Optimized Single-Mode Cavity for Ceramics Sintering

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***Abstract*— In this paper, we present a novel method of optimization of a single-mode cavity used for sintering ceramics. The goal of the optimization is to *maximize* the electric energy stored in the dielectric and *minimize* spatial variations of tashis field in the domain occupied by the material. Results of simulations of two configurations are discussed.**

I. Introduction

IN RECENT years, there has been an increasing interest in using microwave power for ceramics sintering. Microwave radiation has several inherent advantages over "conventional" heating such as power transfer efficiency, reduced sintering times [1], [2], lower temperature requirements, and improved material characteristics [2], [3]. This type of processing allows precise and controlled heating, and offers processing possibilities not available before [4]. Conventional sintering relies on thermal diffusion of heat from the surface of the sample to its center [5], therefore, relatively steep temperature gradients can develop and they can lead to the cracking of the sample [2]. Microwave heating, on the other hand, is a volume process; therefore, with proper insulation and a smooth enough electric field, uniform heating can be achieved. Although the proposed method seems simple, implementation of such a microwave system is not trivial. Today, most of the research in this field relies on multimode cavities primarily because a multipurpose furnace is required for processing samples of various geometries and dielectric coefficients. The disadvantage of multimode cavities is that due to constructive interference of the electromagnetic (EM) waves propagating inside the cavity hot spots are formed [1]. These hot spots are responsible for a phenomenon called "thermal runaway," in which a local increase in temperature causes a local increase of ohmic losses, which, in turn, causes the temperature to rise even more, thereby completing a vicious cycle that can destroy the whole sample. Nonuniform heating is one of the major limitations of multimode cavities and has been dealt with by applying various sintering techniques (e.g., preheating the sample in an indirect manner such as in the "picket fence" configuration [3]) and introducing field modification devices (e.g., the mode stirrer that is supposed to ensure that the time averaged value of the electric field be uniform throughout the sample). Another possible solution offered was to go up to higher frequencies, but this solution is probably not economically feasible [1], [2].

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An alternative altogether is to turn to single-mode cavities where there is much more control over the field distribution inside the cavity and there is the additional advantage of being able to optimize the geometry of the cavity to fit the parameters of the sample. Single-mode cavities have been proven as adequate for sintering in [6]. Other authors, e.g., Jackson *et al.* [7], have developed a microwave absorption model for a cylindrical sample filling the entire axis of a cylindrical cavity and compared power absorption for the lower TM_{lm0} modes. Iskander [8] and Manring *et al.* [9] have developed numerical codes for solving the fields in single-mode-type resonators and explored the effects of these fields on the sintering process. It has been widely accepted that uniform and efficient heating are both crucial for successful sintering and it is the goal of this paper to present a systematic procedure for optimizing the performance of a single-mode cavity with regards to these two criteria. However, before we go into the details of the optimization process, the following three questions need to be addressed: 1) Are the parameters of the problem stable enough to render such an optimization procedure possible? 2) What is the meaning of optimization in the context of ceramics sintering? and 3) Is there an optimum geometry?

The answer to the first question is relatively simple. With regard to the geometry, we typically know or control the range of parameters, therefore, we do not consider the geometry as a difficulty. The parameter that complicates the situation is the dielectric coefficient since, even if we know its value at room temperature, this value may change quite substantially during the sintering process and what is even worse is that we do not know exactly how this value changes as a function of the temperature. This is a problem that we do not attempt to solve in this paper (i.e., ϵ is assumed to be real), however, we wish to point out that in many cases the change in the imaginary part of ϵ is substantially larger than the change in its real part. As a result, the shift in the resonant frequency (due to the change in the real part of ϵ) is smaller than the reduction of the quality factor of the cavity that is controlled primarily by the increased losses. Consequently, although the source is no longer at resonance, the performance of the cavity is expected to be reasonable because the effective bandwidth of the cavity increases. Furthermore, even if such a shift occurs, it happens on a time scale of seconds or more, therefore, we have the possibility of tuning the cavity mechanically in parallel to the heating process [6]. For these reasons, in this paper, we consider only a given set of parameters.

As for the other two questions, in order to establish a logical optimization procedure, one has to determine the constraints that have to be maximized or minimized. In the framework of the method presented in this paper, we identify two such major constraints: efficient heating, which imposes the delivery of maximum RF power to the sample, and uniform heating, which restrains the variations of the electric field across the sample. Delivering *maximum power* implies that the electric field stored in the sample has to be maximized relative to the total EM power stored in the cavity. Thus, clearly the general trend would be to increase the volume of the sample and, if possible, have it fill the entire volume of the cavity. However, when the sample fills the entire cavity, we run into severe problems of *field uniformity* because of the boundary conditions imposed by the cavity, which, in turn, translate into significant spatial variations of the EM field. Thus, the general trend, if only the uniformity was to guide us, would be to choose a sample geometry that is much smaller than the radiation wavelength in the structure. On grounds of these two countering trends, we developed the optimization method presented in this paper. Its essence is to determine two functions that quantify the power delivered to the sample and provide a measure of the electric-field variations across the sample. Based on these two functions, we can construct a variety of optimum criteria—two of which are presented.

The method to be presented here is general and can be formulated for a complex geometry, however, it is more instructive to illustrate the problem for a relatively simple geometry. Therefore, we present in this paper the design and optimization of a single-mode pillbox cavity. Two specific cases are considered: in the first, the dielectric fills the entire length of the cavity, whereas in the second, it occupies only a fraction of this length. This paper is organized as follows. In Section II, we quantify the criteria of optimization. Section III is a detailed account of the EM analysis, and the optimization process is presented in Section IV.

II. Criteria of Optimization

The criteria to be presented in this section are independent of the geometry of the cavity or sample. Nevertheless, we believe the reader will benefit if we introduce a typical geometry to demonstrate the concept using the simplest possible configuration that still possesses the main physical features of the system under consideration. Two cavity configurations were explored. The first configuration consists of an axi-symmetric cylindrical cavity of radius R and width d, loaded with a dielectric cylinder (ϵ) of radius R_d and width g, as illustrated in Fig. 1(a). The second configuration is just the limiting case of the former when $g = d$ [see Fig. 1(b)]. Furthermore, to define the problem completely, we shall assume that the sample's dielectric coefficient ϵ and width g are given and fixed as are the cavity's width d and resonance frequency. Therefore, the parameters we may vary in order to optimize the cavity's performance are the cavity's radius R and the sample's radius R_d.

The methods for calculation of the resonant frequency are described in Section III and, for the moment, we shall assume that the EM field in the cavity is known. With this assumption in mind, we proceed to determine the optimization criteria.

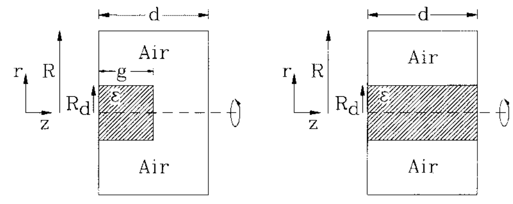

Fig. 1. (a) Pillbox cavity with a dielectric "disk." (b) Pillbox cavity with a dielectric "cylinder."

As already indicated, the goal is to *maximize* the amount of energy deposited in the ceramic volume and *minimize* the spatial variations of the electric field. In order to quantify these requirements, the following two functions are defined. The first function provides a measure of the heating efficiency in the cavity. For its definition, we rely on the fact that the heat is proportional to the power dissipated in the sample, which, by virtue of Ohm's law, is proportional to the amount of electric energy stored in the dielectric denoted by $U_E^{(D)}$. In turn, this energy has to be compared to the total EM energy in the cavity—denoted by $U^{(T)}$. Therefore, we define the *energy efficiency function* as the ratio $E_{\text{eff}} = U_E^{(D)}/U^{(T)}$, which is to be maximized.

The second function to be defined is the *smoothness function* (S). Since it is the electric field that is responsible for heating the sample, a smooth electric field will result in a uniform heating process. The degree of uniformity of the electric field Δ is determined by calculating the standard deviation of the electric field in the volume (V_d) of the dielectric normalized to average electric field intensity. The average of an arbitrary function $G(\mathbf{r})$ is defined as

$$\langle G \rangle \equiv \frac{1}{V_d} \int_{V_d} dV G(\mathbf{r}) \tag{1}$$

thus, Δ

$$\Delta \equiv \frac{\sqrt{\langle |\vec{E}|^2 \rangle - |\langle \vec{E} \rangle|^2}}{\langle |\vec{E}| \rangle} \tag{2}$$

and, consequently, we may define the smoothness function as

$$S \equiv 1 - \frac{\Delta}{\Delta_{\max}} \tag{3}$$

where $\Delta_{\max}$ is the maximum value of Δ in the range of parameters of interest. The smoothness function is unity when the field is completely uniform and zero in the case of maximum variations. Bearing in mind that uniform distribution of the field is critical for uniform heating, we would like this function to be as close to unity as possible, i.e., it should be maximized. This definition is somewhat arbitrary in the sense that it is not unique and perhaps other definitions can be considered, however, it has been chosen firstly because standard deviation is a very natural way of quantifying the variations in any function, and secondly, the variance of the electric field is connected with the square of the electric field and this, in turn, is proportional to the energy; therefore, requiring that the variance be as small as possible is actually a statement on the spatial distribution of energy in the dielectric, which, as indicated above, should be as smooth as possible.

III. The EM Problem

In order to calculate the two functions introduced above, it is necessary to establish the EM field in the entire volume of the cavity. We begin with an *analytical* solution of the fields for the case where the ceramic fills the entire width of the cavity [Fig. 1(b)]. This is followed by a *variational* calculus approach for a partially filling sample and we conclude this section with a description of the *numerical* approach used to simulate the same configuration.

A. Analytic Approach

The basic configuration [10] is illustrated in Fig. 1(b). It is assumed that the width (d) of the cavity is much smaller than the wavelength in vacuum. Furthermore, the system is assumed to be azimuthally symmetric, therefore, only the TM_{010} is excited, i.e., $\partial_z \sim 0$. This mode is determined by the z component of the magnetic vector potential (A_z). Subject to the symmetry of the system, two components of the EM field E_z and H_ϕ can be derived from this component. The solution of the magnetic vector potential reads

$$A_z(r) = \begin{cases} AJ_0\left(\frac{\omega}{c} nr\right), & 0 \le r \le R_d \\ B\left[J_0\left(\frac{\omega}{c} r\right)Y_0\left(\frac{\omega}{c} R\right) - Y_0\left(\frac{\omega}{c} r\right)J_0\left(\frac{\omega}{c} R\right)\right], & R_d \le r \le R \end{cases} \tag{4}$$

where $n = \sqrt{\epsilon}$, ω is the angular frequency in the system, c is the phase velocity of an EM wave in vacuum, and $J_0(x)$, $Y_0(x)$ denote the zero-order Bessel function of the first and second kind, respectively. Imposing the boundary conditions at $r = R_d$ leads to the following expression for the dispersion relation:

$$\frac{J_0\left(\frac{\omega}{c} R_d\right)Y_0\left(\frac{\omega}{c}R\right) - Y_0\left(\frac{\omega}{c}R_d\right)J_0\left(\frac{\omega}{c}R\right)}{J_1\left(\frac{\omega}{c} R_d\right)Y_0\left(\frac{\omega}{c}R\right) - Y_1\left(\frac{\omega}{c}R_d\right)J_0\left(\frac{\omega}{c}R\right)} = \frac{J_0\left(\frac{\omega}{c} nR_d\right)}{nJ_1\left(\frac{\omega}{c} nR_d\right)} \tag{5}$$

where $J_1(x)$, $Y_1(x)$ are the first-order Bessel functions of the first and second kind. This dispersion relation determines all the azimuthally symmetric $(\partial_\phi \sim 0)$, radial modes (TM_{0s0}). However, by careful choice of parameters and an adequate feeding system, the discussion can be confined to the first mode, i.e., $s = 1$. Once the parameters of the system are chosen such that (e.g., at 2.45 GHz) this relation is satisfied the expressions for the smoothness function S and the energy efficiency function E_{eff} can be determined analytically. Note that the boundary conditions impose a relation between the two amplitudes of (4) (A and B) and, consequently, the two functions are independent of the absolute value of the amplitude of the EM field. Furthermore, (5) provides one constraint, which limits the five degrees of freedom (ω, R, R_d, ϵ, and d). For example, once ω is set, and ϵ is chosen, R can be imposed by (5) for an almost arbitrary choice of R_d (the range being limited by the single-mode constraint). The details of the optimization procedure will be presented in the following section. We proceed now by presenting two alternative methods of solving the EM field.

B. The Variational Approach

For the case in which the dielectric material only partially fills the width of the cavity [see Fig. 1(a)], separation of variables cannot be directly applied. Among the quasi-analytic approaches, we chose the variational approach. In order to satisfy the boundary conditions at the metallic walls, we consider a solution of the wave equation that has the following form:

$$A_z(r, z) = \sum_{s=1, m=0}^{\infty} \alpha_{s,m} J_0\left(\frac{p_s}{R} r\right) \cos\left(\frac{\pi m}{d} z\right) \tag{6}$$

where p_s are the zeros of the zero-order Bessel function of the first kind $[J_0(p_s) = 0]$. This will be used as a trial function in the Ritz variational method [11]. Our first step is to define the Lagrange density for the magnetic vector potential

$$\mathcal{L} = \frac{1}{2}\left|\vec{\nabla} \cdot \vec{A}\right|^2 - \frac{1}{2}\left(\frac{\omega}{c}\right)^2 \epsilon(r, z)\left|\vec{A}\right|^2 \tag{7}$$

where it can be readily checked that Lagrange equation of motion (with this choice of Lagrangian density) is completely equivalent to the homogeneous wave equation for the magnetic vector potential. Rather than following the differential approach, here we adopt the integral one in the sense that we define the action

$$\mathcal{I}(\alpha) = \int_V dV \mathcal{L}(\vec{r}, \alpha) \tag{8}$$

where $\alpha = \{\alpha_i\}$ denotes the set of unknown coefficients $\alpha_{s,m}$, and V denotes the entire volume of the cavity. Since an exact solution of the wave equation implies that this integral has an extremum, we determine the approximate solution of the problem by requiring that

$$\frac{\partial \mathcal{I}(\alpha)}{\partial \alpha} = 0. \tag{9}$$

As $\mathcal{I}$ is a quadric form in the variables $\alpha_{s,m}$, this expression can be formulated in the following matrix form:

$$\begin{pmatrix} U_{11} & U_{12} & \cdots & U_{1M} \\ U_{21} & U_{22} & \cdots & U_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ U_{M1} & U_{M2} & \cdots & U_{MM} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \cdots \\ \alpha_M \end{pmatrix} = 0 \leftrightarrow U \cdot \alpha = 0 \tag{10}$$

where formally

$$U_{i,j} = U_{i,j}(R, R_d, \omega, \epsilon_r) \equiv \frac{\partial}{\partial \alpha_j}\left(\frac{\partial \mathcal{I}}{\partial \alpha_i}\right). \tag{11}$$

The dispersion relation in this case is determined from the condition that for a nontrivial solution $(\alpha \ne 0)$ the determinant of the matrix U must nullify, i.e.,

$$\det(U) \equiv \begin{vmatrix} U_{11} & U_{12} & \cdots & U_{1M} \\ U_{21} & U_{22} & \cdots & U_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ U_{M1} & U_{M2} & \cdots & U_{MM} \end{vmatrix} = 0. \tag{12}$$

From this point on, the procedure is identical with the analytic approach and for a set of geometrical parameters that satisfy the dispersion relation above, the smoothness function S and the energy efficiency function E_{eff} can be determined analytically. As in the previous case, S and $E_{\rm eff}$ are independent of the absolute value of the vector α.

When dealing with function series, as presented in (6), the question of how many terms to take naturally arises. An estimate of the number of Bessel harmonics needed to be taken in (6) can be derived by considering the step function

$$f(r) = \begin{cases} 1, & 0 \le r \le R_2 \\ 0, & R_2 < r \le R_1. \end{cases} \tag{13}$$

In principle, this function may also be represented by an infinite series of Bessel function. In practice, the number of terms considered is finite, therefore, the immediate question is what is the number of terms necessary to bring the error (in the total "energy") associated with this representation below, say, 2%. For example, in order for the error to drop below 2%, for the case when $R_2/R_1 = 0.5$, at least 20 Bessel harmonics are required in the sum [12]. Since the potential function is expected to be continuous, this error will provide us with an upper bound of the error in our problem.

A similar approach may be employed in order to estimate the error associated with the finite number of cosine harmonics, as in (6). Consider the step function

$$f(z) = \begin{cases} 1, & 0 \le z \le g \\ 0, & g < z \le d \end{cases} \tag{14}$$

that, in principle, may be represented by an infinite series of cosine harmonics. When a finite number of terms are considered then for $d = g/2$, the relative error in the "energy" (square of the function) drops below 1% only if the number of terms is larger than 21. Thus, for a typical geometry $R_d/R \simeq 0.5$ and $d/g \simeq 0.5$, over 400 terms are necessary for an error smaller than 2%.

C. The Numerical Approach

Another approach to determine the magnetic vector potential in the cavity is to numerically solve the wave equation. For this purpose we used the "Poisson-SuperFish" codes written at Los Alamos National Laboratory. The output file containing the fields specified at the points of intersection on a user defined grid was used as an input for a Matlab program, which processed the data and calculated the smoothness function and the energy efficiency function. The grid in both directions was 10^{-4} m, therefore, at 2.45 GHz and for $\epsilon = 10$, the error is estimated to be less than 2%, and this error scales as $\sqrt{\epsilon}$.

The procedure was tested against the analytical solution for a dielectric that fills the entire length of the cavity, and a very good agreement was found. Fig. 2 illustrates two typical field distributions for the two geometries of interest.

The results of the variational method at the limit of full occupancy fits well with the analytic results and SuperFish. For the case when the dielectric partially fills the width of the cavity, the agreement of SuperFish with the variational method was not satisfactory since we took only approximately 100 terms (20 radial harmonics and five longitudinal harmonics). For an error of less than 2%, approximately 500 terms are required, in which case, the numerical errors became quite large. For this reason, we considered only the numerical solution. In the remainder of this paper, we present results of the analytic and numerical methods.

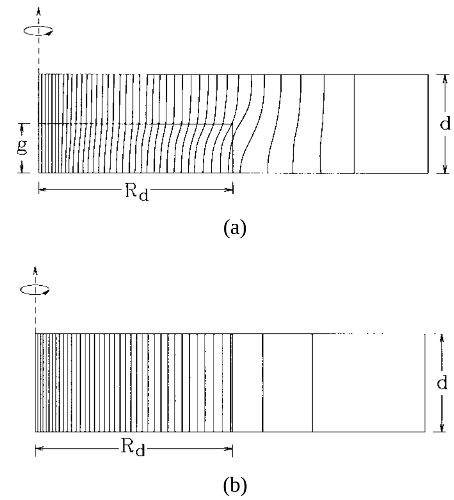

Fig. 2. (a) Equipotential curves of A_z as calculated by SuperFish, for the case where $\epsilon = 4$, $g = d = 1$ cm, $R \simeq 2.6$ cm, and $R_d \simeq 1.3$ cm. As can be seen, the vector potential is independent of the z coordinate, is maximum at $r = 0$, and decreases monotonically until the conductive outer surface. (b) Equipotential curves of A_z for the case $\epsilon = 4$, $g = 0.5$ cm, $d = 1.0$ cm, $R \simeq 2.9$ cm, and $R_d \simeq 2$ cm. The geometry of the sample dictates that the vector potential is z, as well as r, dependent. Note the discontinuity associated with the normal electric field.

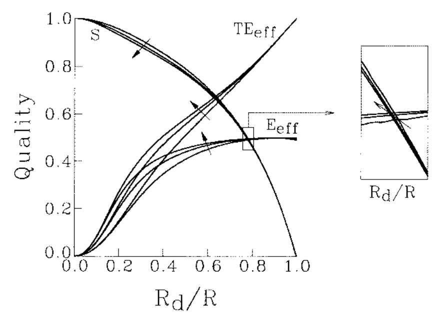

Fig. 3. Smoothness function (S), energy efficiency function $(E_{\rm eff})$, and the total energy efficiency function plotted as a function of the ratio (R_d/R) for three different dielectric coefficients ($\epsilon = 2, 3, 4$); the arrows point toward increasing values of ϵ. The adjacent frame illustrates the intersection region enlarged; the arrow in this frame points toward increasing values of ϵ.

IV. Optimization Procedure

A. Analytic Approach

After defining the dispersion relation and the EM field in the cavity, the smoothness function (S) and energy efficiency function $(E_{\rm eff})$ can be established, thus we can examine the variation of these two functions when the parameters are changed—subject to the constraint associated with the dispersion relation. Fig. 3 illustrates both the smoothness and energy efficiency function when the ratio R_d/R is varied for three different dielectric coefficients ($\epsilon = 2, 3, 4$); the width of the cavity and the dielectric is $d = 1$ cm. The intersection region is enlarged in the upper right-hand-side frame; the arrows point toward an increasing ϵ. Let us examine this plot in the following systematic way.

It is evident from this figure that the smoothness function *decreases* monotonically from unity, for which the radius of the dielectric approaches null and drops to zero when the dielectric fills the entire cavity.

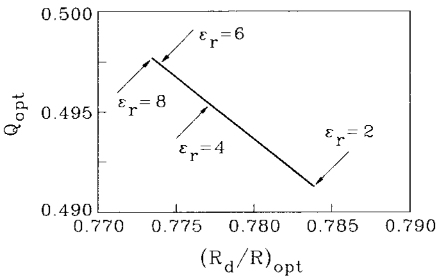

Fig. 4. Quality of the optimization as a function of $(R_d/R)_{\rm opt}$ calculated for a dielectric coefficient, which varies from 2 to 8.

The smoothness function does not change dramatically with the dielectric coefficient.

The energy efficiency function *increases* as a function of R_d/R and approaches an asymptotic value that is almost independent of the dielectric coefficient. For comparison, we also plot the *total energy efficiency function* $\rm TE_{eff}$, which is the ratio between the total energy stored in the dielectric domain (including the magnetic energy and the total energy in the cavity. As expected, this function approaches unity for $R_d/R = 1$.

For intermediary values, $R_d/R \sim 0.3$; both $E_{\rm eff}$ and $\rm TE_{eff}$ are quite significantly ϵ dependent. This fact can become critical when the system operates away from the optimal range of parameters to be defined later.

As indicated, when we introduced S and $E_{\rm eff}$, our purpose is to design a system for which both functions are maximized. At this point, we have a variety of possibilities in choosing the optimum: at the intersection of the two lines, at the location of the maximum of the product of the two functions, etc. We shall begin with the first possibility, which is simpler, and continue in the following section to explore the second.

With this choice of criterion, we observe that, in the intersection region, both S and $E_{\rm eff}$ are almost independent of the dielectric coefficient (ϵ) and at the intersection $R_d/R \sim 0.78$. For values of ϵ, which vary from 2 to 8, the "quality" factor varies linearly with R_d/R, as illustrated in Fig. 4. Note that the variation in quality is very small.

It is interesting to point out that for intermediary values of R_d/R, $E_{\rm eff}$ increases with ϵ since more energy can be stored in the sample, while at the same time, the smoothness function decreases. This change in the smoothness function can be intuitively explained by the fact that as ϵ becomes higher, the discontinuity between the air's dielectric constant and that of the sample increases, therefore, the distortion in the electric field is larger.

Although our main goal in this paper is optimization of a cavity, it is important to point out that Fig. 3 provides us with important information regarding nonoptimized regimes of operation.

Consider, for example, the case when *uniform heating* is crucial. According to Fig. 3, a reasonable choice is $0.2 \geq R_d/R \geq 0.1$.

On the other hand, if fast heating is the primary concern, even at the expense of field uniformity, then the constraints become more flexible and the radii ratio can vary as $1.0 > R_d/R > 0.5$.

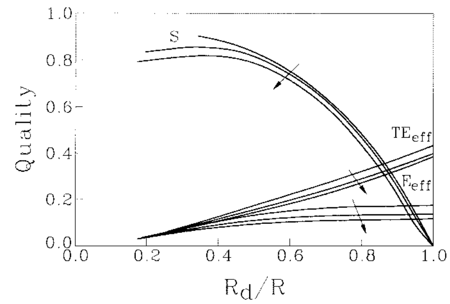

Fig. 5. Smoothness function (S), energy efficiency function $(E_{\rm eff})$, and the total energy efficiency function plotted as a function of the ratio R_d/R for three different dielectric coefficients ($\epsilon = 2, 3, 4$); arrows point toward increasing values of ϵ.

A. Numerical Approach

For the case that $g = d/2$ (d is still 1 cm), the optimization is performed using the EM field as calculated with SuperFish. A typical field distribution in such a cavity is illustrated in Fig. 2. Note the "fringe" effects in the vicinity of the edge of the dielectric; this type of field distribution can be a major drawback when uniform heating is a critical constraint. We have repeated the same calculation, which provided the results presented in Fig. 3, for the partially filling sample, and the results are illustrated in Fig. 5. As before, the following main results are pointed out:

- All three curves are more sensitive to variations in ϵ.
- The optimization quality is $0.2 \geq Q_{\rm opt} \geq 0.1$, compared to the order of 0.5 in the previous case.
- The *optimal* value of R_d/R is closer to unity ($R_d/R \sim 0.95$), compared to 0.78 in the previous case.
- A careful inspection of the curves reveals that, contrary to the case where the dielectric fills the whole width of the cavity, the energy efficiency function *decreases* with increasing ϵ. This behavior is caused by the "mirror effect" created by exposing the top surface of the dielectric disk to the fields propagating inside the cavity. Therefore, the higher the dielectric constant of the material, the better this "mirror" reflects the waves.

V. Discussion and Conclusions

The specific system under consideration has six degrees of freedom: the frequency, the two radii, the width of the cavity and the sample, and finally, the dielectric coefficient. In the analysis presented above, it was assumed that four of these parameters are given and, in practice, there are only two degrees of freedom, namely, the radius of the cavity R and that of the sample R_d. The ratio between these two quantities is set by the dispersion relation, e.g., (5). In order to determine every one of the radii, one has to introduce an additional constraint. In the previous discussion, the constraint that was suggested was the *intersection* point of the smoothness function with the energy efficiency function. As already indicated, the main advantage of this criterion is that, at least for the $g = d$ case, the dependence on the dielectric coefficient is negligible. However, we found that it is quite sensitive to the width of the ceramic (g).

Another valid criterion is the *product* of the two functions, i.e., $F = E_{\rm eff} \times S$. It is evident that this function has a

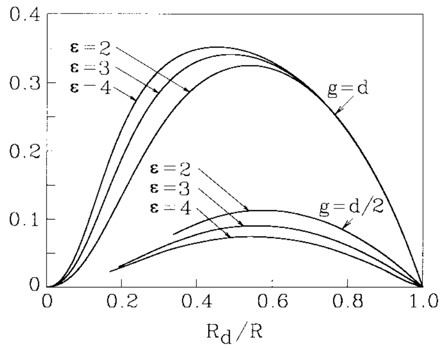

Fig. 6. The product criterion $F = S \times E_{\rm eff}$ as a function of the ratio R_d/R for three different dielectric coefficients.

maximum since, at $R_d/R \approx 0$, the energy efficiency function vanishes, whereas at $R_d/R \approx 1$, the smoothness function is zero. Since we found that these are positive defined functions, we expect at least one extremum between these two points. This maximum is clearly revealed in Fig. 6 where we plot F for the three cases presented in Fig. 3. Note that for dielectric coefficients in the range $2 \le \epsilon \le 4$, the peak varies slowly between $R_d/R \approx 0.5$ to 0.55. In addition, the same criterion for $g = d/2$ peaks for the same range of radii. With this regard, the product criterion (F) seems advantageous compared to the intersection criterion.

In conclusion, in this paper, we have presented a method to optimize a single-mode cavity used for ceramic sintering. It is instructive to present the method in terms of a simple pillbox cavity. The advantage of this simple model consists of the analytic expressions for the EM field. Although a more general case was formulated by means of variational calculus, its accuracy was not sufficient for our purposes, therefore, we resorted to the numerical code SuperFish for evaluation of the eigenfrequency and the field components in the cavity. With the frequency and the various fields established in the entire space of the cavity, we have devised two functions that allowed us to formulate, in a mathematical way, an optimization criteria. The first function, called the "smoothness function" (S), represents the variation in the electric energy in the dielectric domain; it is defined in such a way that it has a maximum when the field is uniform. The second function, called the "energy efficiency function" ($E_{\rm eff}$), represents the electric-field energy stored in the dielectric domain relative to the total EM energy stored in the cavity. For best heating, we aim toward maximum electric field in the dielectric. As expected, the results of the optimization are dependent on the criterion that is being used. For the criterion presented above (F), which we regard as advantageous, the ratio is typically $R_d/R \sim 0.5$. Another important conclusion derived is that, for the case where the sample fills only part of the width of the cavity ($g = d/2$), the "quality" factor of the optimization yielded much lower values than for the case where $g = d$ ($Q_{\rm opt} \sim 0.15$ versus ~ 0.5).

REFERENCES

[1] M. C. L. Patterson, P. S. Apte, R. M. Kimber, and R. Roy, "Batch process for microwave sintering of Si_3N_4," *Microwave Processing Mater. III,* vol. 269, pp. 2911–300, 1992.

[2] J. G. P. Binner and T. E. Cross, "Applications for microwave heating in ceramic sintering: Challenges and opportunities," *J. Hard Mater.,* vol. 4, pp. 177–185, Dec. 1993.

[3] M. A. Janney, C. L. Calhoun, and H. D. Kimrey, "Microwave sintering of zirconia," *Ceramic Trans.,* vol. 21, pp. 311–318, 1991.

[4] W. H. Sutton, "Microwave processing of ceramics—An overview," *Microwave Processing Mater. III,* vol. 269, pp. 3–20, 1992.

[5] M. A. Janney, Hal D. Kimrey, and James O. Kiggans, "Microwave processing of ceramics: Guidelines used at the Oak Ridge National Laboratory," *Microwave Processing Mater. III,* vol. 269, pp. 173–185, 1992.

[6] D. S. Patil and B. C. Mutsuddy, "Microwave sintering of alumina ceramics in a single-mode applicator," *Ceramic Trans.,* vol. 21, pp. 301–309, 1991.

[7] H. W. Jackson, M. Barmatz, and P. Wagner, "Microwave power absorption profile in a cylindrical sample contained in a resonant cylindrical cavity," *Microwave Processing Mater. IV,* vol. 347, pp. 317–323, 1994.

[8] M. F. Iskander, "Modeling the microwave process—Challenges and new directions," *Ceramic Trans.,* vol. 36, pp. 167–187, 1993.

[9] E. B. Manring and J. Asmussen, Jr., "Numerical model for the modes of a lossy, coaxially loaded cylindrical cavity," *Ceramic Trans.,* vol. 36, pp. 201–212, 1993.

[10] R. E. Collin, *Field Theory of Guide Waves,* 2nd ed. New York: IEEE Press, 1991, pp. 430–438.

[11] L. Cairo and T. Kahan, "Variational techniques in electromagnetism," *Blackie,* pp. 1–67, 104–108, 1965.

[12] L. Schächter, *Beam Wave Interaction in Periodic and Quasi-Periodic Structures.* Berlin, Germany: Springer-Verlag, 1996, pp. 63–70.

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