Relativistic quantum mechanical analysis of a free electron laser

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A free electron laser based on a magnetic undulator in its linear regime is analyzed using the Klein–Gordon equation to describe the electron dynamics. Special attention is paid to an electromagnetic wave with a general polarization and an arbitrary spectrum. The influence of high harmonics is investigated. The effect of decreasing periodicity of the wiggler on the analysis is investigated too.

I. INTRODUCTION

In the last ten years the search for new radiation sources has increased significantly. A variety of methods were proposed, in most of them electron-beam interacts with external electromagnetic field, thus direct contact with matter—as in the present lasers—is avoided. From all existing proposals the magnetic wiggler seems to be the most attractive. Its gain was calculated both classically and quantum mechanically in several different ways. Madery’ in the early seventies, had calculated the gain using quantum considerations, in an electron rest frame of reference. The quantities of interest such as, frequency, differential (or total) cross section, etc., could be readily “translated” to an observer in a laboratory frame of reference. Others prefer the laboratory frame and thus a Dirac formalism is necessary. A classical approach—as pointed out by Madery—cannot be used because, indeed it is widely used.

With the progress in the FEL (free electron laser) theory, some new questions arise such as: (a) How an arbitrarily polarized electromagnetic wave interacts in such a device? (b) Are the higher harmonics always negligibly small? In this context, it is legitimate to neglect the higher harmonics in the microwave regime, which is much lower (in frequency scale) than the frequency we deal with. Moreover, in microwave devices the energy stored in the higher harmonics increases with increasing frequency, thus the assumption of one propagating harmonic should be better justified.

Another question which will be only partially treated in this work is: (c) Can we decrease the spatial wiggler periodicity without a basic change in the analysis?

In the study proposed, particle dynamics is governed by a wave function which is a solution of the Klein–Gordon equation in the presence of electromagnetic potentials, which themselves are solutions of inhomogeneous wave equations derived from Maxwell equations. The coupling between these two waves is through a suitable identification of the Klein–Gordon current four-vector with the one-particle electromagnetic one. The macroscopic electromagnetic current is defined assuming that the initial distribution is uniform in the usual space and extremely peaked in the momentum space.

II. DYNAMIC CONSIDERATIONS

We shall consider a one-dimensional system such that the only spatial changes involve the x coordinate (\( \partial / \partial y = \partial / \partial z = 0 \)). The wiggler is modelized by a circularly polarized static magnetic field which can be derived from the following vector potential:

\[
A_\omega = (B_\omega / k_\omega) [I_y \cos(k_\omega x) + I_x \sin(k_\omega x)],
\]

(1)

where \( k_\omega = 2\pi / L \) and \( L \) is the wiggler spatial periodicity. A typical undulator design has a spatial periodicity \( L \approx 1 / 3 \) cm and a magnetic strength \( B = 0 \pm 10 \) T.

Bearing in mind the quantum electron dynamics on one hand and the classical treatment of the radiation field on the other, the latter will be described hereafter by two potentials \( A \) and \( \phi \). The magnetic vector potential \( (A_{\text{rot}}) \) is formally represented by a superposition of plane waves:

\[
A_x = \int d\omega \, dk \, L(\omega, k) e^{j\omega t - jkx},
\]

\[
A_y = \int d\omega \, dk \, \Lambda(\omega, k) e^{j\omega t - jkx},
\]

\[
A_z = \int d\omega \, dk \, \Gamma(\omega, k) e^{j\omega t - jkx}.
\]

(2)

Together with the Lorentz gauge

\[
\nabla \cdot A + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0,
\]

we are able to define the electric scalar potential:

\[
\phi = c^2 \int d\omega \, dk \frac{k}{\omega} L(\omega, k) e^{j\omega t - jkx}.
\]

(2a)

From earlier communications we know that \( \phi \) is significant only in those cases where the electron–electron interaction is important. We shall limit our analysis to low-density beams, thus \( \phi \) is negligible. The transverse components of the magnetic vector potential are the dominant ones \( (A_x \) is linearly dependent on \( \phi \); see Eq. (2)) in our problem, and their dynamics is governed by a nonhomogeneous wave equation (MKSA units):

\[
\left( \frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) A_x = -\mu_0 j_1.
\]

(2b)

The notation \( A_x \), \( j_1 \) is introduced to stress the fact that we deal with the transverse (to the initial direction of motion) components only. The electron motion is ruled by the superposition of these two real vector potentials:

\[\text{[Equation]}\]