Exponential gain in a Smith–Purcell amplifier

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We study here the gain of an amplifier based on the Smith–Purcell effect. The grating is characterized by a reflection matrix and the gain, in the exponential region, is calculated as a function of the beam height and thickness.

In the present letter we investigate the gain of an amplifier, based on the Smith–Purcell (SP) effect, in the regime where the gain is exponential. We consider a metallic grating: its surface in the $x$-$y$ plane, the $y$ axis is parallel to the grooves, and its length in the $x$ direction much larger than the period $L$ of the grating. When a monochromatic electromagnetic (EM) plane wave is incident upon the diffraction grating in the $x$-$z$ plane, the wave is scattered by the surface in the same plane into a manifold of harmonics, whose wave vectors are related both to the grating periodicity and the incident wave parameters. Some of these harmonics are propagating waves (PW's), which travel away from the surface, and the rest are evanescent surface harmonics (ESH), which travel along the grating surface with phase velocity smaller than $c$, the speed of light. When an electron beam propagates near the surface, parallel to the grating, and along the $x$ axis, it can move synchronously with one of the ESH, and exchanges energy with it. This resonant ESH may deliver some of its energy to the PW's, and the latter would be amplified along the grating. It is the gain of the propagating waves in the presence of the beam that concerns us here, and thus this system is identified as a Smith–Purcell amplifier (SPA). If the grating is long enough, the gain would be exponential, and the system is similar to a traveling-wave amplifier (TWA).

Free-electron amplifiers (and lasers) based on the SP effect have been suggested in recent years. Rusin and Bogomolov were probably the first to propose a device based on the SP effect. More recently Yariv and Shih and Wachtell studied independently the gain of SP-like amplifier and found it to be not exponential. In their analysis they explicitly assumed that the synchronous wave does not change along the beam, much in the spirit of the so-called bremsstrahlung free-electron lasers (FEL's) of Hopf et al. Gover and Livni have investigated this amplifier as a waveguide (i.e., a closed system slow wave structure) and solved the beam-wave equations in a self-consistent manner to find an exponential gain.

This letter presents a study of an open SPA, where the gain of the scattered propagating waves is investigated explicitly as part of the complete system of incident wave, evanescent harmonics, interacting beam, and the diffracted waves. The PW's are analyzed in terms of the linear response of the SP system to the exciting incident wave. The poles of the response function determine in a self-consistent way the characteristics of the amplified waves. The growth rate is calculated in terms of the height $h$ of the beam above the grating, and its thickness $D$ (see Fig. 1).

Our system is composed of three parts: EM waves, grating surface, and the electron beam.

(a) The EM fields above the periodic structure are described as a superposition of Floquet harmonics (FH). Since the grooves are uniform along the $y$ direction we consider only TM waves, and write the magnetic component of the EM field as

$$H_y(x,z,\omega) = \sum_n \int_{-\pi/L}^{\pi/L} dk \left[ F_n^{(inc)}(k,\omega) e^{-i\beta_n z} + F_n^{(exc)}(k,\omega) e^{i\beta_n z} \right] e^{-i\omega z},$$

where only a single frequency, $\omega$, is assumed (i.e., that of the incident wave). In Eq. (1) the summation is over the FH indices $n = 0, \pm 1, \pm 2, \ldots$, the integration is carried over the $k$'s in the first Brillouin zone (along the $k_x$ axis only), $k_n = \pi + 2\pi n/L$ and $\beta_n = [k_n^2 - (\omega/c)^2]^{1/2}$ can be either real for ESH, or imaginary for PW.

(b) The EM properties of the grating surface are described in terms of a characteristic reflection matrix, $R_{nm}$, which prescribes the amplitudes of the scattered waves when the amplitudes of the incident waves are given. If we divide the total field of Eq. (1) into incident harmonics, $F_n^{(inc)}$, i.e., $\beta_n$ positive, and scattered harmonics, $F_n^{(exc)}$, with negative $\beta_n$, the boundary conditions on the grating surface specify

$$F_n^{(exc)}(k,\omega) = \sum_m R_{mn}(k,\omega) F_m^{(inc)}(k,\omega).$$

Notice that the matrix $R_{nm}$ connects both radiating and evanescent harmonics. We assume here that $R$ is given, and thus the grating is completely specified electromagnetically.

(c) The electron beam is described here using the fluid model equations. The electrons are restricted to move only along the $x$ axis, and their average velocity is $V_0$. The beam is

![FIG. 1. Configuration of the Smith–Purcell amplifier.](image-url)