update()$;
C10 ***if*** $(Z_i \leq N_i(l')\ \forall l' \in G_i)\ level()$;
}
}
}
D1 $initialize()$
D2 { ***for*** $(k \in \overline{V})\{$
D3 $d_i^k \leftarrow \mid \overline{V} \mid$;
D4 $p_i^k \leftarrow nil$;
}
D5 ***for*** $(l' \in G_i)\ N_i(l') \leftarrow 0$;
D6 $m_i \leftarrow 1$;
D7 $d_i^i \leftarrow 0$;
D8 ***for*** $(k \in G_i)\{$
D9 $d_i^k \leftarrow 1$;
D10 $p_i^k \leftarrow k$;
}
}
E1 $update()$
E2 { $N_i(l) \leftarrow N_i(l) + 1$;
E3 ***for*** $(k \in LIST)\{$
E4 ***if*** $(d_i^k > N_i(l) + 1)\{$
E5 $d_i^k \leftarrow N_i(l) + 1$;
E6 $p_i^k \leftarrow l$;
}
}
}
F1 $level()$
F2 { $Z_i \leftarrow Z_i + 1$;
F3 $LIST_i \leftarrow \{k \mid d_i^k = Z_i\}$;
F4 ***for*** $(k \in G_i)$ send $MSG(R_i, LIST_i)$ to k;
F5 ***if*** $(LIST_i = \emptyset)\ m_i \leftarrow 0$;
}

Note that <B1> and <C3> here correspond to <A2> in MH1. The properties of EMH1 are:

Theorem 8.4 *(EMH1) Consider an arbitrary* finite *sequence of topological events with arbitrary timing and location and let (E,V) denote a connected subnetwork in the final topology. Then there is a finite time after the sequence is completed after which no messages travel in (V,E) and all nodes $i \in V$ will have $m_i = 1$ with the same cycle number R_i, with correct d_i^k and p_i^k for all $k \in V$ and with $d_i^k = \mid \overline{V} \mid$, $p_i^k = nil$ for all $k \notin V$.*

Proof: From <B2>, each topological event increments the cycle counter R_i at the node i adjacent to the

change. Let $\{i_n\}$ be the collection of nodes that register change of status of an adjacent link, and let $\{t_n\}$ be the corresponding collection of times when the status change is registered. Since there is a finite number of topological events, the collections $\{i_n\}$, $\{t_n\}$ are finite. Let $R = \max\{R_{i_n}(t_n+)\}$ over all n. Then R is the highest cycle number ever known in the network and the cycle with number R is started by (one or more) nodes $i \in \{i_n\}$ that increment their R_i to R as a result of a topological event. These nodes can be considered as if they receive START in the MH protocol and, indeed, the network covered by the cycle with number R registers no more topological events, since no counter number R_i is ever increased to $(R+1)$. Moreover, from the Follow-up property of DLC follows that in the final topology, $l \in G_i$ if and only if $i \in G_l$, so that the assumption of bidirectionality (Assumption a) in Sec. 3.1) holds in the final topology. Consequently, the evolution of the cycle with sequence number R is the same as in protocol MH1 and therefore Theorem 8.1 holds here, completing the proof.

Problems

Problem 8.2.1 In a network with all delays constant and equal to 1, can we use bounded sequence numbers in EMH1?

Problem 8.2.2 Does EMH1 work in a network where all parts of *data reliability* properties, except for *crossing*, hold?

Problem 8.2.3 Consider a network with nodes a, b, c, d, e, links and delays (in both directions) as follows:
a,b - 3
a,c - 5
a,e - 3
b,c - 3
c,d - 2
c,e - 4
d,e - 2
Suppose all nodes come up and all links enter connected state at time $t = 0$, and afterwards the following happen :
At time $t = 5$ link (a, b) fails and both ends enter Initialization state.
At time $t = 9$ link (e, c) enters Initialization state at node e.
At time $t = 10$ link (e, c) enters Initialization state at node c, and link (a, b) enters connected state at node a.
At time $t = 12$ link (e, c) enters connected state at node c, and link (a, b) enters connected state at node b.
At time $t = 15$ link (e, c) enters connected state at node e and no more topological changes happen afterwards.
The nodes perform EMH1:

1. Indicate the values of the variables $(G_i, d_i^k, p_i^k, Z_i, R_i)$ as a function of time at each node.
2. At what time are there no messages traveling in the network?
3. What is the highest sequence number reached by the network? Suggest other timings for the same topological changes in order to get a higher maximal sequence number, suggest timings to get a lower maximal sequence number.

Problem 8.2.4 Specify the code of ECT5 (protocol CT5 extended to handle topological changes using sequence numbers).

8.3 Another Version (MH2)

A slight change in Protocol MH1 [Gal82] reduces the time complexity from $O(\mid V \mid^2)$ to $O(\mid V \mid)$, without affecting the communication cost. Instead of collecting the identities of all nodes at a given distance and send them in one message, we send the identities in separate messages as they become available. To enable neighbors to distinguish between levels, after having sent all identities of nodes at a given distance, a node sends a $SYNCH$ message to all neighbors.

Protocol MH2

Messages

$MSG(k)$ - message sent by node i with identity of node k
$SYNCH$ - message designating beginning of new level
$START$ - $SYNCH$ from nil

Variables

d_i^k - distance from i to k; set initially to $\mid \overline{V} \mid$ for all k (values $0, 1, \ldots \mid \overline{V} \mid$)
p_i^k - preferred neighbor of i for k for all k
Z_i - state of node i showing distance covered by the protocol up to now (values $-1, 0, 1, \ldots, \mid \overline{V} \mid -1$)
m_i - shows if node i is currently participating in the protocol (values $0, 1$)
$N_i(l)$ - level of last message received on link (i, l) (values $-1, 0, \ldots, \mid \overline{V} \mid -1$), for $l \in G_i$
L - shows if messages MSG have been sent since Z_i was last incremented

Initialization

- just before node i enters algorithm, holds $Z_i = -1$
- after entering the protocol, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

```
A1    receive SYNCH from l ∈ G_i ∪ {nil}
A2    {   if (Z_i = −1){
A3            initialize();
A4            level();
          }
A5        if (m_i = 1){
A6            N_i(l) ← N_i(l) + 1;
A7            if (Z_i ≤ N_i(l') ∀l' ∈ G_i) level();
          }
      }
B1    receive MSG(k) from l ∈ G_i ∪ {nil}
B2        if (m_i = 1) update();
      }
C1    initialize()
C2    {   for (k ∈ V̄){
C3            d_i^k ←| V̄ |;
C4            p_i^k ← nil;
          }
C5        for (l' ∈ G_i) N_i(l') ← −1;
C6        m_i ← 1;
C7        L ← 1;
C8        d_i^i ← 0;
C9        for (k ∈ G_i){
C10           d_i^k ← 1;
C11           p_i^k ← k;
          }
      }
D1    update()
D2    {   if (d_i^k > N_i(l) + 1){
D3            d_i^k ← N_i(l) + 1;
D4            p_i^k ← l;
          }
D5        if (d_i^k = Z_i + 1){
D6            for (l' ∈ G_i) send MSG(k) to l';
D7            L ← 1;
          }
      }
E1    level()
E2    {   Z_i ← Z_i + 1;
E3        for (k ∈ G_i) send SYNCH to k;
E4        if (L = 0) m_i ← 0;
E5        else L ← 0;
E6        for (k' | d_i^k' = Z_i + 1){
E7            for (l' ∈ G_i) send MSG(k') to l';
E8            L ← 1;
          }
      }
```

Problems

Problem 8.3.1 How do Lemmas 8.2 and 8.3 change when proving MH2?

8.4 The Fixed Topology Distributed Bellman-Ford Minimum Hop Protocol (MH3)

This protocol establishes minimum hop paths from all nodes in the network to a given destination s. A node i keeps an estimate d_i of its shortest path to the destination and estimates $D_i(l)$ of the shortest path via each neighbor l. When the estimate d_i changes, i sends a message containing the new estimate to all neighbors. When i receives a message from a neighbor l, it updates its estimate $D_i(l)$ of the shortest path via that neighbor and its estimate d_i of its shortest path to the destination. If the new d_i is different from the old one, a message containing the new value is sent to all neighbors. The distance from a node i to s can be no larger than $\mid \overline{V} \mid -1$, where $\mid \overline{V} \mid$ is the maximum number of nodes potentially in the network. It is shown that a finite time after the protocol is started, it terminates. At that time, all nodes in V have correct estimated distances: if $s \in V$, then all $d_i \leq \mid \overline{V} \mid -1$ and are correct; if $s \notin V$, then all $d_i = \mid \overline{V} \mid$. In practice, the protocol is repeated independently for every destination s. Also, to save overhead, messages belonging to several protocols (for different destinations) may be combined in one message.

This is the ARPA-1 routing protocol [MW77], specialized to the case when link weights are 1, except that the updates are performed on an event driven basis rather than periodically. It is also the fixed topology part of the [1] MERIT network routing protocol [Taj77].

Protocol MH3

Messages

$MSG(d)$ - message sent by node i, containing i's estimated distance to s, (values $0, 1, \ldots, \mid \overline{V} \mid$)

Variables

d_i - estimated distance from i to s (values $0, 1, \ldots, \mid \overline{V} \mid$)
p_i - preferred neighbor of i
$D_i(l)$ - estimated distance from i to s via neighbor l, (values $0, 1, \ldots, \mid \overline{V} \mid$)

Initialization

holds $d_s = 0$ and for all $i \neq s$ and $l \in G_i$, the variables d_i and $D_i(l)$ satisfy:

a) $d_i = \min D_i(l')$ over $l' \in G_i$
b) $D_i(l) = \min (d_l + 1, \mid \overline{V} \mid)$ or there is at least one MSG on (l, i) and the last $MSG(d)$ on (l, i) contains $d = d_l$

Note: an example of a set of variables and messages that satisfy the above is: $d_i = D_i(l) = \mid \overline{V} \mid$ for all $i \neq s$ and all $l \in G_i$, there is a $MSG(0)$ on all links (s, l), for all $l \in G_s$ and there is no MSG after it.

Algorithm for node s

```
A1      do nothing
```

Algorithm for node i ≠ s

```
B1      receive MSG(d) from l ∈ G_i
B2      {   D_i(l) ← min(d + 1, | V̄ |);
B3          update();
        }
C1      update()
C2      {   k* ← node that achieves minD_i(l') over l' ∈ G_i;
C3          if (d_i ≠ D_i(k*)) {
C4              p_i ← k*;
C5              d_i ← D_i(k*);
C6              for (k ∈ G_i) send MSG(d_i) to k;
            }
        }
```

[1] introduce periodical updates

Lemma 8.5 *At all times holds $d_s = 0$ and for all $i \neq s$ and $l \in G_i$, the variables d_i and $D_i(l)$ satisfy:*
a) $d_i = min\ D_i(l')$ over $l' \in G_i$

b) $D_i(l) = min\ (d_l + 1, |\ \overline{V}\ |)$ or there is at least one MSG on (l,i) and the last $MSG(d)$ on (l,i) contains $d = d_l$

c) all variables $d_i, D_i(k)$ take on values between 0 and $|\ \overline{V}\ |$; all messages $MSG(d)$ contain $0 \leq d \leq |\ \overline{V}\ |$

Proof: Note that d_s stays 0 forever and *a*) and *c*) are obviously correct. To prove part *b*), consider a node i and some $l \in G_i$. Recall from assumption a) in Sec. 3.1 that $l \in G_i$ if and only if $i \in G_l$. Then *b*) is correct at initialization by assumption and if l changes its estimated distance d_l, it sends a $MSG(d_l)$ to all nodes in G_l, and in particular on (l,i). By FIFO, unless l changes again its d_l, this is the last MSG on (l,i), until it arrives at i, in which case the latter sets $D_i(l) = \min\ (d_l + 1, |\ \overline{V}\ |)$. qed

Theorem 8.6 *(MH3) There is a finite time after which no messages travel in (V,E) and: i) if $s \in V$, then all nodes $i \in V$ have $d_i =$ shortest hop distance to s and $p_i =$ first link on the minimum hop path to s; ii) if $s \notin V$, then all nodes $i \in V$ have $d_i = |\ \overline{V}\ |$.*

Proof: We prove the theorem via several lemmas.

Lemma 8.7 *If all message activity ceases, then the $d_i, D_i(l), p_i$ entries are correct for all $i \in V$ and all $l \in G_i$.*

Proof: Suppose first that $s \in V$. For $i \in V$, let

$d_i^* =$ shortest hop distance from i to s

$K =$ set of nodes in V for which $d_i < d_i^*$

$j =$ node in K with minimum d_i, i.e. holds $d_j \leq d_i, \forall i \in K$

Since there are no messages on the links, Lemma 8.5 implies that $d_j = D_j(p_j) = d_{p_j} + 1$, hence $p_j \notin K$, so $d_{p_j} \geq d_{p_j}^*$. But since j and p_j are neighbors, the triangle inequality implies that $d_j^* \leq d_{p_j}^* + 1$. Hence, $d_j = d_{p_j} + 1 \geq d_{p_j}^* + 1 \geq d_j^*$, contradicting the fact that $j \in K$. Therefore K is empty.

Now let

$K' =$ set of nodes in V for which $d_i > d_i^*$

$j =$ node in K' closest to s, i.e. holds $d_j^* \leq d_i^*, \forall i \in K'$

$j* =$ the next neighbor of j on the minimum hop path to s.

Holds $d_j^* = d_{j*}^* + 1$. Since $j*$ is closer to s than j and hence $j* \notin K'$, holds $d_{j*} \leq d_{j*}^*$ (in fact, since we have shown already that K is empty, the latter holds with equality). Moreover, d_j is selected as the minimum of $D_j(l)$ over all neighbors l of j and $D_j(j*) = d_{j*} + 1$. Therefore, holds $d_j \leq d_{j*} + 1$. Hence $d_j \leq d_{j*} + 1 \leq d_{j*}^* + 1 = d_j^*$, contradicting the fact that $j \in K'$. Therefore K' is also empty and therefore all nodes have correct entries.

Now suppose that $s \notin V$. Suppose that message activity had ceased and $\exists i \in V$ such that $d_i <| \overline{V} |$. Let j be the node in V with minimum d_i, i.e. $d_j \leq d_i\ \ \forall i \in V$. From Lemma 8.5 follows that there is a neighbor k of j, such that $d_k + 1 = D_j(k) = d_j <| \overline{V} |$. This means that $d_k < d_j$, contradicting the fact that j is the node with minimum d_i. qed

Lemma 8.8 *Let $MSG(d_i')$, $MSG(d_i)$ be two consecutive messages sent out by node i, the second as a result of receiving $MSG(d)$. Then either $d_i = d + 1$ or $d_i > d_i'$ (or both).*

Proof: Message $MSG(d_i)$ is sent out only if at the same time d_i changes. If d_i is decreased, then the new minimum is $d + 1$. If it is increased, then d_i' is the value just before the increase. qed

Lemma 8.9 *Message activity ceases in finite time.*

Proof: Messages $MSG(d)$ with $d = 0$ can only exist on links at initialization. Hence there is only a finite number of such messages. Suppose a node i sends out an infinite number of messages $MSG(1)$. By Lemma 8.8, every time it does that except maybe for the first time, it either receives a message $MSG(0)$ or the previous message sent out by i had $d < 1$, i.e. it had $d = 0$. Hence, node i either sends or receives an infinite number of messages with $d = 0$, contradiction. It is shown in the same way by induction that there are only a finite number of messages with $d = 2, 3, ..., \mid \overline{V} \mid$. Since there is only a finite number of possible values of d, there is only a finite number of messages in the network[2]. qed

Theorem 8.6 follows from Lemmas 8.7 and 8.9. qed

Problems

Problem 8.4.1 The proof of Lemma 8.7 consists of two steps : in the first step, the node j that is used to prove that K is empty is defined as the node in K with minimum d_i; in the second step, the node j' that is used to show that K' is empty is defined as the node in K' closest to s. Explain why we cannot use for j' the same definition as in the first step, namely $j' =$ the node in K' with minimum d_i.

Problem 8.4.2 Does MH3 still work properly if the FIFO property of DLC does not hold?

Problem 8.4.3 Give examples where if the required initial conditions of MH3, do not hold, the protocol does not work.

[2]Message complexity????
Time complexity????

8.5 The Changing Topology Distributed Bellman-Ford Minimum-Hop Protocol (EMH3)

One of the attractive properties of the Bellman-Ford minimum hop distributed protocol is that the changing topology version needs no reinitialization after every topological event, as other protocols, like MH1 and the Dijkstra distributed protocols do. This is due to the fact that the fixed topology versions work with quite general initial conditions. The only requirements are that at initialization, d_i minimizes the entries $D_i(l)$ and, in addition, either $D_i(l)$ reflects the minimum distance d_l or there is a message on the link (l,i) that reflects that distance and that is the the last message on (l,i). Therefore, proper operation of the changing topology version is ensured if the latter preserves those properties after every topological event and operates identically to the fixed topology version in a fixed topology network. As in the fixed topology version, we specify the algorithm for each destination separately. In practice, the algorithm is performed for all destinations in parallel.

Protocol EMH3

Messages

$MSG(d)$ - message sent by node i, containing i's estimated distance to s

Variables

G_i - set of neighbors, i.e. $l \in G_i$ if (l,i) is in *Connected* state at i
d_i - estimated distance from i to s (values $0, 1, \ldots \mid \overline{V} \mid$)
p_i - preferred neighbor of i
$D_i(l)$ - estimated distance from i to s via neighbor l, (all $l \in G_i$)

Algorithm for node s

```
A1      Node s becomes operational
A2      {   d_s ← 0;
        }
B1      Link (s,l) enters Connected state
B2      {   send MSG(0) to l;
        }
```

Algorithm for node $i \neq s$

```
C1    Node i becomes operational
C2    {   d_i ←| V |;
      }
D1    Link (i,l) enters Connected state
D2    {   D_i(l) ←| V |;
D3        send MSG(d_i) to l;
      }
E1    Link (i,l) enters Initialization mode
E2    {   if (l = p_i) update_fail();
      }
F1    receives MSG(d) from l ∈ G_i
F2    {   D_i(l) ← min(d + 1, | V |);
F3        update();
      }
G1    update()                                        /* same as in MH3 */
G2    {   k* ← node that achieves minD_i(l') over l' ∈ G_i;
G3        if (d_i ≠ D_i(k*)) {
G4            p_i ← k*;
G5            d_i ← D_i(k*);
G6            for (k ∈ G_i) send MSG(d_i) to k;
          }
      }
H1    update_fail()
H2    {   k* ← node that achieves min D_i(l') over l' ∈ G_i;
H3        p_i ← k*;
H4        if (d_i ≠ D_i(k*)) {
H5            d_i ← D_i(k*);
H6            for (k ∈ G_i) send MSG(d_i) to k;
          }
      }
```

Lemma 8.10 *At all times holds $d_s = 0$ and for all $i \neq s$ and l such that $l \in G_i$ and $i \in G_l$ (i.e. $(i,l) \in E$), the variables d_i and $D_i(l)$ satisfy:*

a) $d_i = \min D_i(l')$ over $l' \in G_i$

b) $D_i(l) = \min(d_l + 1, | \overline{V} |)$ or there is at least one MSG on (l,i) and the last $MSG(d)$ on (l,i) has $d = d_l$.

c) messages $MSG(d)$ always contain $0 \leq d \leq | \overline{V} |$

Proof: Note that d_s stays 0 forever and that *a*) and *c*) are obviously correct. To prove part *b*), consider a node i and some l such that $l \in G_i$ and $i \in G_l$, at some time t. Let t_l, t_i denote the last time before t when respectively i joined G_l and l joined G_i, namely the last time when (i,l) entered Connected state at l and i respectively. At time t_l, node l sends to i a message $MSG(d_l)$ (see Fig. 8.1). If $t_l > t_i$, then at time $\max(t_l+, t_i+) = t_l+$, this is the last and only MSG on (l,i), therefore *b*) holds at that time. On the other hand, if $t_i > t_l$, then by the Crossing property of DLC, (i,l) was in Initialization Mode at i on the entire interval (t_l, t_i), so that the MSG sent by l to i at time t_l and all possible subsequent MSG's due to possible changes of d_l have not been delivered to i by time t_i. Therefore, by the Confirm property of DLC, those MSG's have not yet been acknowledged and thus are still on the link (l,i). Therefore *b*) holds at $\max(t_l+, t_i+) = t_i+$. If subsequently, l changes its estimated distance d_l, it sends a $MSG(d_l)$ to all its neighbors, and in particular on (l,i). By FIFO, the last MSG on (l,i) carries d_l, until it arrives at i, in which case the latter sets $D_i(l) = \min(d_l + 1, | \overline{V} |)$. qed

Theorem 8.11 *(EMH3) Consider an arbitrary* finite *sequence of topological events with arbitrary timing and location and let (V,E) denote a connected subnetwork in the final topology. Then there is a finite time after the sequence is completed after which no messages travel in (V,E) and:*

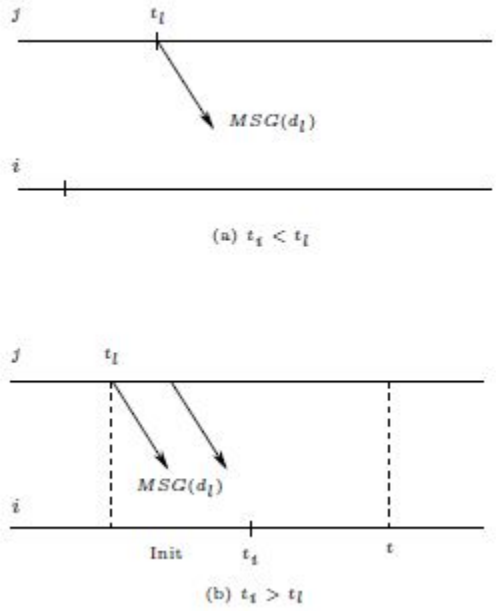


Figure 8.1: Diagram for proof

a) if $s \in V$, then all nodes $i \in V$ have d_i = shortest hop distance to s and p_i = first link on the minimum hop path to s;

b) if $s \notin V$, then all nodes $i \in V$ have $d_i =\mid \overline{V} \mid$.

Proof: The properties of Lemma 8.10 hold at all times and in particular immediately after the last time $\hat{t}$ when <A1>,<B1>,<C1>, <D1> or <E1> occur. After that time only <F1>-<G6> are operational, which is exactly Protocol $MH3$, on a fixed topology network. From the Follow-up property of DLC follows that in the final topology, $l \in G_i$ if and only if $i \in G_l$, so that the bidirectionality assumption 3.1 in Sec. 3.1 holds in the final topology. Since by Lemma 8.10, the initialization conditions are as required in Sec. 8.4, Protocol $EMH3$ behaves after time $\hat{t}$ exactly as $MH3$ and therefore has the same convergence properties. qed

We have here the opportunity to demonstrate for the first time for a nontrivial case that the DLC properties are essential for ensuring proper operation of the higher-level protocols. It is trivial to indicate here, as well as in the fixed topology protocols of the previous chapters, situations where if FIFO or Delivery do not hold, the Network Protocol does not work. The following example shows that the same is true for the Crossing property. Consider the network of Fig. 8.2 and assume that the protocol has converged. Suppose that link (b, a) enters Initialization Mode at b at time t_1, returns to Connected state at b at time t_2 and node a never enters Initialization Mode for link (b, a). At time t_1 node b sets $d_b = 3$ and sends $MSG(3)$ to c. At t_2 it sets $D_b(a) =\mid \overline{V} \mid$ and sends $MSG(3)$ to a. During the entire protocol, node a sends nothing to node b. When node a receives the message, it sets $D_a(b) = 4$, and since d_a does not change, it sends no message. Consequently, in steady state, node b will have $D_b(a) =\mid \overline{V} \mid$, $d_b = 3$ and $p_b = c$, which is incorrect.

Problems

Problem 8.5.1 Give an example to show that Protocol $EMH3$ with line <D3> removed does not work.

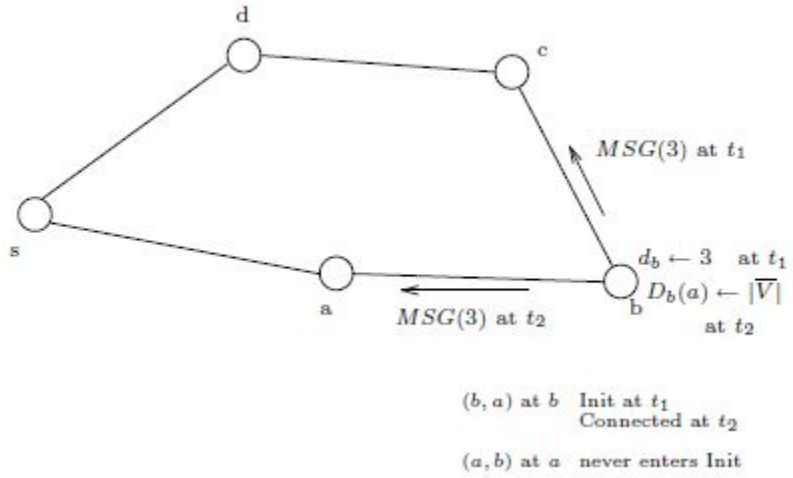


Figure 8.2: Example for Crossing

Problem 8.5.2 We gave above an example where Crossing is necessary in order for EMH3 to work. Give examples where other DLC properties are necessary in order for EMH3 to work.

Problem 8.5.3 Describe a situation where if FIFO of DLC does not hold, a Network Protocol does not work after some topological change.

Chapter 9

PATH-UPDATING PROTOCOLS

In the protocol of [SMG78], [MS79b], each node maintains a path to each other node in the network and updating cycles allow these paths to be changed so that they are improved in each cycle and, in addition, the collection of paths to any given node form at any given time a loop-free pattern (i.e. a tree). Here we present first the fixed-topology part of the path-updating protocol and then show that protocol $CT2$ can be used to initialize it.

9.1 Protocol PU1

The protocol updates paths from all nodes in the network to a given node s in terms of some possibly time varying link weights $\{d_{il}\}$ and can be repeated independently to update paths to each of the other nodes. Therefore, we can present only the protocol for a given destination node s. The protocol is very similar to the PIF protocol, except for two features: first, a tree is initially available and the protocol moves first up and then down on that tree, and second, when moving downtree, the protocol updates the initial tree, so that the resulting paths provide an improvement over the old ones.

Protocol PU1

Messages

$MSG(d_i)$ - message carrying the estimated distance from i to s, MSG from nil to s contains $d_s = 0$

Variables

G_i - set of neighbors of i
m_i $= 1$ after performing $phase1()_i$ and before performing $phase2()_i$; $= 0$ otherwise
$e_i(l)$ - number of MSG's sent to l - number of MSG's received from l, for all $l \in G_i$
d_{il} - distance from node i to neighbor l as measured at the time it is needed by the algorithm; can be time-varying (values: any *strictly positive* real number), $l \in G_i$; $d_{s,nil} \equiv 0$
d_i - estimated distance from i to s on the preferred path
p_i - preferred neighbor of i for s
$D_i(l)$ - storage for $d_l + d_{il}$, for $l \in G_i$

Initialization

We use superscript 0 to denote values of variables just before $START$ is delivered to s. Then all connected nodes i have:

a) p_i^0, d_i^0 with the property that the collection of links (i, p_i^0) form a directed tree rooted at s and also $d_i^0 > d_{p_i^0}^0$, i.e. d_i^0 is strictly decreasing while moving downtree.
b) $m_i^0 = 0, e_i^0(l) = 0$ for all $l \in G_i$.

Algorithm for node i

A1 receive $MSG(d)$ from $l \in G_i \cup \{nil\}$
A2 { $e_i(l) \leftarrow e_i(l) - 1$;
A3 $D_i(l) \leftarrow d + d_{il}$;
A4 ***if*** $(l = p_i)$ $phase1()$;
A5 ***if*** $(e_i(k) = 0 \ \ \forall k \in G_i - \{p_i\})$ $phase2()$;
}
B1 $phase1()$
B2 { $m_i \leftarrow 1$;
B3 $d_i \leftarrow \min D_i(l')$ over $\{l' \mid e_i(l') = -1\}$;
B4 ***for*** $(k \in G_i - \{p_i\})$ {
B5 send $MSG(d_i)$ to k;
B6 $e_i(k) \leftarrow e_i(k) + 1$;
}
}
C1 $phase2()$
C2 { $d_i \leftarrow \min D_i(l')$ over $l' \in G_i$;
C3 send $MSG(d_i)$ to p_i;
C4 $e_i(p_i) \leftarrow e_i(p_i) + 1$;
C5 $p_i \leftarrow$ node that achieves $\min D_i(l')$ over $l' \in G_i$;
C6 $m_i \leftarrow 0$;
}

Theorem 9.1 *(PU1)*

Suppose the Initialization assumptions a) and b) given in the protocol hold. Then:

a) all nodes $i \in V$ will perform the event $phase1()_i$ in finite time and exactly once; in addition, for all i holds $t(phase1()_i) > t(phase1()_{p_i^0})$.

b) all nodes $i \in V$ will perform $phase2()_i$ in finite time and exactly once; moreover $t(phase2()_i) < t(phase2()_{p_i^0})$; node i receives no messages after time $t(phase2()_i)$; also, at the time when node s performs $phase2()_s$, all nodes in V have completed the algorithm, i.e. have performed $phase2()$, and there are no messages traveling in the network.

c) exactly one MSG travels on each link in (V, E) in each direction.

d) The collection of links $\{(i, p_i)\}$ forms at all times a tree rooted at s with the following properties:

(i) $m_i \leq m_{p_i}$

(ii) *if $m_i = m_{p_i} = 0$, then $d_i > d_{p_i}$.*

e) For each link (i, l) the distance d_{il} is measured exactly once by node i; at the end of the protocol, all nodes will have paths to s that are no longer than before the protocol starts, where the length of a path is the sum of the weights of the links in terms of the measured $\{d_{il}\}$; if the initial tree T^0 defined by $\{(i, p_i)\}$ is not identical to the shortest-path-tree in terms of the measured $\{d_{il}\}$, then there is a nonempty set of nodes that did not have optimal paths in the initial tree and do have optimal paths in the new tree T^1.

Proof: Observe that the present protocol is identical to $PIF2$, except that $phase1()$ is performed by a node i only when MSG is received from p_i (and not as soon as the first MSG is received, as in $PIF2$), the new quantities d_i, $D_i(l)$, d_{il} are introduced and the preferred neighbor p_i is changed in $phase2()$. Now, $phase1()$ and $phase2()$ propagate here exactly as in $PIF2$, provided that in that protocol a MSG traverses any link in T^0 much faster than any other link. Since Theorem 3.5 holds for arbitrary link travel times, assertions *a*), *b*), *c*) follow.

Before continuing, we introduce several definitions:

t_0 - time when the protocol starts

$t'_i = t(phase1()_i)$

$t''_i = t(phase2()_i)$

T^0 - the initial tree

T^1 - the new tree

T^* - the shortest path tree in terms of the measured $\{d_{il}\}$

T^0_i, T^1_i, T^*_i - the corresponding tree paths from i to s

$p^0_i = p_i(t_0)$ - the initial preferred neighbor

p^1_i - the new preferred neighbor

p^*_i - the father of i in T^*

$B^0_i = \sum_{T^0_i} d_{jk}; B^1_i = \sum_{T^1_i} d_{jk}; B^*_i = \sum_{T^*_i} d_{jk}$.

Note that by *a*) and *b*), a node i calculates its estimated distance d_i exactly twice, when it performs $phase1()_i$ and $phase2()_i$ respectively. Also, from *c*), for each neighbor l the estimated distance $D_i(l)$ through l is calculated exactly once. From <C5>, <C2> and <B3>, holds

$$D_i(p^1_i) = d_i(t''_i+) \leq d_i(t'_i+) \leq D_i(p^0_i) \tag{9.1}$$

and from <C3> and <B5>,

$$D_i(j) = \begin{cases} d_j(t'_j+) + d_{ij} & \text{if } i \neq p^0_j \\ d_j(t''_j+) + d_{ij} & \text{if } i = p^0_j \end{cases} \tag{9.2}$$

In order to prove *d*), suppose the assertions in *d*) hold in the entire network up to time $t-$ and we want to show that if $phase1()$ or $phase2()$ happens at time t at some node i, the assertions continue to hold.

First suppose that $phase1()_i$ happens at time t, i.e. $t = t'_i$. The preferred neighbor p_i is not changed in $phase1()_i$ and hence the tree property continues to hold. Also, *d*)ii) is not affected by $phase1()_i$ because m_i becomes 1, thus we only have to check that the ordering of m stated in *d*)i) continues to hold. Since $m_i(t-) = 0$, we have by the induction hypothesis $m_j(t) = 0$ for any j for which $p_j(t) = i$ and hence *d*) i) continues to hold for such j and i after time t. It remains to check that *d*)i) continues to hold for i and $p_i(t) = p^0_i$. When performing $phase1()_i$, node i receives MSG from p^0_i, so that p^0_i must have performed $phase1()$ before t and has not performed yet $phase2()$ since i has not yet sent any message. Thus, $m_{p^0_i}(t) = 1$ and, since $m_i(t+) = 1$, assertion *d*) i) continues to hold after t for i and p^0_i as well.

Now suppose $phase2()$ happens at some node i at time t, i.e. $t = t''_i$. Observe that at that time, i had already received MSG from all neighbors and it performs $m_i \leftarrow 0$. Consider first any node j such that $p_j(t) = i$. If $p^0_j = i$, then receipt of MSG at i from j means that j had performed $phase2()$ before time t, i.e. $t''_j < t$. If $p^0_j \neq i$, then j has changed p_j before time t and again this shows that it had performed $phase2()$ before time t. Consequently, $m_j(t) = 0$ and hence *d*) i) continues to hold after time t for j and i. At time t''_j, node j had selected $p_j \leftarrow i$ and had set $d_j \leftarrow D_j(i)$ (see <C2>, <C5>). Thus, from Eq. 9.1,

$$d_j(t+) = d_j(t''_j+) = D_j(i) = d_i(t'_i+) + d_{ji} > d_i(t'_i+) \geq d_i(t''_i+) = d_i(t+)$$

where the third equality follows from the first part of Equation 9.1, because from *b*), the fact that $t''_j < t''_i$ implies $j \neq p^0_i$. Thus *d*)ii) continues to hold at time $t+$ for j and i. Now, consider the pair i and $p_i = p_i(t''_i+)$.

Assertion *d*) i) holds trivially after t for i and p_i since $m_i \leftarrow 0$ at time t, while assertion *d*) ii) holds because $d_i(t+) = D_i(p_i)(t+) \geq d_{p_i}(t) + d_{ip_i} > d_{p_i}(t)$. Now (i, p_i) cannot close a loop since by *d*) i), all nodes l in such a loop must have $m_l = 0$, and going around the loop this would imply by *d*) ii) that $d_i > d_i$.

Now we prove *e*). By induction on T^0, holds $d_i(t'_i+) \leq B^0_i$, since $d_i(t'_i+) \leq D_i(p^0_i) = d_{p^0_i}(t'_{p^0_i}+) + d_{ip^0_i} \leq B^0_{p^0_i} + d_{ip^0_i} = B^0_i$ (the first inequality follows from Eq. 9.1 and the first equality holds because $p^0_{p^0_i} \neq i$, i.e. the preferred neighbour of the preferred neighbour of i cannot be i because of the tree property). Also, by induction on T^1, holds $d_i(t''_i+) \geq B^1_i$, since $d_i(t''_i+) = D_i(p_i) \geq d_{p_i}(t''_{p_i}+) + d_{ip_i} \geq B^1_{p_i} + d_{ip_i} = B^1_i$ (see Eq. 9.1). Thus, since $d_i(t'_i+) \geq d_i(t''_i+)$, follows that $B^1_i \leq B^0_i$, i.e. the new path for any i cannot be worse than the old one. This is the first part of *e*).

To prove the second part, we first show that if for a node k holds $T^0_k \equiv T^*_k$, then $d_k(t''_k+) = d_k(t'_k+) = B^0_k = B^1_k = B^*_k$ and k does not change its preferred neighbor, namely $p^1_k = p^0_k$. We prove this by induction on T^*. Suppose the above holds for $p^*_k = p^0_k$. Then

$$D_k(p^*_k) = d_{p^*_k}(t'_{p^*_k}+) + d_{kp^*_k} = B^*_{p^*_k} + d_{kp^*_k} = B^*_k$$

Also, $\forall m \in G_k$ holds: $D_k(m) \geq d_m(t''_m+) + d_{km} \geq B_m + d_{km} \geq B^*_k$. Hence $d_k = B^*_k$ and $p^1_k = p^0_k$.

Now, let i be a node with the property $p^0_i \neq p^*_i$ and $T^0_{p^*_i} = T^*_{p^*_i}$, namely, the father of i in the shortest path tree already has the best path when the protocol starts, but i does not. There must be such an i if $T^0 \neq T^*$. We have $d_{p^*_i} = B^*_{p^*_i}$, implying that $D_i(p^*_i) = d_{p^*_i} + d_{ip^*_i} = B^*_{p^*_i} + d_{ip^*_i} = B^*_i$. Also, as before, for any $m \in G_i$ holds $D_i(m) \geq B^*_i$. Hence $p^1_i = p^*_i$ and thus $B^1_i = B^*_i$. But $B^0_i > B^*_i$ because $p^0_i \neq p^*_i$ and therefore $B^0_i > B^1_i$. Thus node i strictly improves its path and the new path is the best path from i to s, which is the second part of *e*). qed

Problems

Problem 9.1.1 Consider Protocol PU1 with line <C2> deleted. Does this still work ? Prove or give counter example. Which version is better ?

Problem 9.1.2 Alter the PU protocol such that the source node s will learn if the new tree is identical or not to the old one. (This may possibly be used by s in order to reduce the frequency of updates). You may use only one additional kind of message. Give a correctness proof, and explain how does your protocol affect the communication and time cost.

Problem 9.1.3 Let G be the network graph described below, with the following values of p_i, d_i :

...

...

...

What is the minimum number of cycles of PU1 in this graph until convergence to the optimal tree T^*? What is the maximum number of cycles of PU1 in this graph until convergence?

9.2 Protocol Path-Updating Initialization

REVISE In order to allow proper evolution of the PU protocol, it is necessary to initialize it in the sense of building the initial trees $\{(i, p_i^j)\}$ for all destinations j in the network. This can be done by using protocol $CT2$ with some simple additions.

Protocol PUI

Messages

$MSG^j(d)$ - message carrying the estimated distance from i to j, MSG from nil to some node contains $d = 0$

Variables

G_i - set of neighbors of node i
m_i - indicates whether i has entered the protocol (values 0,1)
m_i^j - indicates whether i has entered PIF^j (values 0,1)
p_i^j - preferred neighbor in PIF^j
$e_i^j(l)$ - number of MSG^j sent to l - number of MSG^j's received from l, for all $l \in G_i$
$D_i^j(l)$ - storage for $d_l^j + d_{il}$, for $l \in G_i$

Initialization

if a node receives at least one MSG, then

- just before the time it receives the first one holds $m_i = 0$
- after receiving the first MSG, node i discards and disregards messages not sent in the present instance of the protocol

Algorithm for node i

A1 receives $MSG^j(d)$ from $l \in G_i \cup \{nil\}$
A2 { $D_i^j(l) \leftarrow d + d_{il}$
A3 **if** $(m_i = 0)$ {
A4 $m_i \leftarrow 1$; /* enter protocol */
A5 $initialize()$;
A6 $phase1^i()$;
}
A7 **if** $(m_i^j = 0)$ $phase1^j()$;
A8 $e_i^j(l) \leftarrow e_i^j(l) - 1$;
A9 **if** $(e_i^j(k) = 0 \; \forall k \in G_i - \{p_i^j\})$ $phase2^j()$;
}
B1 $phase1^j()$ /* same as PIF1 */
B2 { $m_i^j \leftarrow 1$;
B3 **if** $(i \neq j)$ $p_i^j \leftarrow l$ **else** $p_i^j \leftarrow nil$;
B4 **if** $(i \neq j)$ $d_i^j \leftarrow D_i^j(l)$ **else** $d_i^j \leftarrow 0$;
B5 **for** $(k \in G_i - \{p_i^j\})$ {
B6 send $MSG^j(d^j)$ to k;
B7 $e_i^j(k) \leftarrow e_i^j(k) + 1$;
}
}
C1 $phase2^j()$ /* same as PIF1 */
C2 { $d_i^j \leftarrow \min D_i^j(l')$ over $l' \in G_i$;
C3 send MSG^j to p_i^j;
C4 $e_i^j(p_i^j) \leftarrow e_i^j(p_i^j) + 1$;
C5 $p_i^j \leftarrow$ node that achieves $\min D_i^j(l')$ over $l' \in G_i$;
C6 $m_i^j \leftarrow 0$
}
D1 $initialize()$
D2 { **for** $(j' \in \overline{V})$ {
D3 $m_i^{j'} \leftarrow 0$;
D4 **for** $(k \in G_i)$ $e_i^{j'}(k) \leftarrow 0$;
}
}

Theorem 9.2 *(PUI) Suppose START is delivered to any node. Then any given node j will perform $phase2()_j^j$ in finite time and at that time the links $\{(i, p_i^j)\}$ will form a directed tree rooted at j, with the property $d_i^j > d_{p_i^j}^j$ for all i. In addition, at that time, all $m_i^j = 0$, and all $e_i^j(l) = 0$.*

Proof: The protocol here evolves as CT2 and hence all properties of CT2 hold here. In particuler, every node j performs $phase2()_j^j$ in finite time. Also, for a given j, action $phase1()^j$ evolves as in PI2, so that Theorem 3.2a) holds. Consequently, $\{(i, p_i^j)\}$ as considered after all nodes perform $phase1()^j$ form a tree rooted at j. Also, by <A2>, <B4>, <C2> and the fact $d_{il} > 0$, the quantities d_i^j are strictly decreasing going downtree. After $phase1()^j$ is performed at all nodes, the protocol for j behaves as in PU1, so that all properties continue to hold until j performs $phase2()_j^j$.

Problems

Problem 9.2.1 When the delay on the links is constant and equals to the weight of these links, will PUI end up finding the best routing tree to s, or will some iterations of PU be needed in some cases?

9.3 The Fixed-Topology Arbitrary-Weight Distributed Bellman-Ford Protocol (PU2)

This protocol establishes shortest paths from all nodes in the network to a given destination s in terms of some link weights $\{d_{ik}\}$. In order to save communication overhead, in some applications messages belonging to protocols corresponding to all destinations may be included in one message, but this is of no concern to us here. A node i keeps an estimate d_i of its shortest path to the destination and estimates $D_i(l)$ of the shortest path via each neighbor l. When the estimate d_i changes, i sends a message containing the new estimate to all neighbors. When i receives a message from a neighbor l, it updates its estimate $D_i(l)$ of the shortest path via that neighbor and its estimate d_i of its shortest path to the destination. If the new d_i is different from the old one, a message containing the new value is sent to all neighbors. Similar actions are taken if a link weight changes. It is shown that if link weight changes stop, a finite time afterwards the protocol terminates at all nodes in the connected network component containing the destination s. At that time, all nodes in that component have correct estimated distances. The protocol does not terminate at nodes disconnected from s. At those nodes the estimated distance goes to infinity.

Protocol PU2

Messages

$MSG(d)$ - message sent by node i, containing i's estimated distance to s, $d \geq 0$.

Variables

d_i - estimated distance from i to s (values $[0, \infty]$)
p_i - preferred neighbor of i
$D_i(l)$ - estimated distance from i to s via neighbor l (values $(0, \infty]$)
d_{ik} - distance from i to neighbor k, possibly changing with time (values $(0, \infty)$)

Initialization

Holds $d_s = 0$ and for all $i \neq s$ and $l \in G_i$, the variables d_i and $D_i(l)$ satisfy:

a) $d_i = \min D_i(l')$ over $l' \in G_i$
b) i) $D_i(l) = d_l + d_{il}$ or
ii) $D_i(l) \geq d_{il}$ and there is at least one $MSG(d)$ on (l, i) and the last $MSG(d)$ on (l, i) has $d = d_l$

Note: an example of a set of variables and messages that satisfy the above is: $d_i = D_i(l) = \infty$ for all $i \neq s$ and all $l \in G_i$, there is a $MSG(0)$ on every link (s, l), all $l \in G_s$ and this is the last message on each such link.

Algorithm for node s

```
A1      do nothing
```

Algorithm for node i ≠ s

```
B1      receive MSG(d) from l ∈ G_i
B2      {   D_i(l) ← d + d_il;
B3          update();
        }
C1      when d_il changes by Δ
C2      {   D_i(l) ← D_i(l) + Δ;
C3          update();
        }
D1      update()
D2      {   k* ← node that achieves minD_i(l') over l' ∈ G_i;
D3          if (d_i ≠ D_i(k*)) {
D4              p_i ← k*;
D5              d_i ← D_i(k*);
D6              for (k ∈ G_i) send MSG(d_i) to k;
            }
        }
```

This is the fixed topology part of the ARPA-1 routing protocol [MW77], except that the updates are

performed on an event driven basis rather than periodically.[1]

Lemma 9.3 *At all times holds $d_s = 0$ and for all $i \neq s$ and $l \in G_i$, the variables d_i and $D_i(l)$ satisfy:*
a) $d_i = min\ D_i(l')$ over $l' \in G_i$

b) $D_i(l) \geq 0, d_i \geq 0$. The contents of a MSG is always nonnegative.

c) $D_i(l) = d_l + d_{il}$ or there is at least one MSG on (l,i) and the last message on (l,i) has $d = d_l$

Proof: Note that d_s stays 0 forever and *a*) is obviously correct. Part *b*) holds at initialization by assumption. Since $d_{il} > 0$ holds at all times, part *b*) is easily proved by a common induction. To prove part *c*), consider a node i and some $l \in G_i$. Recall from assumption a in Sec. 3.1 that $l \in G_i$ if and only if $i \in G_l$. Then *c*) here is correct at initialization by assumption and if l changes its estimated distance d_l, it sends a $MSG(d_l)$ to all its neighbors, and in particular on (l,i). By FIFO, unless l changes again its d_l, this is the last MSG on (l,i), until it arrives at i, in which case the latter sets $D_i(l) = d_l + d_{il}$. qed

Theorem 9.4 *(PU2) Suppose weight changes stop. If $s \in V$, then there is a finite time after which no messages travel in (V,E) and all nodes $i \in V$ have d_i = shortest distance to s and p_i = first link on the shortest path to s. If $s \notin V$, then $d_i \to \infty$ for all $i \in V$.*

Proof: We prove the theorem via several lemmas.

Lemma 9.5 *If weight changes stop and message activity ceases, then the $d_i, D_i(l), p_i$ entries are correct for all $i \in V$ and all $l \in G_i$.*

Proof: If all d_i entries are correct, then obviously so are the $D_i(l)$ and p_i entries. Therefore it is sufficient to consider only the estimated distance entries d_i. For $i \in V$, let

d_i^* = shortest distance from i to s (may be ∞)
K = set of nodes in V for which $d_i < d_i^*$
j = node in K with minimum d_i, i.e. holds $d_j \leq d_i, \forall i \in K$

Since there are no messages on the links, Lemma 9.3 implies that $d_{p_j} = d_j - d_{jp_j}$, hence $p_j \notin K$, so $d_{p_j} \geq d_{p_j}^*$. But since j and p_j are neighbors, holds $d_j^* \leq d_{p_j}^* + d_{jp_j}$. Hence, $d_j = d_{p_j} + d_{jp_j} \geq d_{p_j}^* + d_{jp_j} \geq d_j^*$, contradicting the fact that $j \in K$. Therefore K is empty.

Now let

K' = set of nodes in V for which $d_i > d_i^*$
j = node in K' closest to s, i.e. holds $d_j^* \leq d_i^*, \forall i \in K'$
$j*$ = the next neighbor of j on the shortest path to s.

Note that a node i with $d_i^* = \infty$ cannot be in K'. In particular, this says that $d_j^* < \infty$, i.e. j is connected to s, and therefore $j*$ is well defined. Holds $d_j^* = d_{j*}^* + d_{jj*}$. Since $j*$ is closer to s than j and hence $j* \notin K'$, holds $d_{j*} \leq d_{j*}^*$ (in fact, since we have shown already that K is empty, the latter holds with equality). Moreover, d_j is selected as the minimum of $D_j(l)$ over all neighbors l of j and $D_j(j*) = d_{j*} + d_{jj*}$. Therefore, holds $d_j \leq d_{j*} + d_{jj*}$. Hence $d_j \leq d_{j*} + d_{jj*} \leq d_{j*}^* + d_{jj*} = d_j^*$, contradicting the fact that $j \in K'$. Therefore K' is also empty and therefore all nodes have correct entries. qed

Lemma 9.6 *If link weight changes stop, then, for every node $i \in V$ and every finite number z, there is a finite number of events when i reduces its estimated distance d_i to a value $\leq z$.*

[1] periodical updates

Proof: Note that after link changes stop, a node i reduces its estimated distance to the value d_i^+ only as a result of receiving a message $MSG(d)$ from some neighbor k with d that satisfies $d_i^+ = d + d_{ik} < D_i(k)$. Therefore to every such event at a node i corresponds a similar event at some neighbor k, where the latter decreases its estimated distance to $d_k^+ = d_i^+ - d_{ik}$. Denote by I the set of nodes that reduce their estimated distance an *infinite* number of times to values $d_i^+ \leq z$. For $i \in I$, denote by δ_i the sequence of values d_i^+ that node i reduces its estimated distance to and by $z_i = \liminf \delta_i$. Clearly, $z_i \leq z$ and let $i*$ be the node that achieves $\min z_i$ over $i \in I$. To every decrease of d_{i*} to a value d_{i*}^+ corresponds a decrease to a value $d_{i*}^+ - d_{ik}$ at some neighbor k. Since $i*$ has only a finite number of neighbors, it must have a neighbor $k*$ that has an accumulation point of δ_{k*} at $z_{i*} - d_{i*k*}$. Therefore, $k* \in I$ and $z_{k*} \leq z_{i*} - d_{i*k*}$, contradicting the fact that z_{i*} is minimal. qed

Lemma 9.7 *If link weight changes stop, then, for every node $i \in V$ and every finite number z, there is a finite number of events when i increases its estimated distance d_i from a value $\leq z$.*

Proof: Note that after link changes stop, a node i increases its estimated distance from the value d_i^- only as a result of receiving a message $MSG(d)$ from its preferred neighbor p_i with d that satisfies $D_i(p_i) = d_i^- < d + d_{ip_i}$. Therefore to every such event at a node i corresponds a similar event at p_i, where the latter increases its estimated distance from the value $d_{p_i}^- = d_i^- - d_{ip_i}$. Denote by I the set of nodes that increase their estimated distance an infinite number of times from values $d_i^- \leq z$. For $i \in I$, denote by δ_i the sequence of values d_i^- that node i increases its estimated distance from and by $z_i = \liminf \delta_i$. Clearly, $z_i \leq z$ and let $i*$ be the node that achieves $\min z_i$ over $i \in I$. To every increase of d_{i*} from a value d_{i*}^- corresponds an increase from a value $d_{i*}^- - d_{ik}$ at some neighbor k. Since $i*$ has only a finite number of neighbors, it must have a neighbor $k*$ that has an accumulation point of δ_{k*} at $z_{i*} - d_{i*k*}$. Therefore, $k* \in I$ and $z_{k*} \leq z_{i*} - d_{i*k*}$, contradicting the fact that z_{i*} is minimal. qed

Lemma 9.8 *If link weight changes stop, then for every node i, either d_i stops changing in finite time, or $d_i \to \infty$.*

Proof: Suppose that d_i never stops changing. Since there are a finite number of instances when a node i increases its d_i from values $\leq z$, for any finite z and the value after the increase is larger than before it, there are a finite number of instances when the node increases its d_i *to* values $\leq z$. Since every value of d_i is either after an increase or after a decrease, there are only a finite number of values of $d_i \leq z$, for every finite z. Hence $d_i \to \infty$. qed

We now proceed with the proof of the Theorem. Clearly, there cannot be two neighbors i, k such that d_i stops changing, but $d_k \to \infty$. This is because a finite time after d_i stops changing, holds $D_k(i) = d_i + d_{ki}$ and always holds $d_k \leq D_k(i)$. Therefore, since $d_s \equiv 0$, if $s \in V$, then d_i stop changing for all $i \in V$. Since messages are sent only when d_i change, message activity stops also. By Lemma 9.5, all entries in V are correct. Now suppose $s \notin V$ and not for all nodes $i \in V$ holds $d_i \to \infty$. Then all d_i stop changing and message activity ceases. This contradicts Lemma 9.5. This completes the proof of the Theorem. qed

From the above follows that Protocol PU2 does not provide a mechanism to detect that i is disconnected from s. The solution in the ARPA-1 routing algorithm is to run in parallel an MH3 Protocol.

Problems

Problem 9.3.1 Prove or give a counterexample: If $d_i \to \infty$, then d_i cannot decrease after a finite time.

Problem 9.3.2 What is the communication complexity of the Bellman-Ford Arbitrary Weight protocol when the delay on each link is constant and equals to the weight of the link?

9.4 The Changing-Topology Bellman-Ford Arbitrary Weight Protocol (EPU2)

The extensions to the arbitrary weight Bellman-Ford protocol to a network with topological changes are similar to the ones for the minimum hop case. For completeness we present here the protocol and state its main properties without proof.

Protocol EPU2

Messages

$MSG(d)$ - message sent by node i, containing i's estimated distance to s

Variables

G_i - set of neighbors, i.e. $l \in G_i$ if (l, i) is in *Connected* state at i
d_i - estimated distance from i to s (values $[0, \infty]$)
p_i - preferred neighbor of i
$D_i(l)$ - estimated distance from i to s via neighbor l (values $[0, \infty]$)
d_{ik} - distance from i to neighbor k, possibly changing with time (values $(0, \infty)$)

Initialization

none

Algorithm for node s

A1 Node s becomes operational
A2 { $d_s \leftarrow 0$;
}
B1 Link (s, l) enters *Connected* state
B2 { send $MSG(0)$ to l;
}

Algorithm for node $i \neq s$

```
C1    Node i becomes operational
C2    {   d_i ← ∞;
      }
D1    Link (i,l) enters Connected state
D2    {   D_i(l) ← ∞;
D3        send MSG(d_i) to l;
      }
E1    Link (i,l) enters Initialization mode
E2    {   if (l = p_i) update_fail();
      }
F1    receives MSG(d) from l ∈ G_i
F2    {   D_i(l) ← d + d_il;
F3        update();
      }
G1    whenever d_il changes by Δ
G2    {   D_i(l) ← D_i(l) + Δ;
G3        update();
      }
H1    update()                                        /* same as in PU2 */
H2    {   k* ← node that achieves minD_i(l') over l' ∈ G_i
H3        if (d_i ≠ D_i(k*)) {
H4            p_i ← k*;
H5            d_i ← D_i(k*);
H6            for (k ∈ G_i) send MSG(d_i) to k;
          }
      }
I1    update_fail()
I2    {   k* ← node that achieves min D_i(l') over l' ∈ G_i
I3        p_i ← k*
I4        if (d_i ≠ D_i(k*)) {
I5            d_i ← D_i(k*);
I6            for (k ∈ G_i) send MSG(d_i) to k;
          }
      }
```

The main properties of Protocol EPU2 are stated in the following Lemma and Theorem whose proof is similar to that of Lemma 8.10 and Theorem 8.11 in Sec. 8.5 and is therefore omitted.

Lemma 9.9 *At all times holds $d_s = 0$ and for all $i \neq s$ and l such that $(i,l) \in E$ the variables d_i and $D_i(l)$ satisfy:*

a) $D_i(l) = d_l + d_{il}$ or there is at least one MSG on (l,i) and the last $MSG(d)$ on (l,i) has $d = d_l$.

b) $d_i = \min D_i(l')$ over $l' \in G_i$

Theorem 9.10 *(EPU2) Consider an arbitrary* finite *sequence of topological events with arbitrary timing and location and let (E,V) denote a connected subnetwork in the final topology. Then there is a finite time after the sequence is completed after which : i) if $s \in V$, then no messages travel in (V,E) and all nodes $i \in V$ have $d_i =$ shortest distance to s and $p_i =$ first link on the shortest path to s; ii) if $s \notin V$, then $d_i \to \infty$ for all nodes $i \in V$.*

9.5 Loop Reducing Protocols

Two main deficiencies of the Bellman-Ford Distributed protocol is formation of loops and slow convergence due to link weight increases. For example, consider the simple network of Fig. 9.1, where the link weights are indicated next to the links and are the same in both directions. Suppose now that the weight of link (a, c) increases from 1 to 100 at time 0 and that the control message delay on each link is 1. Then at times 0,2,4, ... ,18, the estimated distance d_a will take on values 4,6,8, ... ,22 and at times 1,3,5, ... ,17, the estimated distance d_b will take on values 5,7,9, ... ,21, and during all this time a and b will point to each other as their preferred neighbors. Obviously this is not a desirable situation, since packets routed according to preferred neighbors p_i will loop around for a very long time, unnecessarily loading up the network. Several protocols, like the split-horizon protocol [Ceg75], [Hag83] and the Predecessor Protocol [Sch81], [SY82] have been designed with the goal of reducing two-link loops, while also speeding up protocol convergence. By definition, two nodes i and l form a two-link loop if $p_i = l$ and $p_l = i$. Those protocols reduce, but do not eradicate, two-link loops and do not address the problem of multi-link loops.

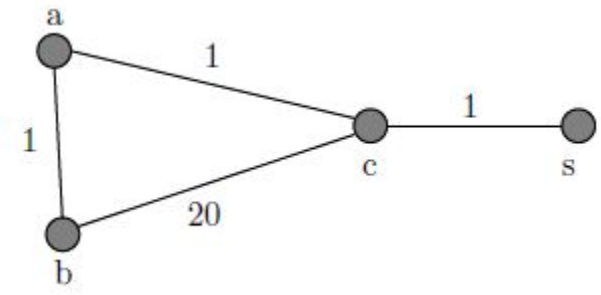


Figure 9.1: check if 61

TSAI????

The main idea of the loop-reducing protocols is that a node i sends to its preferred neighbor $j = p_i$ a value that is higher than i's estimated distance d_i. The latter is still sent to the other neighbors. In the predecessor protocol, this higher value is ∞, while in the split horizon protocol, it is the minimum of $D_i(l)$ over $l \in G_i - \{p_i\}$. The motivation for sending ∞ is to inhibit j from selecting i as its preferred neighbor, in case the link from j to its own preferred neighbor suffers a serious degradation. After the time when i selects a new preferred neighbor, j is free to select i as its preferred neighbor. The motivation behind sending to $j = p_i$ the minimum taken over all links except the preferred one, is to give j an estimate of the expected delay from i to s (referred in [Ceg75] as the horizon), on the best route that does not go through j. This is the delay that i can offer to j in case the best route from j to s suffers a serious degradation, that will make j desire to select i as its preferred neighbor.

In the predecessor protocol, if the network of Fig. 9.1 is in steady state before time 0, holds $D_a(b) = \infty$. At time 0, node a sets $d_a \leftarrow 101$ and sends $MSG(101)$ to b and $MSG(\infty)$ to c. This will set, at time 1, the value $D_b(a) = 102$, causing b to switch its preferred neighbor to c, set $d_b \leftarrow 21$ and send $MSG(21)$ to a. At time 2, node a will set $d_a \leftarrow 22$ and $p_a \leftarrow b$, completing the protocol. One can see that in this case no loop is formed. In the split-horizon protocol, before time 0 holds $D_a(b) = 22$, so when d_{ac} becomes 100 at time 0, node a will switch to $p_a = b$, thereby forming a two-link loop. But this loop will be resolved after a short duration, because at time 1, node b will receive $MSG(101)$ from a, causing it to switch to $p_b \leftarrow c$. Note that, at least for this example, the predecessor protocol forms no loops, but converges slower than the split-horizon protocol. The predecessor protocol is not free of two-link loops either. For example, if in Fig.

9.1, the weight of (b, c) were 1, and at time 0, the weight d_{cs} increased to 10, then at time 1, both a and b will receive MSG(11) from c and will switch to each other as their preferred neighbor. At time 2, when they receive from each other $MSG(\infty)$, they will switch back to c. Therefore a two-link loop will exist between time 1 and time 2.

In the following we specify the split-horizon and the predecessor protocols and prove convergence of the estimated distances to the correct values.

9.5.1 The split-horizon and the predecessor protocols

Protocol SH

Messages

$MSG(d)$ - message

Variables

d_i - estimated distance from i to s (values $[0, \infty)$)
p_i - preferred neighbor of i
$D_i(l)$ - estimated distance from i to s via neighbor l (values $(0, \infty)$)
d_{ik} - distance from i to neighbor k, possibly changing with time (values $(0, \infty)$)

Initialization

For all i, denote $S_i = \{l \mid p_l = i\}$ (node i does not know S_i). Holds:

- $d_s = 0$, $p_s = nil$.
- for $i \neq s$, p_i is arbitrary provided that $p_i \in G_i \cup \{nil\}$
- for all $i \neq s$ and $l \in G_i$, the variables d_i and $D_i(l)$ satisfy:

a) $d_i = \min D_i(l')$ over $l' \in G_i$
b) if $l \notin S_i$, then $D_i(l) = d_l + d_{il}$ or there is a MSG on (l, i) and the last $MSG(d)$ on (l, i) has $d = d_l$
c) if $l \in S_i$, then $D_i(l) \geq d_l + d_{il}$ or there is a MSG on (l, i) and the last $MSG(d)$ on (l, i) has $d \geq d_l$
d) $\boxed{d'_i = \min_{l' \in G_i - p_i} D_i(l') \text{ ?????}}$

Note: an example of a set of variables and messages that satisfy the above is:

(i) for all $i \neq s$ and all $l \in G_i$, and there are no MSG's on (i, l)
(ii) there is only one MSG on every link (s, l), for all $l \in G_s$ and this is MSG(0).

Algorithm for node s
A1 do nothing

Algorithm for node $i \neq s$

```
B1      receives MSG(d) from neighbor l
B2      {   D_i(l) ← d + d_il;
B3          update();
        }
C1      when d_il changes by Δ
C2      {   D_i(l) ← D_i(l) + Δ;
C3          update();
        }
D1      update()
D2      {   k* ← node that achieves min D_i(l') over l' ∈ G_i;
D3          k' ← node that achieves min D_i(l') over l' ∈ G_i − {k*};
D4          if (d_i ≠ D_i(k*)) {
D5              p_i ← k*;
D6              d_i ← D_i(k*);
D7              d'_i ← D_i(k');
D8              send MSG(d'_i) to p_i;
D9              for (k ∈ G_i − {p_i}) send MSG(d_i) to k;
            }
D10         else if (d'_i ≠ D_i(k')) {
D11             d'_i ← D_i(k');
D12             send MSG(d'_i) to p_i;
            }
        }
```

Protocol PRED

Messages

$MSG(d)$ - message

Variables

d_i - estimated distance from i to s (values $[0, \infty)$)
p_i - preferred neighbor of i
$D_i(l)$ - estimated distance from i to s via neighbor l (values $(0, \infty)$)
d_{ik} - distance from i to neighbor k, possibly changing with time (values $(0, \infty)$)

Initialization

For all i, denote $S_i = \{l \mid p_l = i\}$ (node i does not know S_i). Holds:

- $d_s = 0$, $p_s = nil$.
- for $i \neq s$, p_i is arbitrary provided that $p_i \in G_i \cup \{nil\}$
- for all $i \neq s$ and $l \in G_i$, the variables d_i and $D_i(l)$ satisfy:

a) $d_i = \min D_i(l')$ over $l' \in G_i$
b) if $l \notin S_i$, then $D_i(l) = d_l + d_{il}$ or there is a MSG on (l, i) and the last $MSG(d)$ on (l, i) has $d = d_l$
c) if $l \in S_i$, then $D_i(l) \geq d_l + d_{il}$ or there is a MSG on (l, i) and the last $MSG(d)$ on (l, i) has $d \geq d_l$

Note: an example of a set of variables and messages that satisfy the above is:

(i) for all $i \neq s$ and all $l \in G_i$, and there are no MSG's on (i, l)
(ii) there is only one MSG on every link (s, l), for all $l \in G_s$ and this is MSG(0).

Algorithm for node s

```
A1      do nothing
```

Algorithm for node $i \neq s$

```
B1    receive MSG(d) from l ∈ G_i
B2    {   D_i(l) ← d + d_il;
B3        update();
      }
C1    when d_il changes by Δ
C2    {   D_i(l) ← D_i(l) + Δ;
C3        update();
      }
D1    update()
D2    {   k* ← node that achieves min D_i(l') over l' ∈ G_i;
D3        if (d_i ≠ D_i(k*)) {
D4            p_i ← k*;
D5            d_i ← D_i(k*);
D6            send MSG(∞) to p_i;
D7            for (k ∈ G_i − {p_i}) send MSG(d_i) to k;
          }
      }
```

9.5.2 Proof of convergence of the split-horizon and predecessor protocols

Lemma 9.11 *Recall the notation $S_i = \{l \mid p_l = i\}$. The following hold for both protocols. At all times holds $d_s = 0$ and for all $i \neq s$ and $l \in G_i$, the variables d_i and $D_i(l)$ satisfy:*

a) $d_i = min\ D_i(l')$ over $l' \in G_i$ $\boxed{\text{and } p_i \text{ achieves minimum ????}}$

b) If $l \notin S_i$, then $D_i(l) = d_l + d_{il}$ or there is at least one MSG on (l,i) and the last $MSG(d)$ on (l,i) has $d = d_l$

c) If $l \in S_i$, then $D_i(l) \geq d_l + d_{il}$ or there is at least one MSG on (l,i) and the last $MSG(d)$ on (l,i) has $d \geq d_l$

Proof: Note that d_s stays 0 forever and that *a*) is obviously correct since d_i is always maintained as $minD_i(l')$ over $l' \in G_i$. To prove parts *b*) and *c*), consider a node i and some $l \in G_i$. Then *b*) and *c*) are correct at initialization by assumption. Next note that a node l can change its preferred neighbor p_l only if at the same time it changes its estimated distance d_l. Consequently, we need to look only at instances when l changes its d_l. Suppose *b*) or *c*) stops being correct at some time for some link (l,i). Since when node l changes its d_l it sends $MSG(d_l)$ to all $k \in G_l - \{p_l\}$ and $MSG(d), d \geq d_l$ to p_l, this can happen only when the last $MSG(d)$ on (l,i) arrives at i. But at that time i sets $D_i(l) \rightarrow d + d_{il}$ and d_l or p_l have not changed since that MSG was sent, so $D_i(l) \geq d_l + d_{il}$ with equality if $p_l \neq i$. Therefore *b*) and *c*) hold. $\boxed{d_i = d_l = \infty \text{ ????}}$ qed

Corollary: Two nodes cannot form a two-link loop if there is no message on the link connecting them.

Proof: If $p_i = l$ and $p_l = i$ and there are no messages on the link (i,l) in either direction, then $d_i = D_i(l) \geq d_l + d_{il} > d_l$ and by a similar argument $d_l > d_i$, leading to a contradiction. qed

Lemma 9.12 *If weight changes stop and message activity ceases, then in both protocols the d_i, p_i entries are correct for all $i \in V$.*

Proof: For $i \in V$, let

d_i^* = shortest distance from i to s (may be ∞)

K = set of nodes in V for which $d_i < d_i^*$

j = node in K with minimum d_i, i.e. holds $d_j \leq d_i, \forall i \in K$

Since there are no messages on the links, the above Corollary implies that $p_j \notin S_j$, so that Lemma 9.11b) and c) implies that $d_{p_j} = d_j - d_{jp_j}$, hence $p_j \notin K$, so $d_{p_j} \geq d^*_{p_j}$. But since j and p_j are neighbors, holds $d^*_j \leq d^*_{p_j} + d_{jp_j}$. Hence, $d_j = d_{p_j} + d_{jp_j} \geq d^*_{p_j} + d_{jp_j} \geq d^*_j$, contradicting the fact that $j \in K$. Therefore K is empty.

Now let

K' = set of nodes in V for which $d_i > d^*_i$
j = node in K' closest to s, i.e. holds $d^*_j \leq d^*_i, \forall i \in K'$
$j*$ = the next neighbor of j on the shortest path to s.

Note that a node i with $d^*_i = \infty$ cannot be in K'. In particular, since $j \in K'$, this says that $d^*_j < \infty$, i.e. j is connected to s, and therefore $j*$ is well defined. Moreover $d^*_j = d^*_{j*} + d_{jj*}$. Since $j*$ is closer to s than j and hence $j* \notin K'$, holds $d_{j*} \leq d^*_{j*}$ (in fact, since we have shown already that K is empty, the latter holds with equality). Note that $D_{j*}(j) \geq d_j + d_{j*j} > d_j > d^*_j = d^*_{j*} + d_{jj*} \geq d_{j*} + d_{jj*} > d_{j*}$. Consequently, $j* \notin S_j$, since that would require $d_{j*} = D_{j*}(j)$, so $D_j(j*) = d_{j*} + d_{jj*}$. Moreover, d_j is selected as the minimum of $D_j(l)$ over all neighbors l of j, so $d_j \leq d_{j*} + d_{jj*}$. Hence $d_j \leq d_{j*} + d_{jj*} \leq d^*_{j*} + d_{jj*} = d^*_j$, contradicting the fact that $j \in K'$. Therefore K' is also empty and therefore all nodes have correct entries. m aybe change notation for $j*$????? qed

After link weight changes cease, an entry $D_i(k)$ can change only as a result of i receiving a message $MSG(d)$ from k, and the new value is $d + d_{ik}$. In the predecessor protocol, this message was sent by k when d_k changes and this can happen only when some entry $D_k(l)$ changes at k. In the split-horizon protocol, that message was sent by k when d_k or d'_k changes, and this happens when some entry $D_k(l)$ changes at k. We shall say that the change in $D_i(k)$ is caused by the change in $D_k(l)$.

Lemma 9.13

a) In the split-horizon protocol, a decrease of an entry $D_i(k)$ to *a value* α *is caused by a decrease of an entry at* k *to the value* $\alpha - d_{ik}$. *An increase of* $D_i(k)$ from *a value* α *is caused by an increase at* k *from the value* $\alpha - d_{ik}$.

b) In the predecessor protocol, a decrease of an entry $D_i(k)$ to *a value* α *is caused by a decrease of an entry at* k to *the value* $\alpha - d_{ik}$ *or by an increase at* k from *a value* $\leq \alpha - d_{ik}$. *An increase of* $D_i(k)$ from *a value* α *is caused by an increase at* k from *the value* $\alpha - d_{ik}$ *or by a decrease* to *a value* $\leq \alpha - d_{ik}$.

Proof: Instead of a hard to follow proof using general variables, we shall illustrate all possible changes in an example. The general situation will be clear from the example. Consider a node i with 4 neighbors x, y, z, w, and with estimated distances $D_i(\bullet)$ as in Fig. 9.2. The preferred neighbor of i is x and $d_i = 5$. Assume that all link weights are 1.

Consider first the split-horizon protocol. The last message sent by i to x was $MSG(6)$ and to the others was $MSG(5)$.

After those messages arrive, since all link weights are 1, holds $D_x(i) = 7$ and $D_y(i) = D_z(i) = D_w(i) = 6$. Table 9.1 includes all possible kinds of changes at i and their effects.

One can see that all decreases to some value α are caused by decreases to $\alpha - 1$. Similarly, all increases from some value α are caused by increases from $\alpha - 1$.

Consider now the predecessor protocol. The last message sent by i to x was $MSG(\infty)$ and to the others was $MSG(5)$. After those messages arrive, holds $D_x(i) = \infty$ and $D_y(i) = D_z(i) = D_w(i) = 6$. The following table includes all possible kinds of changes at i and their effects.

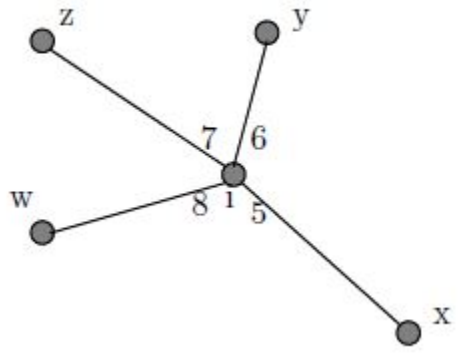


Figure 9.2: Example for the proofs.

Link	Change	Effect
$D_i(z)$	[7-9]	–
	[7-6.5]	–
	[7-5.5]	$D_x(i)$[7-6.5]
	[7-4]	$D_x(i)$[7-5], $D_y(i)$[6-5], $D_w(i)$[6-5]
$D_i(y)$	[6-7.5]	$D_x(i)$[7-8]
	[6-6.5]	$D_x(i)$[7-7.5]
	[6-5.5]	$D_x(i)$[7-6.5]
	[6-4]	$D_x(i)$[7-5], $D_z(i)$[6-5], $D_w(i)$[6-5]
$D_i(x)$	[5-7.5]	$D_y(i)$[6-8], $D_z(i)$[6-7], $D_w(i)$[6-7]
	[5-5.5]	$D_y(i)$[6-6.5], $D_z(i)$[6-6.5], $D_w(i)$[6-6.5]
	[5-4]	$D_y(i)$[6-5], $D_z(i)$[6-5], $D_w(i)$[6-5]

Table 9.1: All possible changes and their effects for the split-horizon protocol

Link	Change	Effect
$D_i(z)$	[7-9]	–
	[7-6.5]	–
	[7-5.5]	–
	[7-4]	$D_z(i)$[6-∞], $D_y(i)$[6-5], $D_w(i)$[6-5], $D_x(i)$[∞-5]
$D_i(x)$	[5-7.5]	$D_y(i)$[6-∞], $D_z(i)$[6-7], $D_w(i)$[6-7], $D_x(i)$[∞-7]
	[5-6.5]	$D_y(i)$[6-∞], $D_z(i)$[6-7], $D_w(i)$[6-7], $D_x(i)$[∞-7]
	[5-5.5]	$D_y(i)$[6-6.5], $D_z(i)$[6-6.5], $D_w(i)$[6-6.5]
	[5-4]	$D_y(i)$[6-5], $D_z(i)$[6-5], $D_w(i)$[6-5]

Table 9.2: All changes and their effects for the predecessor protocol

One can see that all decreases to some value α are caused either by a decrease to $\alpha - 1$ or by an increase from a value $\leq \alpha - 1$. Also, every increase from some value α is caused either by an increase from $\alpha - 1$ or by a decrease to some value $\leq \alpha - 1$.

Lemma 9.14 *In both protocols, if link weight changes stop, then, for every node i and, every $k \in G_i$ either $D_i(k)$ stops changing in finite time or $D_i(k) \to \infty$.*

Proof: Denote by L the set of links (i,k) such that $D_i(k)$ never stops changing, but $D_i(k) \not\to \infty$. Let $z_{ik} = liminf D_i(k)$. For $(i,k) \in L$ holds $z_{ik} < \infty$ and let $(i*, k*)$ be the link that achieves $min z_{ik}$ over $(i,k) \in L$. For every $\epsilon > 0$, there are an infinite number of instances when $D_{i*}(k*)$ is decreased to a value $\leq z_{i*k*} + \epsilon$ or $D_{i*}(k*)$ is increased from a value $\leq z_{i*k*} + \epsilon$. By Lemma 9.13, to every decrease to a value $\leq z$ or increase from a value $\leq z$ of $D_{i*}(k*)$ corresponds a decrease to a value $\leq z - d_{i*k*}$ or an increase from a value $\leq z - d_{i*k*}$ at $k*$. Since $k*$ has only a finite number of neighbors, it must have a neighbor l such that $D_{k*}(l)$ takes on an infinite number of times values $\leq z_{i*k*} + \epsilon - d_{i*k*}$. Therefore, $(k*, l) \in L$ and $z_{k*l} \leq z_{i*k*} - d_{i*k*}$, contradicting the fact that z_{i*k*} is minimal in L. qed

Theorem 9.15 *Suppose weight changes stop. In both the split-horizon and the predecessor protocols, if $s \in V$, then there is a finite time after which no messages travel in (V,E) and all nodes $i \in V$ have $d_i =$ shortest distance to s and $p_i =$ first link on the shortest path to s. If $s \notin V$, then $d_i \to \infty$ for all $i \in V$.*

Proof: From Lemmas 9.14 and 9.11a) follows that for every node i, either d_i stops changing or $d_i \to \infty$. Now observe that there cannot be two neighbors i, k such that d_i stops changing, but $d_k \to \infty$. $\boxed{d_i = \infty??}$ This is because the latter implies $D_i(k) \to \infty$ or, in the predecessor protocol, $D_i(k) = \infty$, so that a finite time after d_i stops changing, $p_i \neq k$ and therefore $D_k(i) = d_i + d_{ki}$ and therefore stops changing. However this is a contradiction, since always holds $d_k \leq D_k(i)$. Therefore, for two neighbors i, k either both d_i and d_k go to ∞ or both stop changing. Since $d_s \equiv 0$, if $s \in V$, then d_i stops changing for all $i \in V$. Now suppose $s \notin V$ and not for all nodes $i \in V$ holds $d_i \to \infty$. Then all d_i stop changing and message activity ceases, contradicting Lemma 9.12. qed

```
topology:believe first
weights:believe last
here we believe last update
in ct3 we believe first
example where without seq.nu. doesn't work
example where without proper init. doesn't work ???
 .-----.------.------.------.
   12     12     12     1
    \lar      \rar     \rar     \lar
```

Problems

Problem 9.5.1 Give an example of the split-horizon and predecessor protocols, in which a loop occurs.

9.6 The Distributed Dijkstra Protocol

The distributed protocol here is based on the Dijkstra algorithm [Dij59] for obtaining shortest paths in a network. An early version of the present distributed protocol was proposed by R.G. Gallager [Gal78] and analyzed by D. Friedman [Fri79]. The version presented here was introduced in [ZS80] and has additional features that produce savings in communication and protocol duration [2].

UPDATE ???? The validation process is based on examination of the decentralized protocol vs. the centralized algorithm, where in the first one we distinguish the communication process from the computation part. The first one deals with the construction of a communication mechanism whose purpose is to enable a node to obtain information that initially resides at other nodes. This mechanism is also designed in such a way that nodes screen and summarize the information prior to its transmission to a neighbor. Once the information is correctly transmitted, the computation part is able to construct shortest paths as in the centralized algorithm. We show that, provided that the centralized algorithm is already known and proved (as in the case of the Dijkstra algorithm), such a separation reduces the validation of the distributed protocol to the proof of correctness of the communication mechanism.

9.6.1 Preliminaries

For our purpose, the nodes in the network are numbered and are referred to by their number. As before, we associate to each direction (i, j) on a link from i to j a strictly positive constant weight d_{ij}, where the weights of opposite directions may be different. A *path* is a sequence of distinct nodes $\{i_0, i_1, \ldots, i_m\}$ such that there is a link connecting i_k and i_{k+1}. Given a path P, we define $D(P)$ as the sum of the weights along the path. For the purpose of the algorithms of this section, it is convenient to define a total order $\prec$ on all paths originating at a given node i, by using the following recursive definition:

Definition 1: We say that two paths P_1, P_2 that originate at a node i are such that $D(P_1) \prec D(P_2)$ if one of the following holds:

a) $D(P_1) < D(P_2)$

b) $D(P_1) = D(P_2)$ and $x_1 < x_2$ where x_1, x_2 are the end nodes of P_1, P_2 respectively.

c) $D(P_1) = D(P_2)$ and $x_1 = x_2$ and $D(P_1') \prec D(P_2')$, where P_1', P_2' are subpaths of P_1, P_2 originating at i and terminating at the nodes x_1', x_2' preceding $x_1 = x_2$ on each of the paths.

We say that P_1 is shorter than P_2 if $D(P_1) \prec D(P_2)$. For any two nonidentical paths P_1, P_2 originating at a node i, either P_1 is shorter than P_2 or P_2 is shorter than P_1. Also, with this definition, there is a unique path connecting two given nodes i and x that is shorter than all other paths connecting i and x, and this will be called the *shortest path.* In addition, this definition ensures that if j is a node on the shortest path P from node i to node x, then the shortest path from i to j and the shortest path from j to x are both subpaths of P. This last property is of importance in the distributed protocol and its validation.

In each of the algorithms below, a node i holds variables d_i^x, f_i^x for each node x, that indicate respectively $D(P)$, where P is a certain path from i to x, and the last node before x on P. Similarly to Definition 1, we use:

Definition 2: We say that $d_i^{x_1} \prec d_i^{x_2}$, where $x_1 \neq x_2$, if one of the following holds:

a) $d_i^{x_1} < d_i^{x_2}$

b) $d_i^{x_1} = d_i^{x_2}$ and $x_1 < x_2$

[2] Check Humblet -Dijkstra [Hum88]

Also, if y is a neighbor of x such that $y \neq f_i^x$, we say that $d_i^y + d_{yx} \prec d_i^x$ if one of the relations below holds:

c) $d_i^y + d_{yx} < d_i^x$

d) $d_i^y + d_{yx} = d_i^x$ and $y < f_i^x$.

Throughout this section, all comparisons will be made according to the $\prec$ relation. For example, a node that achieves $\min_x d_i^x$ is the unique node $\hat{x}$ for which $d_i^{\hat{x}} \prec d_i^x$ for all x. Other notations are:

G_i = set of neighbors of node i
Π_i^{*x} = shortest path from i to x (in the sense of Definition 1)
$D_i^{*x} = D(\Pi_i^{*x})$
p_i^{*x} = first node after i on Π_i^{*x}
f_i^{*x} = last node before x on Π_i^{*x} (father of x for i)
$\Pi_i^{*x}(cond)$ = shortest path from i to x under condition *cond*
$D_i^{*x}(cond) = D(\Pi_i^{*x}(cond))$
on U : let $U \subseteq \overline{V}$ and $i_0 \in U$; then a path $\{i_0, i_1, \ldots, i_{m-1}, i_m\}$ is on U if $i_k \in U$ for $k = 0, 1, \ldots, m-1$ (but not necessarily for $k = m$).
$S_i^*(x) = \{y \mid f_i^{*y} = x\}$ = set of sons of x on the tree of shortest paths from i.

Note that Definition 1 ensures that for a given i, every node is the son of exactly one node. Note also that if $j = p_i^{*x}$, then $S_i^*(x) \subseteq S_j^*(x)$.

9.6.2 The Centralized Dijkstra Algorithm (CDA)

The Dijkstra algorithm starts with knowledge at a node i of the topology of the graph and the weights of the links, and computes shortest distances and paths from a given node i to all other nodes in the network. At each stage, the algorithm divides the nodes in three categories: P_i - set of nodes to which i has permanent distance, or in short, set of permanent nodes, T_i - set of nodes with tentative distance, or in short, set of tentative nodes and the rest forms the set of nodes with unknown distance or unknown nodes. The tentative nodes are the neighbors of permanent nodes that are not permanent themselves. At any given instant, the algorithm knows the shortest path and distance from i to all permanent nodes $x \in P_i$ and also the shortest path and distance *on* P_i from node i to all tentative nodes. The Dijkstra algorithm is based on the following observation: for the node $\hat{x} \in T_i$ with shortest distance *on* P_i from i to $\hat{x}$, it can be shown that this distance is in fact the shortest (unconstrained) distance. Therefore, $\hat{x}$ can be made permanent, i.e. transferred to P_i, its neighbors that are *unknown* can be made *tentative*, and the distance *on* P_i to all tentative neighbors of $\hat{x}$ can be updated to reflect the fact that $\hat{x}$ was made permanent.

In order to facilitate comparison with the distributed protocol, we imagine a main processor at node i that performs the main algorithm, helped by a *slave* (also located at node i) that has access to the topology and weight database. Let G_x denote the set of neighbors of node x. For $k \in G_x$, recall that d_{xk} denotes the weight of link (x, k). An *adjacency array* of a node x is defined as (Λ, Δ), where $\Delta \subseteq G_x$ and $\Delta = \{d_{xk}, k \in \Lambda\}$. The role of the slave is to extract from the memory the adjacency array of a given node containing all its neighbors and forward it to the main processor. ASK^x denotes a request by the main processor to the slave asking for the adjacency array of node x, containing all neighbors of x. The assumption is that whenever such a request is submitted and only as a response to such a request, a message $ANS^x(\Lambda, \Delta)$ is returned by the slave, in finite time, where $\Lambda = G_x$. The code of the Dijkstra algorithm is given below, except that all references to the ORACLE should be disregarded and Assumption ii) should be changed to $\Lambda = G_x$. Also

note that in the Centralized Dijkstra Algorithm no ASK is released before the ANS to the previous ASK is received, so that only one node can be in state $s_i^x = 2$ at any given time.

The Dijkstra algorithm can be implemented without change in a distributed environment, but it turns out that without much added computation complexity one can implement a slightly extended version that saves considerably in communication and time complexity. It is convenient to describe this extended version in a centralized situation first, although the centralized version is not implementable. The following two changes will be made:

a) Suppose that i is informed by some $ORACLE$ that the shortest path *on* P_i to some node $x \in T_i$ is in fact the shortest, unconstrained, path to x. Even if x does not minimize the distance *on* P_i over all $x \in T_i$, node x can be made permanent. We denote the event of i being informed by the $ORACLE$ that x can be made permanent by $ORACLE^x$.

b) The slave does not necessarily have to return the adjacency array with $\Lambda = G_x$. Any subset of the neighbors of x containing all sons of x, i.e. $S_i^*(x) \subseteq \Lambda \subseteq G_x$, is sufficient.

In a centralized environment, these two alterations seem mystical, and in fact they are, since there is no obvious mechanism to implement them. However we shall see that in a decentralized protocol, where all nodes collaborate to built their shortest path trees, such information often becomes available. The Dijkstra algorithm with the above changes is:

Protocol CDA

Messages

ASK^x = message to slave requesting the adjacency array of node x
$ANS^x(\Lambda, \Delta)$ = message from slave returning an adjacency array (Λ, Δ) of x (recall that Δ contains the weights $\{d_{xk}, k \in \Lambda\}$)
$START$ = command given to the main processor to start algorithm.

Variables

s_i^x - status of node x (all $x \in \overline{V}$)

3 = permanent
2 = tentative for which ASK has been released, but ANS has not been returned yet
1 = other tentative
0 = unknown

d_i^x - estimated distance to x (all $x \in \overline{V}$)
f_i^x - identity of predecessor (father) of x on the path from i to x (all $x \in \overline{V}$)
$\hat{x}$ - the node to be made permanent next.

Initialization

holds:

$s_i^x = 0$, $d_i^x = \infty$, $f_i^x = nil$, for all x

Assumptions:

i) $ANS^x(\Lambda, \Delta)$ is returned in finite time after ASK^x is released
ii) $S_i^*(x) \subseteq \Lambda \subseteq G_x$
iii) $ORACLE^x$ can occur only if $s_i^x = 1$ and $\Pi_i^x = \Pi_i^{*x}$, where $\Pi_i^x = (i = i_0, i_1, \ldots, i_m = x)$, where $i_{n-1} = f_i^{i_n}$

Algorithm for node i

A1 When receiving $START$
A2 $d_i^i \leftarrow 0$
A3 $f_i^i \leftarrow nil$
A4 $s_i^i \leftarrow 3$
A5 $\forall y \in G_i$ ***do***
A6 $s_i^y \leftarrow 1$
A7 $d_i^y \leftarrow d_{iy}$
A8 $f_i^y \leftarrow i$
A9 $\hat{x}$ achieves $min\{d_i^y \mid s_i^y = 1\}$
A10 $s_i^{\hat{x}} \leftarrow 2$
A11 $ASK^{\hat{x}}$
B1 When receiving $ORACLE^x$ /* $s_i^x = 1$ */
B2 $s_i^x \leftarrow 2$
B3 ASK^x
C1 When receiving $ANS^x(\Lambda, \Delta)$
C2 $\forall y \in \Lambda$***do***
C3 ***if*** $s_i^y < 2$ ***and*** $d_i^x + d_{xy} \prec d_i^y$ ***then***
C4 $s_i^y \leftarrow 1$
C5 $d_i^y \leftarrow d_i^x + d_{xy}$
C6 $f_i^y \leftarrow x$
C7 $s_i^x \leftarrow 3$
C8 ***if*** $s_i^y = 0$ or 3 $\forall y$ ***then*** STOP
C9 ***else***
C10 $\hat{x}$ achieves $min\{d_i^y \mid s_i^y = 1 \text{ or } 2\}$
C11 ***if*** $s_i^{\hat{x}} = 1$ ***then***
C12 $s_i^{\hat{x}} \leftarrow 2$
C13 $ASK^{\hat{x}}$

9.6.3 The Distributed Dijkstra Protocol (DDP)

The Distributed Dijkstra Protocol works in principle as the centralized algorithm presented in the previous subsection. The role of the slave to which a node i forwards the request ASK^x for an adjacency array of x is played by $j = p_i^x$. When it receives $ANS^x(\Lambda, \Delta)$ from j, Λ contains all nodes in $S_j^*(x)$, namely all sons of x in the shortest path tree from j to all nodes. We shall show that this implies that Λ contains all nodes in $S_i^*(x)$. Obviously, the fact that Λ need not contain all neighbors of x saves communication. The role of the $ORACLE^x$ at node i is played by the event of i receiving ASK^x while $s_i^x = 1$. We shall show that in this case x is ready to be made permanent, although x does not necessarily minimize $d_i^y, y \in T_i$. Then ASK^x is sent, $s_i^x \leftarrow 2$ and when $ANS^x(\Lambda, \Delta)$ is received, x is made permanent. Obviously, this saves time at individual nodes. For example, in the network of Fig. 9.3, with unity weights on all links, if link (3,4) is slow, then nodes 5 and 6 can be made permanent at 3, without waiting for 4 to be made first permanent. If we implement the Dijkstra algorithm without the $ORACLE$, then the order in which nodes are made permanent at 3 must be 1,2,4,5,6.

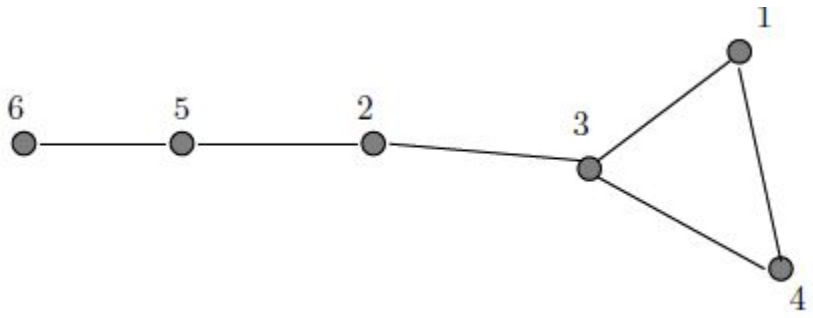


Figure 9.3: Dijkstra

Protocol DDP

Messages

ASK^x = message requesting adjacency array of node x
$ANS^x(\Lambda, \Delta)$ = message returning adjacency array (Λ, Δ) of node x
$START$ = command given to node i to start protocol.

Variables

s_i^x - status of node x (all $x \in \overline{V}$)

3 = permanent
2 = tentative for which ASK has been released, but ANS has not been returned yet
1 = other tentative
0 = unknown

d_i^x - estimated distance to x (all $x \in \overline{V}$)
f_i^x - identity of predecessor (father) of x on the path from i to x (all $x \in \overline{V}$)
p_i^x - identity of the neighbor from which $ANS^w(\Lambda, \Delta)$ has arrived, where $w = f_i^{*x}$ (all $x \in \overline{V}$)
F_i^x - set of nodes from which ASK^x have been received and to which $ANS^x(\Lambda, \Delta)$ has not been returned yet

Initialization

holds

$s_i^x = 0$, $d_i^x = \infty$, $f_i^x = nil$, $p_i^x = nil$, $F_i^x = \Phi$, for all $x \in \overline{V}$

Algorithm for node i

A1 When receiving $START$ or $WAKE$
A2 **if** $s_i^i = 0$ **then**
A3 send $WAKE$ to all $k \in G_i$
A4 $d_i^i \leftarrow 0$
A5 $f_i^i \leftarrow nil$
A6 $s_i^i \leftarrow 3$
A7 $p_i^i \leftarrow nil$
A8 $\forall y \in G_i$ **do**
A9 $s_i^y \leftarrow 1$
A10 $d_i^y \leftarrow d_{iy}$
A11 $f_i^y \leftarrow i$
A12 $p_i^y \leftarrow y$
A13 $\hat{x}$ achieves $min\{d_i^y \mid s_i^y = 1\}$
A14 send $ASK^{\hat{x}}$ to $\hat{x}$
A15 $s_i^{\hat{x}} \leftarrow 2$
A16 $F_i^{\hat{x}} \leftarrow \Phi$
B1 When receiving ASK^x from l /* Comment: $s_i^x \neq 0$ */
B2 **if** $s_i^x = 3$ **then**
B3 $\Lambda^x \leftarrow \{y \mid f_i^y = x\}$
B4 $\Delta^x \leftarrow \{d_i^y - d_i^x \mid y \in \Lambda\}$
B5 send $ANS^x(\Lambda, \Delta)$ to l
B6 **else**
B7 **if** $s_i^x = 2$ **then**
B8 $F_i^x \leftarrow F_i^x \cup \{l\}$
B9 **else**
B10 $s_i^x \leftarrow 2$
B11 $F_i^x \leftarrow \{l\}$
B12 send ASK^x to p_i^x
C1 When receiving $ANS^x(\Lambda, \Delta), x \neq i$ from l
C2 $\forall y \in \Lambda$**do**
C3 **if** $s_i^y < 2$ **and** $d_i^x + d_{xy} \prec d_i^y$ **then**
C4 $s_i^y \leftarrow 1$
C5 $d_i^y \leftarrow d_i(x) + d_{xy}$
C6 $f_i^y \leftarrow x$
C7 $p_i^y \leftarrow l$
C8 $s_i^x \leftarrow 3$
C9 $\Lambda \leftarrow \{y \mid f_i^y = x\}$
C10 $\Delta \leftarrow \{d_i^y - d_i^x \mid y \in \Lambda\}$
C11 send $ANS^x(\Lambda, \Delta)$ to all $k \in F_i^x$
C12 **if** $s_i^y = 0$ or $3, \forall y$ **then** STOP
C13 **else**
C14 $\hat{x}$ achieves $min\{d_i^y \mid s_i^y = 1$ or $2\}$
C15 **if** $s_i^{\hat{x}} = 1$ **then**
C16 send $ASK^{\hat{x}}$ to $p_i^{\hat{x}}$
C17 $s_i^{\hat{x}} \leftarrow 2$
C18 $F_i^{\hat{x}} \leftarrow \Phi$

For the proof of the Distributed Dijkstra Protocol, we need some preliminary notations and definitions.

Notations and definitions

$P_i = \{y \mid s_i^y = 3\}$, set of permanent nodes

$T_i = \{y \mid s_i^y = 1$ or $2\}$, set of tentative nodes

$A_i = \{y \mid s_i^y = 2\}$, ASK^y has been sent, $ANS^y(\Lambda, \Delta)$ has not been returned yet

$\Pi_i^x = (i = i_0, i_1, \ldots, i_m = x)$, where $i_{n-1} = f_i^{i_n}$, path to x known by i (for $x \in P_i \cup T_i$)

$\overline{V} - (P_i \cup T_i) = \{y \mid s_i^y = 0\}$, set of nodes unknown by i

$x \in P_i \cup T_i$ is *strongly known* DIFFERENT TERM? by i if $\Pi_i^x = \Pi_i^{*x}$

Note that x can be strongly known by i without i being aware of this. Only when $x \in P_i \cup A_i$ is i aware that it strongly knows x.

Lemma 9.16

a) For all x and i, if $f_i^x \neq nil$, then $f_i^x \in G_x$.

b) In any $ANS^x(\Lambda, \Delta)$, holds $\Lambda \subseteq G_x$.

Proof: We prove both parts by a common induction on the events in the entire network. When i enters the protocol, it sets $f_i^x \leftarrow i$ for all $x \in G_i$, and obviously holds $i \in G_x$. Suppose both a) and b) hold until some time $t-$, where t is some time when either ANS is sent or f_i^x is changed for some i and some x, or both. We show that *a)* and *b)* hold also after time t. If at time t, $ANS^x(\Lambda, \Delta)$ is sent in <B5>, then Λ consists of nodes y, such that $f_i^y(t-) = x \in G_y$. But $x \in G_y$ implies $y \in G_x$, so that $\Lambda \subseteq G_x$. If at time t, node i executes <C1>, then, if $f_i^y \leftarrow x$, then $y \in \Lambda(t-) \subseteq G_x$, hence $f_i^y(t+) = x \in G_y$. Afterwards, when $ANS^x(\Lambda(t+), \Delta(t+))$ is sent, $\Lambda(t+)$ consists of nodes y such that $f_i^y = x$ and the same argument as before shows that $\Lambda(t+) \subseteq G_x$.

Lemma 9.17

a) Node i can receive $ANS^x(\Lambda, \Delta)$ from node j only if i has previously sent ASK^x to j.

b) For $x \neq i$, the status s_i^x can change only by increments of 1.

c) For all $x \in P_i \cup T_i$ (i.e. $s_i^x > 0$), holds $f_i^x \in P_i$ and $d_i^x = D(\Pi_i^x)$.

d) When $ANS^x(\Lambda, \Delta)$ is sent by a node i, holds $\Delta = \{d_{xy}, y \in \Lambda\}$.

e) If x is strongly known by i at some time t, i.e. $\Pi_i^x = \Pi_i^{*x}$, then it is strongly known by i at any time after t.

f) If $x \in T_i - A_i$ (i.e. $s_i^x = 1$), then $\Pi_i^x = \Pi_i^{*x}$(on P_i), i.e. the path to x known by i is the shortest on P_i.

Proof:

a) $ANS^x(\Lambda, \Delta)$ is sent by j to i either in <B5> when it receives ASK^x from i or in <C11> to nodes in F_j^x. A node enters F_j^x only upon j receiving ASK^x from it.

b) follows from the algorithm and the fact that $ANS^x(\Lambda, \Delta)$ can be returned only after ASK^x is issued.

c) The fact that $f_i^x \in P_i$ follows from the fact that f_i^x becomes y only if at the same time $s_i^y \leftarrow 3$. The rest of *c)* is proved by induction on the events in the network, since at any time when $f_i^x \leftarrow y$, and only at those times, also $d_i^x \leftarrow d_i^y + d_{yx}$. Hence, if *b)* holds in the entire network until before this event, then $d_i^x = D(\Pi_i^y) + d_{yx} = D(\Pi_i^x)$, and therefore *c)* holds also after the event.

d) $ANS^x(\Lambda, \Delta)$ is sent in $<$C11$>$, and the entries in Δ are $d_i^y - d_i^x$. But from *c)*, $d_i^x = D(\Pi_i^x)$ and $d_i^y = D(\Pi_i^y)$, and from <C6>, $f_i^y = x$, so that $d_i^y - d_i^x = d_{xy}$.

e) In view of *c)*, if x is strongly known by i, the inequality $d_i^y + d_{yx} \prec d_i^x$ can never hold in the future, hence f_i^x and d_i^x will never change.

f) If $f_i^x \leftarrow i$ in <A11>, then $P_i = \{i\}$, and the statement is obvious. The only other event when Π_i^y, for some y, is changed is in <C6>, when $f_i^y \leftarrow x$, at time t say. We show now that if *f)* holds until time $t-$, it also holds at time $t+$. If $s_i^y(t-) = 0$, then $\Pi_i^y(t+)$ is the only path *on* $P_i(t+)$ to y, hence *f)* holds trivially at

time $t+$. If $s_i^y(t-)=1$, then $\Pi_i^y(t-)=\Pi_i^{*y}(on\ P_i(t-))$ and by *c*), $d_i^y(t-)=D_i^{*y}(on\ P_i(t-))$. Since at time t, node x enters P_i and $d_i^x+d_{xy}\prec d_i^y(t-)$, the path to y via x is $\Pi_i^{*y}(on\ P_i(t+))$, so that $f_i^y\leftarrow x$ establishes $\Pi_i^y(t+)=\Pi_i^{*x}(on\ P_i(t+))$.

qed

Theorem 9.18 *(DDP)*

a) ASK^x can be received by i from j only if $i=p_j^{*x}$ and only if x is strongly known by i.

b) $ANS^x(\Lambda,\Delta)$ can be received by i from j only if $j=p_i^{*x}$ and then $S_i^*(x)\subseteq\Lambda$.

c) At all times holds $\cup_{y\in P_i}S_i^*(y)\subseteq P_i\cup T_i\subseteq\cup_{y\in P_i}G_y$ and if $x\in P_i$ and $y\in S_i^*(x)$, then $f_i^y=x$.

d) If $x\in A_i\cup P_i$ (i.e. $s_i^x=2$ or 3), then $\Pi_i^x=\Pi_i^{*x}$, i.e. x is strongly known *by i.*

e) If x is strongly known by i, then $p_i^x=p_i^{*x}$ and x is also strongly known by p_i^{*x}.

f) At the time when ASK^x is sent by i to j, holds $j=p_i^{*x}$ and both i and j know x strongly.

Proof: The proof proceeds by a common induction. We assume that *a*)-*f*) hold in the entire network until time $t-$ and proceed to show that they also hold at time $t+$, for any event that happens at time t.

a) If ASK^x is received by i from j at time t, let $t'<t$ be the time when j has sent the message. By the induction assumption, *f*) holds at time t', hence $i=p_j^{*x}$ and x is strongly known by i. From Lemma 9.17e) follows that x is strongly known by i at time t as well.

b) Let $t'<t$ be the time when j has sent the $ANS^x(\Lambda,\Delta)$ message. At time t' holds $s_j^x=3$ and $\Lambda=\{y\mid f_j^y=x\}$. Hence *c*) applied at time t' implies that $S_j^*(x)\subseteq\Lambda$. Now Lemma 9.17a) implies that j receives from i an ASK message at or before $t'-$, so that *a*) implies that $j=p_i^{*x}$. Moreover, since $j=p_i^{*x}$ implies $S_i^*(x)\subseteq S_j^*(x)$, also holds $S_i^*(x)\subseteq\Lambda$.

c) From <C1>, holds $P_i\cup T_i=\cup_{x\in P_i}\Lambda\cup\{i\}$, check this ($\Lambda$) and the first part of *c*) follows from Lemma 9.16b) and part *b*) above. Now, if $y\in S_i^*(x)$ and $x\in P_i$, then y was received in $ANS^x(\Lambda,\Delta)$ and since $D_i^{*y}=d_i^x+d_{xy}$, at that time $f_i^y\leftarrow x$, $d_i^y\leftarrow D_i^{*y}$, and these entries never change afterwards.

d) Suppose that x enters A_i (i.e. $s_i^x\leftarrow 2$) at time t. If this happens in <B10>, then $\Pi_i^x=\Pi_i^{*x}$ because of *a*). Suppose that at time t, x enters A_i in <C17>. Consider the situation just before $s_i^x\leftarrow 2$ is executed. Suppose $\Pi_i^x\neq\Pi_i^{*x}$, i.e. $D(\Pi_i^x)>D_i^{*x}$, and let z be the first node not in P_i on Π_i^{*x}. Since $f_i^{*z}\in P_i$ and $z\in S_i^*(f_i^*(z))$, part *c*) above implies $z\in T_i$. Therefore,

$$d_i^z=D_i^{*z}(onP_i)\leq D_i^{*x}<D(\Pi_i^x)=d_i^x$$

which implies $z\neq x$. But the above contradicts the fact that x minimizes $\{d_i^y,y\in T_i\}$. Hence *d*) holds for $x\in A_i$. This completes the proof for $x\in A_i$. Since in the transition from A_i to P_i, the path Π_i^x and the preferred neighbor p_i^x do not change, *d*) holds also for $x\in P_i$.

e) Let t be the time when x becomes strongly known by i. If this happens in <A11>, then $f_i^x\leftarrow i$ and at the same time $p_i^x\leftarrow x$. Since this is the last time when f_i^x is set and at t node x becomes strongly known by i, holds $p_i^{*x}=x$. Therefore after time t holds $p_i^x=p_i^{*x}$. If at t, node x becomes strongly known in <C6>, at which time f_i^x is set to w say, then $ANS^w(\Lambda,\Delta)$ is received, from p_i^{*w} (by *b*)). Therefore $p_i^x\leftarrow p_i^{*w}$. But since f_i^x does not change after t and at t node x is strongly known by i, holds $f_i^{*x}=w$ and therefore $p_i^{*w}=p_i^{*x}$, so that after t holds $p_i^x=p_i^{*x}$. Next we show that at time t node x is strongly known

by $j = p_i^{*x}$ as well. Since a node is strongly known by itself, we need consider only the case $j \neq x$. When the $ANS^w(\Lambda, \Delta)$ received by i from j at time t was sent, held $w \in P_j$ and $f_j^x = w$. This together with $w = f_i^{*x} = f_j^{*x}$, implies that x was strongly known by j when ANS was sent, and from Lemma 9.17e), also at time t.

f) If ASK^x is sent by i at time t, to j say, then at the same time $s_i^x \leftarrow 2$, i.e. x enters A_i. The statement therefore follows from *d)* and *e)* above.

qed

Theorem 9.19 *(DDP)*
a) If $T_i \neq \emptyset$, then $A_i \neq \emptyset$.

b) If a node enters A_i, it enters P_i a finite time afterwards. If ASK^x is sent by i to j, then $ANS^x(\Lambda, \Delta)$ is received in finite time by i from j.

c) At any node i, STOP occurs in finite time and when this happens $P_i = V$, $T_i = \Phi$, $\overline{V} - V = \{x \mid s_i^x = 0\}$.

Proof:

a) When T_i becomes for the first time nonempty, in <A9>, one of the nodes, $\hat{x}$, enters also A_i. Later, A_i can empty out only in <C8>, but if this happens and there are still nodes in T_i, then again one node, $\hat{x}$ in <C17>, enters A_i at the same time.

b) When a node x enters A_i, i.e. $s_i \leftarrow 2$, node i sends ASK^x to $j = p_i^{*x}$. From Theorem 9.18a) follows that when ASK^x arrives at j, then $s_j^x > 0$. If the ASK^x finds j such that $s_j^x = 3$, then j sends $ANS^x(\Lambda, \Delta)$ back to i and when this is received x enters P_i. If the ASK^x finds $s_j^x = 1$, then j sends ASK^x and sets $s_j^x \leftarrow 2$. Continuing on Π_i^{*x}, since $s_x^x = 0$ or 3, there must be a node k that receives ASK^x while $s_k^x = 3$. That node returns $ANS^x(\Lambda, \Delta)$, and when a node between i and k receives $ANS^x(\Lambda, \Delta)$, it sends $ANS^x(\Lambda, \Delta)$. Consequently, i will also receive $ANS^x(\Lambda, \Delta)$.

c) Since any node in A_i eventually goes to P_i at least one node in T_i must be in A_i and there are only a finite number of nodes in $\overline{V}$, eventually T_i must empty out, so STOP occurs at i. Since obviously if $x \in \overline{V} - V$, then $s_i^x = 0$, we need to show that for all $x \in V$ holds $s_i^x = 3$. Suppose this is not true and let $V' = \{x \in V \mid s_i^x = 0\}$. Since both V' and P_i are nonempty, there exist two neighbors x and y such that $x \in P_i$, $y \in V'$ and $f_i^{*y} = x$. When $s_i^x \leftarrow 3$, i has received $ANS^x(\Lambda, \Delta)$ and since $y \in R^*(x)$, y was in the received Λ. At that time, if s_i^y was 0, it was set to 1, contradicting the fact that s_i^y remains 0 forever.

qed

The communication complexity C of the Distributed Dijkstra Protocol can be calculated as follows. Each node i sends exactly one ASK^x message and one $ANS^x(\Lambda, \Delta)$ message, for each $x \neq i$ in V. Those messages travel on the tree of shortest paths to x. Therefore, the ASK messages are responsible for $\mid V \mid (\mid V \mid -1)$ elementary quantities (node identities). The list $\Lambda(x)$ contains S_i^{*x} and is contained in G_x. Therefore the total number of elementary quantities (weights or node identities) sent in ANS messages bounded from above by $\mid V \mid (\mid V \mid -1)(2 \mid E \mid / \mid V \mid +1)$ elementary quantities. Similarly, if $\Lambda = S_i^*(x)$ in all ANS messages, then each identity x travels exactly twice in ANS messages on each edge (i, k) of the shortest paths tree to x, once in ANS^x and once in Λ, where $y = f_i^{*x}$. In the later case, ANS also carries the weight d_{yx}, hence the total number of elementary quantities in ANS messages is bounded from below by $3 \mid V \mid (\mid V \mid -1)$.

Problems

Problem 9.6.1 What is the complexity of each one of the PU protocols (PU, Bellman-Ford, Dijkstra) when the delay on the links is constant and equals to the weight of these links?

Chapter 10

CONNECTION MANAGEMENT

10.1 Low speed networks

In this section, we present a reliable distributed protocol for management of connections in data networks, that was introduced in [SJ86]. The protocol ensures that the connection is set up properly unless a failure is encountered, data messages are delivered to their destinations unless they encounter a takedown process and the connection is taken down and all used network resources are released after a failure or connection completion.

10.1.1 Background

One of the common methods for routing messages in data-communication networks is virtual circuit-switching [SBP72]. This method consists of first setting up a virtual channel (VC) for any given session between the originating node and the destination node, routing all data messages corresponding to this session on the VC and then cancelling the VC when the session is completed.

The simplest method to identify the VC's at intermediate nodes is by using a unique "global path identifier" (GPID) that includes the origin node, destination node and a number (see Fig. 10.1). If the origin node assigns a different number to each new VC at set-up time, then the GPID identifies the VC uniquely networkwide. If the routing tables at each node keep the next node of the path in both directions for any active GPID and the data messages carry the GPID in their header, then the data can be appropriately dispatched from node to node until it arrives at the terminal node. The GPID carried in the message is also used by the destination node to assign the received data to the correct VC.

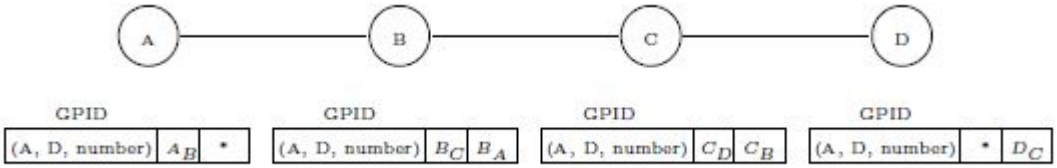


Figure 10.1: Routing tables for the GPID method

This method has, however, two serious drawbacks: the size of the GPID must be very large to guarantee uniqueness, resulting in large tables and long headers. Moreover, the access into an unstructured routing

table using a GPID is difficult. One solution to this is based on the observation that the number of active VC's traversing any given node at any given time is only a small portion of the total number of VCs existing in the network and to use Local Path ID's [MM81], also known as Path Number [SS80] or Logical Record Number [Rin] or Virtual Channel Identifier (VCI) [Bou92]. In order to concur with the terminology of modern ATM networks, we shall use the term Virtual Channel Identifier (VCI). Generally speaking, each node along the VC assigns a local number (VCI), to be used in connection with the VC. Each node knows, for every VCI entry in its VCI-Table, the next node on the VC and the VCI used by the next node for this VC (see Fig. 10.2). When a data message is sent by node X to the next node Y, it carries the VCI of node Y corresponding to the VC and when it is received by Y, the latter accesses the corresponding entry in its VCI-Table. In this entry Y reads the identity and the VCI of the next node, replaces the received VCI by the new one and forwards the message on the corresponding link. The advantages of this method are: a) the number of bits required to specify an VCI is much smaller, resulting in smaller headers and routing tables, and b) the access into the routing table does not require any search because the VCI may be used to directly index into the entry in the table . The number of bits necessary to represent the VCI is $\log_2$ of the maximum number of VCs that traverse a node at any given time. This follows from the fact that when a VC is cancelled, the corresponding entry (VCI) in the node VCI-Table can be released for reuse for any new VC that will traverse the given node.

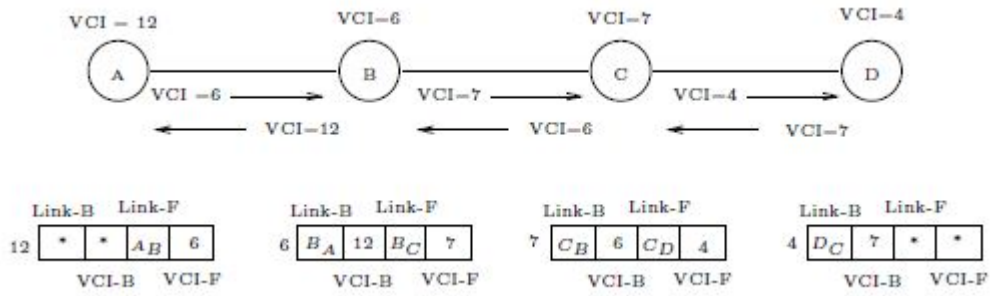


Figure 10.2: Routing tables for the VCI method

Given the basic idea of routing using VCI, this section introduces a protocol that establishes and cancels VC's in the network in a distributed way and ensures their proper operation. The required properties of the protocol are:

1) The VC set-up procedure completes in finite time unless it encounters a failure and permits transmission of data on the VC by the end nodes.

2) All data sent by a node on a particular VC arrives at its destination on the same VC, unless a failure or VC takedown occurs; in case of a failure or takedown, the data may be destroyed, but may not be accepted by a different destination or by the same destination along a different VC; all data messages leave the network in finite time after being sent.

3) All VCI table entries corresponding to a VC that has failed or has been cancelled are released in finite time.

The protocol must have the stated properties in presence of failures of nodes or links in the network.

10.1.2 The basic model and the protocol

In this subsection we shall describe the protocol for setting-up and cancelling a given VC connecting a source node named ORIG to a destination node DEST. Although in ATM VC's are unidirectional, we look at our

VC's in this section as bi-directional in the sense that data messages can be sent in either direction. Only ORIG is allowed to establish it however. Takedown of the VC may be initiated asynchronously by either end at the time of session completion and/or by intermediate nodes when failures occur.

The VC management protocol must be supported by a path determination protocol that determines the path over which the VC is to be established. Two types of such protocols may be considered. In one type, the entire path is available at ORIG before the set-up procedure is started. This may be the case if the network uses static routing, as for example in SNA 4.2 [SS80], [Atk82], or centralized adaptive routing, as in TYMNET [Rin], [Tym81]. The network may have one or more facilities that run centralized path determination protocols and send the path to ORIG upon request or, alternatively, ORIG itself may be able to determine the route to DEST based upon knowledge of the entire topology. The second type of protocols that may be considered consists of distributed path determination protocols, where the entire path is not available at any node, nodes maintain only next-node tables, and the path of the virtual circuit is built on a node by node basis by the set-up message itself. Both types of protocols can be used, but it is convenient for concreteness to first present the VC management protocol while assuming that the entire path of the VC,denoted by *VC-path* is available at ORIG. We then show in Sec. 10.1.5 that certain protocols of the second type can also be used. Henceforth, until otherwise said, we assume that the path is available at ORIG and is carried in its entirety in the header of the SET-F message that sets up the VC.

The formal algorithm performed by each node to implement the protocol appears in Sec. 10.1.3. Next we provide an informal description by first introducing the main ideas and then giving the details of the algorithm. We use the notation F to indicate messages that travel "forward" on the path, namely from ORIG to DEST and B to indicate those messages that travel in the opposite ("backward") direction. A basic assumption is that associated with each link there is a Data-Link Control protocol that ensures Data Reliability (see Chap. 2).

The routing tables in the nodes on the path are dynamically created by the setup SET-F and SET-B messages (see Fig. 10.3). The set-up message SET-F carries *VC-path*, travels from ORIG to DEST and is also used by each node to record the next link on the path in the forward and backward direction, denoted by Link-F and Link-B respectively. A reply message SET-B travels from DEST to ORIG to inform ORIG that the path has been set up successfully. The SET-F and SET-B are also used by each node to set the variables VCI-B and VCI-F respectively, that indicate the VCI selected by the backwards and forwards next nodes for this VC (see Fig. 10.3). After SET-F is received at DEST, the latter may start using the VC for data messages, denoted by data-B . Similarly, receipt of SET-B at ORIG is the green light for sending data messages on the VC in the forward direction, denoted by data-F. In this way we comply with requirement 1) in Sec. 10.1.1.

Since Link-B, VCI-B and Link-F are available at each node after the time SET-F traverses it and VCI-F is available after receipt of SET-B, all data messages will arrive safely at the corresponding end node as long as they do not encounter a failed link or a VC takedown process. This is part of requirement 2) in Sec. 10.1.1.

A takedown process may be initiated by any node on the path at any time. Takedown may be triggered by a desire for session completion at either of the end nodes, by failures of elements along the path or if the set-up procedure encounters a failed link or node. The VC is taken down by CNCL-F and CNCL-B messages that propagate on the VC. Two basic problems exist with the naive method of blindly cancelling the VC and propagating the cancelling messages in both directions after failure. The first is that data messages belonging to the VC may still be in the network (see Fig. 10.3). If a node (node X in Fig. 10.3) takes down the VC upon receipt of CNCL-B and then sets up a new VC using the same VCI, possibly to a different destination, then the data message may get to the wrong destination, violating requirement 2) of Sec. 10.1.1.

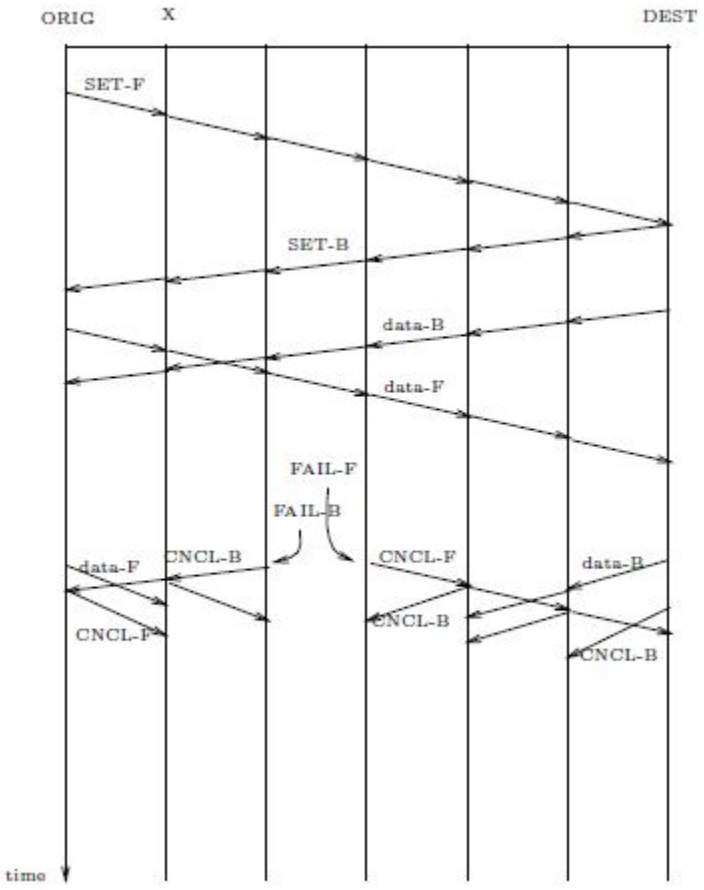


Figure 10.3: The setup, cancel and data message flow

The second problem is that CNCL-F cannot be forwarded if SET-B has not arrived yet, since the latter sets VCI-F, that is needed for CNCL-F (see Fig. 10.4). To handle the second problem, the algorithm requires a node that is about to send a CNCL-F and has not yet received SET-B, to wait until the latter arrives. To correct the first problem, nodes will send a response CNCL-F to a received CNCL-B and a CNCL-B in response to CNCL-F, if such a message has not been sent before. As proved in Sec. 10.1.4, the VCI-Table entry can be released upon receipt of the second CNCL, since it is guaranteed that no further messages belonging to the old VC will arrive at the node. In this way we comply with the rest of requirement 2) and with requirement 3) in Sec. 10.1.1.

The state diagram for each node is given in Sec. 10.1.3. The meaning of the states are given in Statement b) [1]. FAIL-F is the message that is delivered by the Data-Link Control protocol to our protocol when the link to Link-B fails, provided that no CNCL-F or FAIL-F has been received before (see Statement g)). Similarly for FAIL-B. A message sent by a node carries its type, its direction F or B, and possibly two VCI's as indicated in Statement c). The variable *sender-VCI* is the sender's VCI and is used to transmit this variable to the receiving node, and the variable VCI is the receiver's VCI and is used by the receiving node to access the correct entry in its own routing table.

In the algorithm for an Arbitrary Node in Fig. 10.4, the Passive-Setup transition is the first step in the set-up action and the Passive-Passive transition is the initialization of the Takedown process when a failed link is encountered by SET-F (see Fig. 10.6). The Setup-Active transition is the propagation of SET-B, the Setup-Passive transition is the propagation of CNCL-B or the initiation of takedown when the link to Link-F fails (i.e. FAIL-B is received) and the Setup-TakedownP transition brings the algorithm into a state where the node waits for SET-B before sending CNCL-F. Transitions Active-TakedownF and Active-TakedownB are the propagation of CNCL while also sending a response message or initiation of CNCL when a failure occurs and transition Active-Active is propagation of data. State TakedownB means that CNCL-B has been sent and the node is waiting for CNCL-F. As said before and proved in Sec. 10.1.4, in this state any one of the messages CNCL-F or FAIL-F is the last possible message belonging to the current VC that may arrive and can be used to release the table entry. State TakedownF is similar to TakedownB in the opposite direction. In state TakedownP, if SET-B comes, takedown is initiated in the forward direction, while CNCL-B or FAIL-B signals that there is no need to initiate this takedown.

10.1.3 The Algorithm

The state diagram of the algorithm perfomed by a node on *VC-path* to implement the VC management protocol is given in Fig 10.4. The state diagram is accompanied by the following statements.

Statements

a) An entry in the VCI-Table contains the variables VCI-F, VCI-B, Link-F, Link-B, *state*. The entry is selected among entries with *state* = Passive upon arrival of SET-F.

b) The values of the variable *state* and their meaning are:

Passive	unused (after the VCI has been released by the previous VC and before it is assigned to a new VC)
Setup	being setup (after (SET-F)
Active	active (after SET-B)
TakedownF	being taken down (waiting for CNCL-B)
TakedownB	being taken down (waiting for CNCL-F)
TakedownP	being taken down (waiting for SET-B)

[1] We write "Statement" as short for "Statement in Sec. 10.1.3".

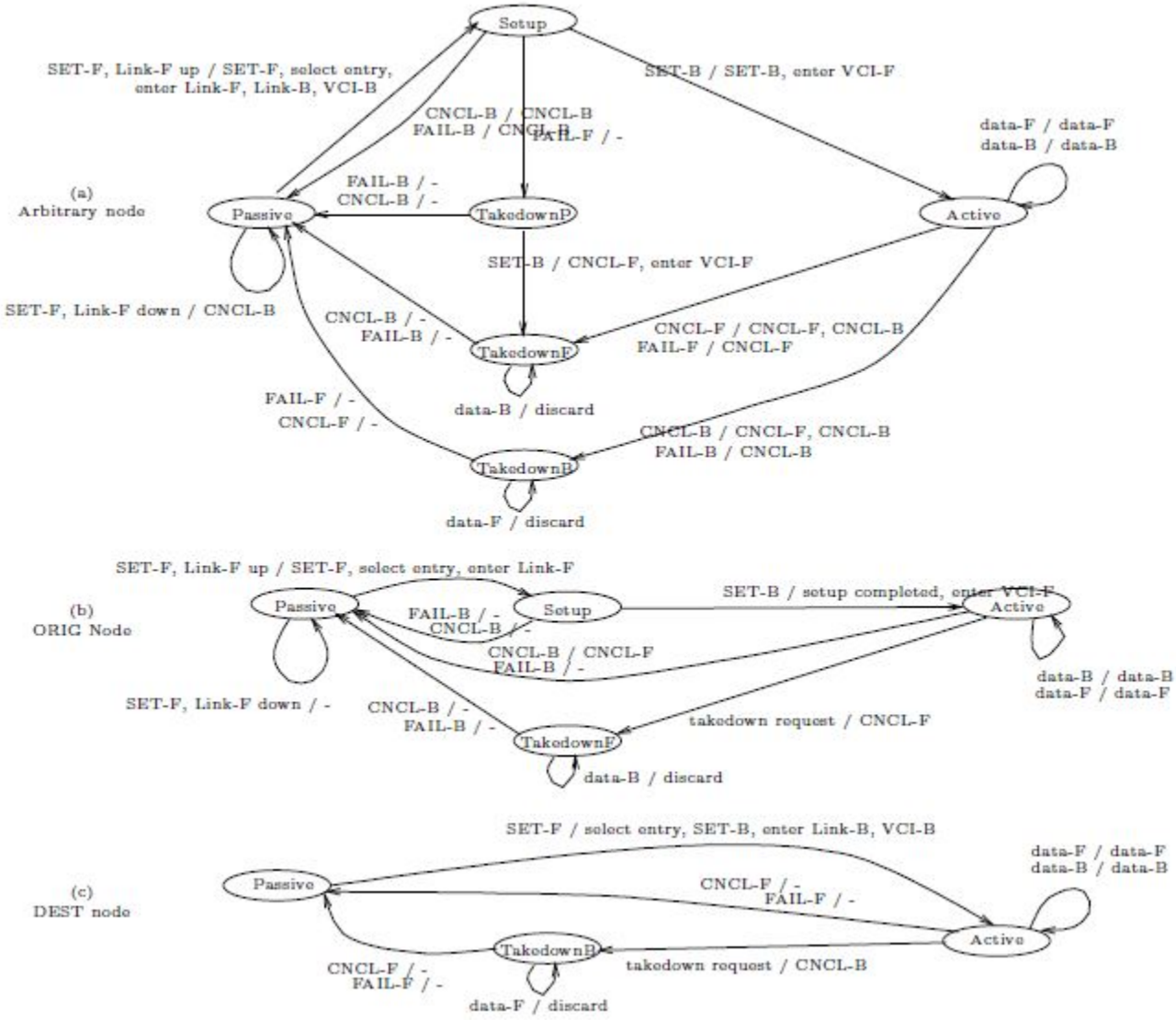


Figure 10.4: The state transition diagrams for the algorithm

The state transition diagram appears in Fig. 10.4.

c) The messages used by the algorithm and the variables carried by them are listed below. sender-VCI is set as the VCI of the sending node. VCI is the entry in the VCI-Table to be accessed by the receiving node.

SET-F (*sender-VCI*, *VC-path*)

SET-B (VCI, *sender-VCI*)

CNCL-F (VCI)

CNCL-B (VCI)

data-F (VCI)

data-B (VCI)

FAIL-B (VCI)

FAIL-F (VCI)

d) VCI-B is set as *sender-VCI* in the incoming SET-F message; VCI-F is set as *sender-VCI* in the incoming SET-B message.

e) B messages are sent on the link to Link-B and F messages on the link to Link-F.

f) On each link there is DLC protocol that ensures data reliability (see Chap. 2).

g) FAIL-F is delivered to the algorithm if if failure notification is received from DLC on Link-B and the algorithm is in state Setup, Active or TakedownB. FAIL-B is delivered to the algorithm if failure notification is received on Link-F and the algorithm is in state Setup, Active, TakedownF or TakedownP.

10.1.4 Main Properties of the Protocol

The main properties of the protocol are given in Theorems 10.8 and 10.9 at the end of this subsection. The rest of this subsection is devoted to the preparation of the proof of those theorems.

Before proceeding, observe at the outset that the Algorithm, as given in Sec. 10.1.3 is not completely specified. This is because the Algorithm does not specify what should a node do when it receives a message that is not specified for a given state in the state diagram. For example, what action should be taken when a node receives a data message in state Setup? As part of the validation proof we shall show that in fact no unspecified message can be received, but for now, in order to have a completely defined protocol, we shall specify that *every unspecified message is discarded without any processing*.

Next, in order to be be able to refer to VC's and messages, we need a unique identification for every VC and for the messages belonging to it. To accomplish that, we temporarily define an additional variable, named GPID, that appears in each entry in the VCI-Table and in each message. All entries and messages belonging to a given VC will contain the same GPID. The GPID of a VC is selected by the higher level so that it is unique networkwide and is delivered to ORIG in the VC setup request. The GPID is carried by the SET-F message in addition to all other variables dictated by the algorithm, is *entered* at each node in the selected VCI upon receiving SET-F and is *erased* when the VCI entry is released, namely when it reenters state Passive. In this way all VCI entries assigned to a given VC are identified by the corresponding GPID.

Every message arriving at or departing from a node is associated with a certain VCI entry: some VCI entry is accessed when a message arrives and every message that is sent draws its variables and direction from an VCI entry. For conciceness, we shall say that the VCI entry *is used* by the message. However, we want to identify messages by the VC they should belong to and not by the VC on which they happen to

propagate. Later we shall prove that the two are the same, but for now we must proceed as if they were not necessarily identical. Consequently, the GPID in messages other than SET-F is defined as follows: messages of types FAIL-F and FAIL-B are received not as a result of being sent by a neighbor and their GPID is defined as the GPID appearing in the VCI entry used by the message at the receiving node; all other types of messages are sent by a node as a result of receiving a message and the GPID in the message to be sent is set as the GPID in the received message. Observe that with this definition, until otherwise proved, a received or sent message does not necessarily carry the GPID of the used VCI entry.

Now consider a given GPID, its corresponding *VC-path* and the messages carrying that GPID. All nodes preceding a given node X in *VC-path* will be referred to as its *predecessors* and all those following X as its *successors*. The *immediate predecessor/successor* is the node appearing just before/after X in *VC-path*. An F or B message carrying the given GPID is said to be *routed according to VC-path* if it is sent on a link corresponding to *VC-path* in the forward/backward direction respectively. In addition, if a message that is received or sent by a node carries the same GPID as the VCI entry used by that message, we say that the message is received/sent *with the correct* GPID. The basic property of our protocol, that implies most of the statements of Theorems 10.8 and 10.9, is that all messages are routed on the VC to which they belong. Formally, this means that they are routed according to *VC-path* and are sent and received with the correct GPID. However, until this fact is proved, it will be useful to consider a second algorithm, that is the same as the given algorithm, except that *for every* GPID, *every message that is received with the incorrect* GPID *is discarded*. The second algorithm will be referred to as *the altered algorithm* as opposed to *the original algorithm* and in the sequel we shall always mention explicitly the algorithm we refer to. Observe that the GPID is not used in the original algorithm explicitly, while the altered algorithm needs the variable GPID in order to discard messages received with incorrect GPID. The structure of the first part of the validation proof will be to first prove certain facts about the altered algorithm, and then to show that due to these facts, it is actually equivalent to the original algorithm.

The following definitions for a given VC in either of the two algorithms will be useful: A node is said *to enter the algorithm* (for the given GPID) if and when it leaves state Passive and sets the given GPID; it is said *to leave the algorithm* or *to release the* GPID when it reenters state Passive (and then it erases the GPID); it is said *to be in the algorithm* for the given GPID in between those times. A node is said *to set* VCI-B *correctly* if the immediate predecessor has previously entered the algorithm and VCI-B is set as the VCI used by the immediate predecessor for the given VC. Similarly for VCI-F.

The validation of the original algorithm consists of two parts. In the first part it is shown that the algorithm routes all messages on the VC they belong to. This corresponds to the so-called "safety" property, that refers to the fact that "bad things do not happen", or in our context, if messages are sent, they are sent on the correct VC. The second part proves the "liveness" part, or "good things do happen", which in our context means that data messages arrive at their destination and, if a failure occurs, the VCI entries are released in finite time.

In the first part, we show that in the altered algorithm all messages are routed according to *VC-path* (Lemma 10.1) and are received with the correct GPID (Lemma 10.2). We then conclude in Lemma 10.3 that these properties also hold for the original algorithm and that in fact both algorithms are identical.

Lemma 10.1 *The following statements are correct for* the altered algorithm *and refer to a given GPID and the corresponding* VC-path *:*

a) Only nodes in VC-path *may enter the algorithm for the given GPID and not more than once. Upon entering the algorithm, a node sets Link-F and Link-B as the immediate successor/predecessor respectively and sets VCI-B correctly. If a node receives SET-F, all its predecessors have entered the algorithm. If a node performs the Passive-Passive transition, it never enters the algorithm.*

b) A message with the given GPID may be sent by a node only if the node is in the algorithm for that GPID and only with the correct GPID.

c) Every message with the given GPID sent by a node is routed according to VC-path*.*

d) If a node sets VCI-F in an entry with the given GPID, it sets it correctly. A node may send a B message with the given GPID only after having set Link-B and VCI-B in the corresponding entry and may send an F message only after having set Link-F and VCI-F.

Proof: The given GPID is unique networkwide, SET-F is sent according to *VC-path* and only once, the variables Link-F and Link-B are set according to *VC-path* and VCI-B is set as *sender-VCI* in the incoming SET-F message. If a node performs the Passive-Passive transition, it will never receive SET-F again with the given GPID and hence will never enter the algorithm. Hence *a*) holds.

In the altered algorithm, a message may be sent by a node only when it receives a message with the correct GPID, since messages received with incorrect GPID are discarded. Since a message that is sent carries the same GPID as the received one, *b*) follows.

Since a B message is sent on the link to Link-B and an F message on the link to Link-F, *a*) and *b*) above imply *c*).

To prove the first part of *d*), suppose a node sets VCI-F in an VCI entry with the given GPID. This can happen only as a result of receiving a SET-B and since in the altered algorithm only messages with the correct GPID are processed, the incoming message must carry the given GPID. From *a*) and *b*) above, the message was sent by the immediate successor and the variable *sender-VCI* in the message was set as the VCI entry selected by the successor for the given GPID. Hence VCI-F is set correctly. Finally, inspection of the state diagram in Fig. 10.4 shows that the rest of *d*) holds. qed

Lemma 10.2 *The following statements are correct for* the altered algorithm *and refer to a given GPID and the messages carrying that GPID.*

a) Except for data, any given type of message can be sent at most once by each node. A node sends no messages after having sent CNCL-B or CNCL-F or both together. A node that performs the Passive-Passive transition, sends no messages.

b) A node that receives a failure notification from Link-F after it has entered the algorithm sends no F messages afterwards (even if the link comes up again). Similarly for B messages and Link-B.

c) No message can arrive at a node prior to SET-F.

d) No B message can arrive at a node after CNCL-B or FAIL-B and no F message can arrive after CNCL-F or FAIL-F.

e) If the algorithm at a node is in state Setup, the algorithm at its immediate predecessor is and has been in states Setup, TakedownP or Passive. If the algorithm at a node performs the Setup-Passive transition, it receives no F message except for SET-F.

f) Every received message is received with the correct GPID.

Proof: A node can enter the algorithm for the given GPID at most once and can send messages with the given GPID only when it is in the algorithm for that GPID (Lemma 10.1). All statements of *a*) follow then by simply inspecting the state diagram of Fig. 10.4.

A failure notification from Link-F after the node has entered the algorithm, either finds the node in the subset of states (TakedownF, Passive) or else a FAIL-B is delivered to the algorithm and this forces the

algorithm into that subset. In either case, the algorithm remains in this subset forever and no F messages can be sent in these states. A failure notification from Link-B after the node has entered the algorithm, either finds the node in the subset of states (TakedownP, TakedownB, Passive) or else a FAIL-F is delivered to the algorithm and this forces the algorithm into that subset. In either case, the algorithm remains in this subset forever and no B messages can be sent in these states. Hence *b*) holds.

To prove *c*), observe that no message carrying the given GPID can travel on a link that does not belong to *VC-path* (Lemma 10.1c)). Let X be the node in *VC-path* where *c*)) is violated for the first time. The received message cannot be of type FAIL-F or FAIL-B because those can be delivered only when the node is in the algorithm for the given GPID. Therefore the message was sent as a result of receiving a message with the same GPID, either by the immediate successor, Y say, or by the immediate predecessor Z. In the first case, Y violates *c*) before X, which is a contradiction. Consider now the second case. In the second case, SET-F was sent by Z, at time t_1 say, before the considered message was sent, at time t_2 say. However by the time the considered message arrives, the SET-F did not arrive. The FIFO property of the DLC procedure (Sec. 2.3), implies that between t_1 and t_2 there was a failure notification at Z from Link-F (and the DLC at Z entered Initialization Mode, see Chap. 2). However this contradicts *b*), since the considered message was sent after the failure notification was received. This completes the proof of *c*).

Next we prove *d*). Since FIFO holds, *a*) above shows that if CNCL-B arrives at a node, no other B message sent by the immediate successor can arrive afterwards. The only B message that can arrive at a node without being sent by the immediate successor is FAIL-B. But, receipt of CNCL-B forces the algorithm into the set of states (TakedownB, Passive) if it is not in this set already, and this set cannot be ever left. But by Statement g) in Sec. 10.1.3, FAIL-B cannot be delivered in these states. Thus FAIL-B cannot be delivered to the node after CNCL-B has been received. The same proof works for CNCL-F, that forces the algorithm in the subset (TakedownP, TakedownF, Passive) if it is not there already. To prove that no B message can arrive after FAIL-B, let X and Y be two consecutive nodes in *VC-path* and suppose FAIL-B arrives at X at time t_1 (Fig. 10.5), in which case, by Statement g), a failure notification is delivered by the DLC on link (X,Y) at X and X is in state Setup, Active, TakedownP or TakedownB. We want to show that this is the last received B message with the given GPID. Suppose this is not the case and let t_2 be the later time when such a message arrives. Since the FAIL-B message forces X into (TakedownF, Passive), no second FAIL-B can be received and thus the message was sent by Y, at time t_3 say. At time t_1, the DLC at X entered Initialization Mode and at t_2 it is in Connected Mode, hence the Crossing property of the DLC (Sec. 2.3) dictates that there is a time t_4 between t_1 and t_2 when the DLC at Y is also in Initialization Mode and $t_4 < t_3$. Since X is in the algorithm at t_1, it has sent SET-F to Y before that time. If the SET-F has not arrived by time t_4, it will never arrive by the Crossing property of DLC, hence Y never enters the algorithm and thus it sends no messages with the given GPID to X. If the SET-F has arrived and Y performed the Passive-Passive transition, then Y will never enter the algorithm. If Y did not perform that transition and has entered the algorithm, it sends no B messages to X after t_4 by *b*). A similar argument shows that no F message can arrive after FAIL-F, completing the proof of *d*).

To prove the first part of *e*), note that if the algorithm in a node is in state Setup, it has sent no B messages. Hence its immediate predecessor, that has entered Setup beforehand, could not have left the set of states (Setup, TakedownP, Passive). The second statement of *e*) then follows since if the algorithm performs the Setup-Passive transition, its predecessor will never leave the above-mentioned set of states and thus will never send any other F message except for the SET-F. The algorithm also cannot receive FAIL-F since if it does, it performs the Setup-TakedownP transition and not the Setup-Passive one.

Next we prove *f*). By inspecting Fig. 10.4, we observe that, except for the case when it performs the Setup-Passive transition, a node that enters the algorithm leaves it after having received both (FAIL-F or

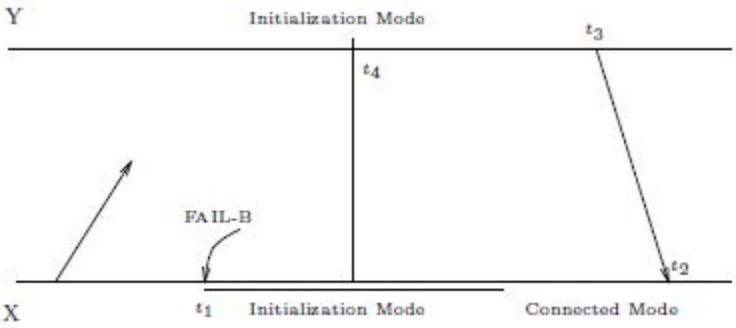


Figure 10.5: Diagram for proof of d)

CNCL-F) and (FAIL-B or CNCL-B). Consequently, *d*)) above shows that no message carrying the given GPID can arrive after the node has left the algorithm for that GPID. If the node does perform the Setup-Passive transition, it leaves the algorithm without waiting for CNCL-F or FAIL-F, but by *e*) it cannot receive afterwards any F message. Thus in this case too, it cannot receive any message with the given GPID after leaving the algorithm. In addition, from *c*), no message carrying the given GPID can arrive before the node enters the algorithm for that GPID. Consequently, every message carrying the given GPID that arrives at a node, does so while the node is in the algorithm for that GPID. It remains to show that all such messages use the entry in the VCI-Table that contains the given GPID. If the received message is of type FAIL-F or FAIL-B, this holds by the definition of GPID. In all other cases, the received message has been sent by the immediate successor or predecessor. Since VCI-B and VCI-F at these nodes have been set correctly before that message was sent (Lemma 10.1a)), d)), the message carries the correct GPID and, upon receipt of the message, the node will access the entry that contains the given GPID. This completes the proof of *f*). qed

Lemma 10.3

a) In the original algorithm, every message is routed according to VC-path *and is sent and received with the correct GPID.*

b) The altered and the original algorithms operate identically and all properties of Lemmas 10.1, 10.2 hold for the original algorithm as well.

Proof: Consider a network operating with the original algorithm and let X be the first node that receives a message with incorrect GPID, at time t say. Recall that the altered algorithm differs from the original only in instances when a message with incorrect GPID is received: the original algorithm processes such a message, while the altered algorithm discards it. Consequently, if the network under consideration used the altered algorithm, the operation up to time $t-$ would have been identical. However this means that with the altered algorithm, node X receives at time t a message with incorrect GPID, contradicting Lemma 10.2f). Therefore, in the original algorithm all messages are received with the correct GPID and the two algorithms are identical. In particular, in both algorithms, all messages are routed according to *VC-path* and all messages that are sent by any node are sent with the correct GPID. This completes the proof of *a*) and *b*). qed

Because of Lemma 10.3b), from now on we need only consider the original algorithm. Observe that up to now it was shown that all messages are routed on the correct VC, but we have not looked yet at the internal operation of the protocol. The latter is treated in the next four lemmas. Lemma 10.4 indicates the VC set-up procedure, Lemma 10.5 leads to the fact that in case of failure or session completion all VCI entries are released in finite time and Lemma 10.6 refers to the propagation of data messages. Recall that

we are still working under the requirement that a node receiving messages of types that are not listed in the Algorithm, discards them. Lemma 10.7 shows that this requirement is superfluous and in fact no such messages can arrive at a node.

From now on we consider a given VC with its associated GPID, *VC-path* and messages. The following definition will be useful: A *chain* is a set of messages *of a given type* and with the given GPID that propagates through the network, each message being sent as a result of receiving the previous message of the chain. Each such node is said to *propagate* the chain, and the node that sends the first message in the chain is said to *initiate* the chain. A node *terminates* the chain if it receives a message of the chain, but does not send a message of the same type. A chain is *interrupted* on a link if one node adjacent to the link propagates the chain, but the other node receives failure notification from that link before receiving the message of the chain (in which case the message is lost).

Lemma 10.4

a) Exactly one chain of SET-F is initiated and only at ORIG. It is propagated along VC-path *while nodes set Link-F, Link-B, VCI-B and exactly one of the following happens: the chain is terminated at DEST, the chain is terminated at a node with the link to the immediate successor being down or the chain is interrupted.*

b) A chain of SET-B is initiated not more than once, only at DEST and only if SET-F arrives there. If initiated, SET-B is propagated backwards along VC-path *by nodes in state Setup while setting VCI-F and exactly one of the following happens: the chain is terminated at ORIG, the chain is terminated at a node in state TakedownP or the chain is interrupted on a link that fails.*

c) If a node sends/receives SET-B, then this is the first B message that is sent/received by the node.

d) No B message can be received in TakedownB or Passive state and no F message can be received in TakedownF, TakedownP or Passive state. No CNCL-F message can be received in Setup state.

e) If a node receives CNCL-B in Setup, then its immediate successor performed the Setup-Passive transition when it has sent the CNCL-B message.

Proof: SET-F is initiated only in state Passive by ORIG and is propagated only while performing the Passive-Setup transition. It is terminated by a node that performs the Passive-Passive transition or at DEST in the Passive-Active transition, hence *a*).

SET-B can be initiated only at DEST in the Passive-Active transition and is propagated only in the Setup-Active transition in the Arbitrary Node algorithm. It is terminated at ORIG or if it arrives at a node not in Setup. It remains to show that the node not in Setup must be in state TakedownP. Observe from *a*) that all nodes have entered Setup before SET-B is initiated, so that the node A that terminates the SET-B chain must have left Setup before receiving SET-B. Now, the transition Setup-Active could not have occured since SET-B could not have been sent twice by the immediate successor (Lemma 10.2a)) and the transition Setup-Passive could not have occured because SET-B cannot arrive after CNCL-B or FAIL-B (Lemma 10.2d)). Consequently, the transition to TakedownP must have occurred and the algorithm could not have left TakedownP because of the same reasons as before, so that SET-B finds the node in TakedownP. This proves *b*).

To prove *c*), observe that if SET-B is sent by a node, that happens in the Setup-Active transition and this is the first B message sent. In addition, because of FIFO, if SET-B is received, no other B message sent by the immediate successor can be received before SET-B. Also, FAIL-B cannot be received before SET-B because of Lemma 10.2d), completing the proof of *c*).

Finally, a node can be in (TakedownB, Passive) only after having received CNCL-B or FAIL-B and no B message can arrive after these messages (Lemma 10.2d)). Similarly for states (TakedownF, TakedownP, Passive) and F messages. Also, if a node is in Setup, it has sent no SET-B or CNCL-B before and its immediate predecessor could not have been in Active. Therefore the possibility that the predecessor has sent CNCL-F in the transitions out of Active state are ruled out. The only other possibility of sending CNCL-F by the immediate predecessor is in the transition TakedownP-TakedownF and this requires receipt of SET-B, which the node in Setup could not have sent. Hence *d*) follows.

To prove *e*), note that if the immediate successor Z of the node X that receives CNCL-B in Setup did not send the CNCL-B in the Setup-Passive transition, Y must have sent it in one of the transitions out of Active state. When it entered Active state, Z has sent SET-B, which did not arrive since X is in Setup. By the Delivery property of DLC (Sec. 2.3), Z has received a failure notification from Link-B and thus, by Lemma 10.2b), it cannot send CNCL-B afterwards. This results in a contradiction. qed

Part (d) of the next Lemma leads to the fact that all VCI entries corresponding to a given VC are released in finite time after a failure or session completion (Theorem 10.9). The other parts are preparatory.

Lemma 10.5

a) Suppose DEST receives takedown request or a node receives FAIL-B. Then all its predecessors will receive CNCL-B or FAIL-B in finite time.

b) If a node in state Setup receives CNCL-B or FAIL-B, its immediate predecessor will also receive CNCL-B or FAIL-B in finite time.

c) A node in TakedownP leaves this state, i.e. receives SET-B, CNCL-B or FAIL-B, in finite time.

d) If a node X receives CNCL or FAIL in either direction in a state other than Setup, then in finite time its immediate successor receives CNCL-F or FAIL-F if it has entered the algorithm, its immediate predecessor receives CNCL-B or FAIL-B and node X receives CNCL or FAIL in the other direction. In this respect, takedown request received at ORIG or DEST acts as CNCL-F and CNCL-B respectively.

Proof: To prove *a*), suppose DEST receives takedown request or or a node receives FAIL-B. Then all its predecessors have entered the algorithm. Going backwards on *VC-path* , let X be the first node that receives no CNCL-B or FAIL-B in finite time. Let Y be its immediate successor. When Y receives CNCL-B or FAIL-B it must be in Setup, Active, TakedownF or TakedownP (by Lemma 10.4d)). If in Setup or Active, it sends CNCL-B. If in TakedownP it has received FAIL-F and if in TakedownF it has either received FAIL-F or sent CNCL-B in the Active-TakedownF transition. In all cases either CNCL-B will arrive at X or else a failure notification will arrive at X (Crossing property in Sec. drefWindow). If the failure notification arrives while X is in TakedownB or Passive state, it has received CNCL-B or FAIL-B beforehand. Otherwise, it finds X is in (Setup, Active, TakedownF, TakedownP), resulting in FAIL-B, hence *a*).

Part *b*) follows from the proof of *a*).

To prove *c*), observe from Lemma 10.4b) that if SET-B is received at ORIG, all nodes enter Active and therefore no node ever enters TakedownP. If the SET-B chain is terminated, no node that propagated SET-B can ever enter TakedownP (because they enter Active) and by Lemma 10.4b), the terminating node is in TakedownP and receives SET-B. Therefore *c*) holds for the node that terminates the SET-B chain and does not apply to its successors. If the SET-B chain is interrupted, *c*) does not apply to the interruption point and its successors. Regarding the predecessors of the termination or interruption point, observe that FAIL-B is delivered to the immediate predecessor of the termination or interruption point, and by *a*) all its predecessors receive either CNCL-B or FAIL-B, whether they are in TakedownP or not, hence *c*).

To prove *d*), suppose first that CNCL-B or FAIL-B is received at a node X in a state other than Setup. Then the proof of *a*) shows that CNCL-B or FAIL-B are received at its immediate predecessor Y. If FAIL-B is received at Y, the corresponding failure notification is delivered to X, and the latter generates FAIL-F if X has not received CNCL-F or FAIL-F before. If CNCL-B is received at Y, Lemma 10.4d) implies that the latter must be in (Setup, Active, TakedownF, TakedownP) and Lemma 10.4e) shows that it cannot be in Setup. If Y is in TakedownF, then CNCL-F was sent before and if it is in Active,then CNCL-F is sent now.

TAKEDOWNP????????

CNCL-F arrives correctly to X or else FAIL-F is delivered, proving that if CNCL-B or FAIL-B is received at X, then CNCL-B or FAIL-B is received at Y and CNCL-F or FAIL-F is received at X. It remains to show that receipt of CNCL-F or FAIL-F at X implies receipt of CNCL-F or FAIL-F at its immediate successor Z and of CNCL-B or FAIL-B at X. By Lemma 10.4d), if X receives CNCL-F it must be in Active or TakedownB and if it receives FAIL-F it must be in Setup, Active or TakedownB. Suppose first that X receives CNCL-F or FAIL-F and is in Active or TakedownB. Then the proof is completely symmetric to the case when it receives CNCL-B or FAIL-B in Active or TakedownF. Finally, suppose X receives FAIL-F in Setup. Then it enters TakedownP and *c*) above applies, namely X receives SET-B, CNCL-B or FAIL-B. If it receives FAIL-B, then FAIL-F is or has been received at Z. If it receives CNCL-B, then this message has been sent by Z and the proof of the first part of *d*) applies with Z replacing X. If it receives SET-B, it acts as if it received FAIL-F in Active state. Therefore the proof of *d*) is complete. qed

Lemma 10.6

a) A forwards chain of data can be initiated only at ORIG and only in state Active. If it is initiated, then all nodes in VC-path *have previously entered Active, the data messages are routed according to* VC-path *and are sent/received at each node with the correct GPID. If and only if the chain is not interrupted and no node has left Active state before the arrival of the data message, the chain is terminated at DEST.*

b) A backwards chain of data can be initiated only at DEST and only in state Active. It propagates backwards on VC-path *and with the correct GPID. If and only if the chain is not interrupted and all nodes on* VC-path *have entered Active and have not left it before the arrival of the data message, the chain is terminated at ORIG.*

Proof: Forward data can be initiated only by ORIG and only in state Active and hence only if the SET-B chain was previously terminated at ORIG. This implies that all nodes in *VC-path* have previously entered Active state (Lemma 10.4b)). Since all messages are routed according to *VC-path* and are sent/received with the correct GPID (Lemma 10.3), the data propagates on *VC-path* as long as it meets nodes in Active. Otherwise, it is discarded, hence *a*) holds.

Backward data can be initiated only by DEST and only in Active. This implies that the SET-F chain was terminated at DEST and nodes in *VC-path* have previously entered Setup state, but not necessarily Active state(Lemma 10.4a)). As before, data-B propagates backwards on *VC-path* as long as it meets nodes in Active, otherwise it is discarded, hence *b*). qed

Lemma 10.7 *In every state of the Algorithm of Fig. 10.4, only the specified types of messages can arrive.*

Proof: The following is a list of states, unlisted messages and the reason why they cannot be received:

state	messages	reason
Passive	other than SET-F	Lemma 10.2c)
Setup	CNCL-F	Lemma 10.4d)
Setup	data-F	Lemma 10.6a)
Setup	data-B	Lemma 10.4c)
TakedownB	all B messages	Lemma 10.4d)
TakedownF	all F messages	Lemma 10.4d)
TakedownF	SET-B	Lemma 10.2a)
TakedownP	all F messages	Lemma 10.4d)

qed

The following two theorems indicate that the VCI management protocol operates correctly. Theorem 10.8 corresponds to requirements 1) and the first part of 2) in Sec. 10.1.1 and Theorem 10.9 implies requirements 2) and 3).

Theorem 10.8

Suppose no failures or VC takedowns occur. Then if ORIG receives a VC setup request from the higher level, the following will hold:

a) All nodes on VC-path *will enter state Active in finite time.*

b) Any data message delivered at ORIG or DEST will arrive at the other end in finite time and on the correct VC.

Proof: Since there are no failures or cancellations, Lemma 10.4 implies that the SET-F chain is terminated at DEST and the SET-B chain is initiated at DEST and terminated at ORIG. Therefore all nodes in *VC-path* enter Active, hence *a*).

Since there are no failures or takedowns, all nodes remain in Active forever. If ORIG is in Active, all nodes in *VC-path* are already in Active and if a data message is delivered, a data-F chain is initiated. By Lemma 10.6a), the chain is propagated on *VC-path* and with the correct GPID (i.e. on the correct VC) and is terminated at DEST. Hence every data message delivered to ORIG arrives at DEST in finite time and on the correct VC. Now consider the case when a data message is delivered to DEST. As before, this can happen only if DEST is in Active and in this case a data-B chain is initiated. By Lemma 10.4c), the SET-B chain precedes the data chain, so that the latter finds every node in Active. Now Lemma 10.6b) and the same reasoning as before show that the data message will arrive at ORIG in finite time and on the correct VC. qed

Theorem 10.9 *??????????????????*

a) If SET-F encounters a failed link or node or is lost in a failure of a link or node (see Fig. 10.6), then no data messages will be sent and all corresponding entries in the VCI-Tables along the path will be released in finite time.

b) If a failure occurs on a link or node after the SET-F, but before the SET-B message was propagated on this link or node, no data message will be sent in the forward direction. Any data message sent in the backward direction will be discarded and all corresponding entries in the VCI-Tables will be released in finite time (see Fig. 10.7).

c) If a VC takedown request is delivered to ORIG or DEST or a failure occurs on a link or node after the SET-B message was propagated on this link or node, then a takedown process is initiated, all data messages that encounter the takedown process are discarded, the other data messages arrive safely at the end node on the correct VC and all corrresponding entries in the VCI-Tables are released in finite time (see Fig. 10.3).

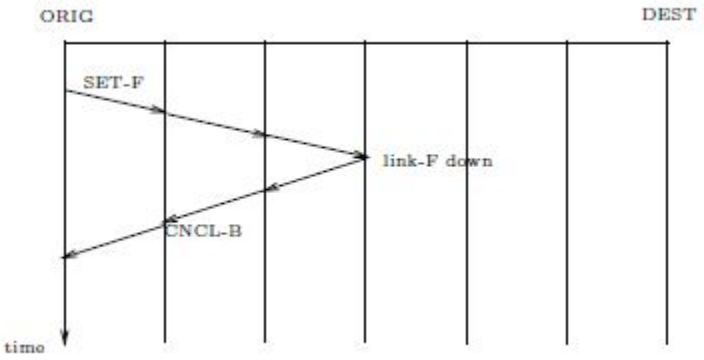


Figure 10.6: SET-F encounters a failed link

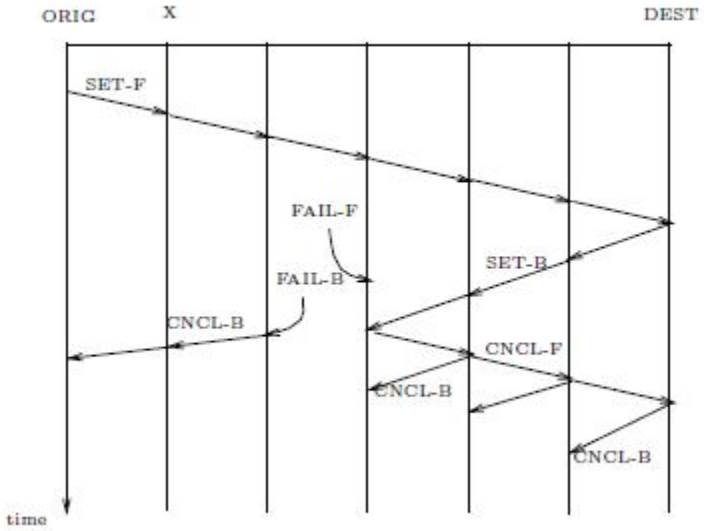


Figure 10.7: Failure occurs before SET-B is received

Proof: First observe that Lemma 10.6a) implies, by induction, that if any node enters TakedownF, TakedownB or TakedownP state, then all nodes that have entered the algorithm will enter Passive state in finite time, at which time their VCI entries are respectively released. Now in case *a*) of this Theorem, the node where SET-F encounters a failed link enters TakedownP or the node adjacent to the link that loses SET-F enters TakedownF.?????????? In cases *b*) and *c*), if a failure occurs, the node immediately preceding the failure in *VC-path* will enter TakedownB. If a takedown request is delivered to ORIG or DEST, the node will enter TakedownF or TakedownB respectively. Consequently, in all cases the condition of Lemma 10.5d)

holds and hence the VCI entries corresponding to the considered VC will be released at all nodes in finite time. The statements concerning data messages follow similarly from Lemma 10.6, completing the proof. qed

10.1.5 The Path Determination Protocol

The path determination protocol is normally referred to in the literature as *the routing protocol* and is responsible for determining the path used for each session. In order to simplify the exposition of the algorithm, we have assumed up to now that it provides the entire route, named *VC-path* , to ORIG, and *VC-path* is carried in the SET-F message. However, we note here that these facts were not used explicitly in any of the proofs. Consequently, all properties of Theorems 10.8 and 10.9 (and all Lemmas in Sec. 10.1.4) hold for any path determination procedure as long as it ensures that the SET-F message arrives eventually at DEST (except if it encounters a failure). Even if the established VC contains loops, as e.g. in the ARPA algorithm [JM80], the correctness properties given in Theorems 10.8 and 10.9 still hold. From the point of view of performance however, loops in established VCs are very non-appealing in circuit switching networks, since the routes remain fixed for the duration of the session. This is in contrast to message switching, where temporary message loops may sometimes be tolerated. Consequently, in circuit switching, the VC set-up protocol should be used only in conjunction with route determination protocols that ensure that the established path is loop-free. Observe that it is not sufficient that the routing tables are loop-free at any given instant of time as in [Seg81], [MS79a], [JM82], since the tables may change during the propagation of SET-F. The requirement is that if the SET-F message reaches DEST, then the established path is loop-free.

In [Seg81], Sec.IV it has been shown that the protocols of [Seg81], [MS79a] can be used in conjunction with the VC set-up protocols to ensure the above requirement. Consider the following combined procedure:

1. Any version of the routing protocols of [Seg81], [MS79a] is used with the addition that a node sets a flag when it loses its preferred neighbor (for a particular destination) and resets the flag when the next routing update cycle (for that destination) is completed at the node (i.e. in the notation of [Seg81], [MS79a], when T21 is performed).

2. SET-F carries only DEST instead of *VC-path* .

3. whenever SET-F arrives and the flag is reset for DEST, transition Passive-Setup is performed with Link-F taken as the current preferred neighbor for DEST; if the flag is set, then the Passive-Passive transition is performed.

4. on each link, SET-F has the same or higher priority than the control messages of the routing protocol.

5. all other operations of the VC management protocol remain unchanged.

Theorem 10.10 *(see [Seg81,Thm.3])*

The above combined procedure satisfies Theorems 10.8 and 10.9 and also guarantees that the established routes are loop-free.

The advantages of this procedure over centralized route determination are that *VC-path* need not be carried in SET-F and no nodes need to have global knowledge of network topology or to perform possibly time-consuming centralized route determination algorithms.

Problems

Problem 10.1.1 In an attempt to simplify the node algorithm, consider the algorithm of Fig. 10.8. Does it work? (*Hint*: What happens if Link-B fails while the node is in Setup, comes up and fails again while the node is in Takedown. (Two FAIL-F, the events take the node into Passive and then Set-B and data-B might still come, and the data will be routed on the new VC, contradicting property .....)

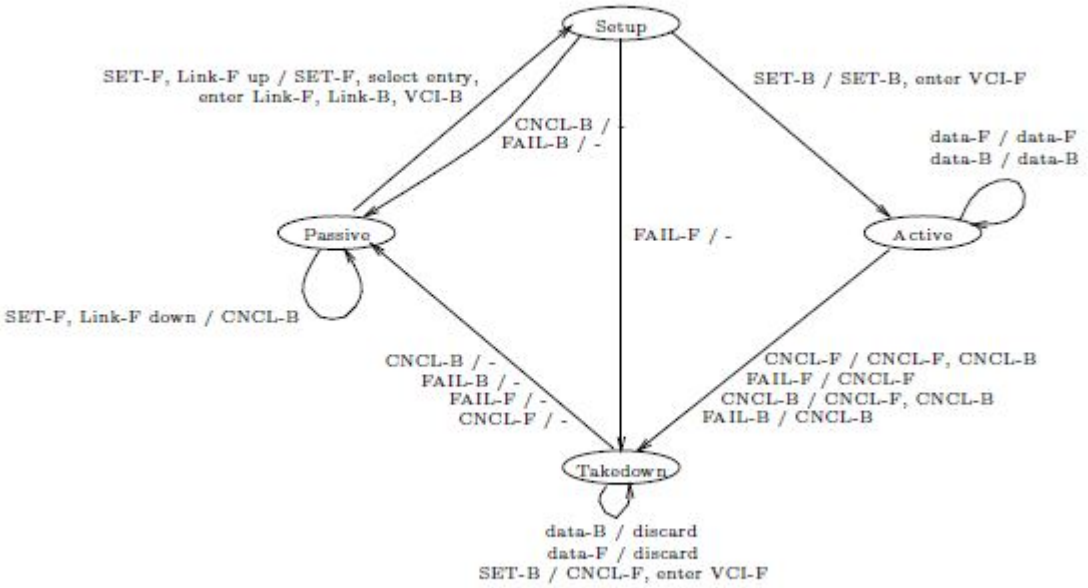


Figure 10.8: A simplified algorithm

Problem 10.1.2 Another attempt appears in Fig. 10.9. Does this work? (Answer: NO, same problem as above)

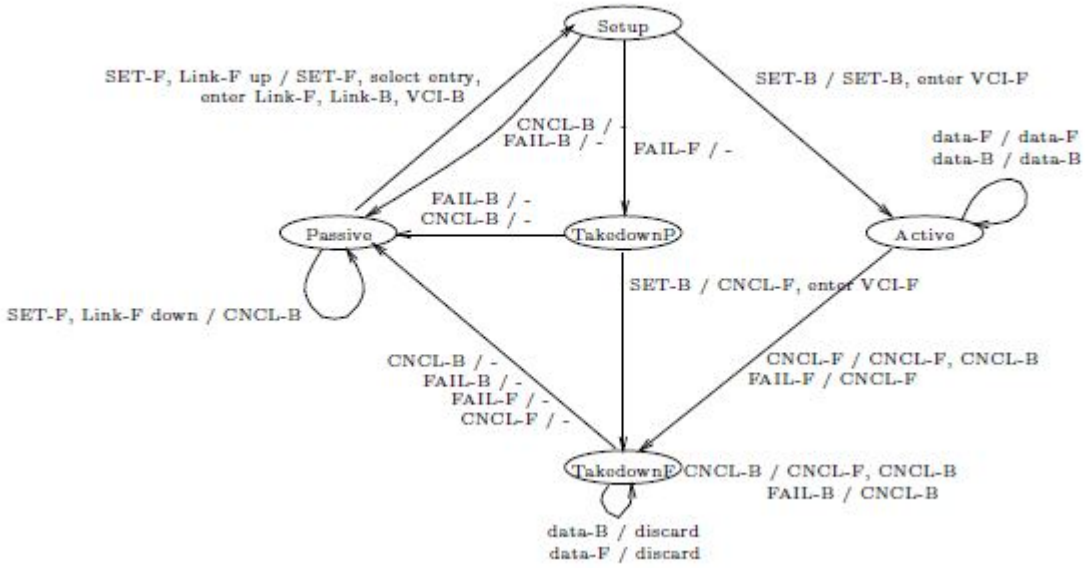


Figure 10.9: Another simplified algorithm

Bibliography

[AAG87a] Y. Afek, B. Awerbuch, and E. Gafni. Applying static network protocols to dynamic networks. In *FOCS 87*, 1987.

[AAG87b] Y. Afek, B. Awerbuch, and E. Gafni. Local failsafe network reset procedure. In J. van Leeuwen, editor, *Distributed Algorithms, 2nd International Workshop, Amsterdam*, pages 197–211, July 1987.

[Atk82] J.D. Atkins. Path control - the network layer of sna. In *P.E. Green, Ed., Computer Network Architecture and Protocols, Plenum Press*, 1982.

[Awe85a] B. Awerbuch. Complexity of network synchronization. *Journal of the ACM*, 22(4):804–823, Oct. 1985.

[Awe85b] B. Awerbuch. A new distributed depth-first search algorithm. *Information Processing Letters*, 20(3), April 1985.

[BC77] G.V. Bochmann and R.J. Chung. A formalized specification of HDLC classes of procedures. In *NTC77*, 1977.

[BG92] D. Bertsekas and R. Gallager. *Data Networks*. Prentice Hall, 1992.

[Bou92] J.-Y. Le Boudec. The asynchronous transfer mode: a tutorial. *Computer Networks and ISDN Systems*, 241, 1992.

[BS88] A.E. Baratz and A. Segall. Reliable link initialization procedures. *IEEE Transactions on Communications*, 36(2):144–152, Feb 1988.

[BSW69] K.A. Bartlett, R.A. Scantlebury, and P.T. Wilkinson. A note on reliable full-duplex transmission over half-duplex links. *Communications of the ACM*, 12(5), May 1969.

[Car82] D.E. Carlson. Bit-oriented data link control. In P.E. Green, editor, *Computer Network Architecture and Protocols*. Plenum Press, 1982.

[Ceg75] T. Cegrell. A routing procedure for the TIDAS message switching network. *IEEE Trans. on Communications*, COM-23(6):575–585, June 1975.

[Cha78] E. Chang. Echo algorithms: Depth parallel operations on general graphs. Memo, Univ. of Toronto, 1978.

[Cha82] E.J.H. Chang. Echo algorithms: Depth parallel operations on general graphs. *IEEE Transactions on Software Engineering*, July 1982.

[Che83] T. Cheung. Graph traversal techniques and the maximum flow problem in distributed computation. *IEEE Trans. on Software Engineering*, SE-9(4):504–512, 1983.

[CL85] K. Mani Chandy and L. Lamport. Distributed snapshots: determining global states of distributed systems. *ACM Transactions on Computer Systems*, 3(1):63–75, Feb Feb.1985.

[CS89] R. Cohen and A. Segall. Distributed query algorithms for high-speed networks. In *Proc. of Workshop on High-Speed Networks, Zurich*, May 1989.

[Dij59] E.W. Dijkstra. A note on two problems in connection with graphs. *Numerische Mathematik*, 1:269–271, 1959.

[DS80] Edsger W. Dijkstra and C.S. Scholten. Termination detection for diffusing computations. *Information Processing Letters*, 11(1), Aug. 1980.

[Eve79] S. Even. *Graph Algorithms*. Computer Science Press, 1979.

[Fin79] S.G. Finn. Resynch procedures and a failsafe protocol. *IEEE Trans. on Communications*, COM-27(6):840–846, June 1979.

[Fri79] D.U. Friedman. Communication complexity of distributed shortest path algorithms. Technical Report LIDS-TH-886, MIT, Feb. 1979.

[Gaf86] Eli Gafni. Perspective on distributed network protocols: a case for building blocks. In *MILCOM '86, Monterey, Calif.*, Oct. 1986.

[Gal76] R.G. Gallager. A shortest path routing algorithm with automatic resynch. March 1976.

[Gal78] R.G. Gallager. Personal communication. 1978.

[Gal82] R.G. Gallager. Distributed minimum hop algorithms. Technical Report LIDS-P-1175, MIT, Jan. 1982.

[GHS83] R.G. Gallager, P.A. Humblet, and P.M. Spira. A distributed algorithm for minimum weight spanning trees. *ACM Transactions on Programming Languages*, 5:66–77, Jan 1983.

[Hag83] J. Hagouel. Issues in routing for large and dynamic networks. Technical Report RC 9942, IBM T.J.Watson Research Center, April 1983.

[HS82] J. Hagouel and M. Schwartz. Correctness proof of a distributed failsafe route table update algorithm. In *3d International Conference on Distributed Computing Systems, Miami*, Oct.1982.

[Hui95] C. Huitema. *Routing in the Internet*. Prentice Hall PTR, Englewood Cliffs, New Jersey, 1995.

[Hum88] Pierre A. Humblet. An adaptive distributed Dijkstra shortest path algorithm. Jan. 1988.

[IBM70] IBM. Synchronous data link control, general information. Technical Report IBM report GA27-3093, 1970.

[ISO81] ISO. Data communication - high level data link control procedures -consolidation of elements of procedures ISO DP 4335, 1981.

[JM80] E.C. Rosen J.M. McQuillan, I. Richer. The new routing algorithm for the arpanet. *IEEE Trans. on Comm.*, COM-28(5):711–719, May 1980.

[JM82] J.M. Jaffe and F.H. Moss. A responsive distributed routing algorithm for computer networks. *IEEE Trans. on Comm.*, COM-30(7, Part II):1758–1762, July 1982.

[LMF88] Nancy Lynch, Yishay Mansour, and Alan Fekete. The data link layer: two impossibility results. 1988.

[LT87] K.B. Lakshmanan and K. Thulasiraman. On the use of synchronizers for asynchronous communication networks,. In J. van Leeuwen, editor, *Distributed Algorithms, 2nd International Workshop, Amsterdam*, pages 257–277, July 1987.

[Lyn68] W.C. Lynch. Reliable full-duplex file transmission over half-duplex telphone lines. *Communications of the ACM*, 11(6), June 1968.

[MM81] G. Markowsky and F.H. Moss. An evaluation of local path id swapping in computer networks. *IEEE Trans. on Comm.*, COM-29(3):329–336, March 1981.

[MRR80] J.M. McQuillan, I. Richer, and E.C. Rosen. The new routing algorithm for the ARPANET. *IEEE Trans. on Communications*, COM-28(5):711–719, May 1980.

[MS79a] P.M. Merlin and A. Segall. A fail-safe distributed routing protocol. *IEEE Trans. Comm.*, COM-29(9):1280–1287, Sept 1979.

[MS79b] P.M. Merlin and A. Segall. A failsafe distributed routing protocol. *IEEE Trans. on Comm.*, COM-27(9):1280–1288, Sept. 1979.

[MW77] J.M. McQuillan and D.C. Walden. The ARPANET design decisions. *Computer Networks*, 1, Aug. 1977.

[Per83] R. Perlman. Fault tolerant broadcast of routing information. In *INFOCOM 83*, 1983.

[Rin] J. Rinde. Tymnet i: An alternative to packet technology. In *Proc. 3d ICCC, Toronto, Canada, August 1976*.

[Ros80] E.C. Rosen. The updating protocol of ARPANET's new routing algorithm. *Computer Networks*, 4:11, 1980.

[SBP72] M. Schwartz, R.R. Boorstyn, and R.L. Pickholtz. Terminal oriented computer communication networks. *Proc. IEEE*, 60:1408–1423, November 1972.

[Sch81] M. Schwartz. Routing and flow control in data networks. In *J.K. Skwirzynski, New Concepts in Multi-user Communication, Proc. NATO Advanced Study Institute, Norwich, U.K.*, 1981.

[SDL80] Synchronous data link control (SDLC) architecture specification. Technical Report SNA-0020-08, Oct 1980.

[Seg81] A. Segall. Advances in verifiable fail-safe routing procedures. *IEEE Trans. Comm.*, COM-29(4):491–497, April 1981.

[Seg83] A. Segall. Distributed network protocols. *IEEE Transactions on Information Theory*, IT-29(1):23–35, Jan. 1983.

[SJ86] A. Segall and J. M. Jaffe. Route setup with local identifiers. *IEEE Trans. on Communications*, COM-34(1), Jan 1986.

[SL83] A.U. Shankar and S.S. Lam. An HDLC protocol specification and its verification using image protocols. *ACM Transactions on Computer Systems*, 1(4):331–368, Nov. 1983.

[SMG78] A. Segall, P.M. Merlin, and R.G. Gallager. A recoverable protocol for loop-free distributed routing. In *Proc. of ICC, Toronto*, June 1978.

[SS80] M. Schwartz and T.E. Stern. Routing protocols. *IEEE Trans. on Comm.*, COM-28(4), April 1980.

[SS91] L. Shabtay and A. Segall. Message delaying synchronizers. In *Distributed Algorithms, 5nd International Workshop, Delphi*, 1991.

[Sta82] IEEE Project 802 Local Area Network Standards. *Logical Link Control, Draft IEEE standard 802.2*, Nov. 1982.

[Sta92] W. Stallings. *ISDN and Broadband ISDN*. Macmillan, 1992.

[SY82] M. Schwartz and T.-K. Yum. Distributed routing in computer-communication networks. In *IEEE Conference on Decision and Control, Orlando, Florida*, Dec 1982.

[Taj77] W.D. Tajibnapis. A correctness proof of a topology information maintenance protocol for a distributed computer network. *Communications of the ACM*, 20(7), July 1977.

[Tym81] L. Tymes. Routing and flow control in tymnet. *IEEE Trans. on Comm.*, COM-29(4):392–399, April 1981.

[ZS80] F.B.M. Zerbib and A. Segall. A distributed shortest path protocol. Technical Report EE Publ. 395, Technion, Israel Institute of Technology, Oct. 1980.