# Mirror manipulation by attractive and repulsive forces of guided waves

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**Abstract:** Two mirrors guiding laser light may experience an either attractive or repulsive force, according to the type of eigenmode they guide. We propose a method for the control over the motion of a mirror by changing the operation wavelength along the dispersion curve of the mode. In addition, a novel method for trapping a mirror in a stable equilibrium, based on a superposition of two modes, is presented. The mirror is then trapped by being exposed to light only from one of its sides.

**OCIS codes:** (260.21100) Electromagnetic theory; (140.7010) Trapping; (130.2790) Guided waves; (230.1480) Bragg reflectors

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## 1. Introduction

Triggered by the work of Ashkin [1], much research has been devoted in the past thirty years to the subject of radiation pressure and optical forces on neutral bodies. In addition to the trapping and manipulation of particles (reviewed in Ref. [2]), there has been a significant interest in the effect of radiation pressure on mirrors. A typical configuration is that of two partially reflective mirrors forming a Fabry-Perot interferometer [3, 4], having one mirror suspended. The suspended mirror is subject to radiation pressure forces leading to a non-linear system that may have states of stable equilibrium. In another setup, the repelling forces of incident radiation were used for the damping of the Brownian motion of a mirror employing a feedback mechanism [5].

Light may not only repel a mirror, but can also attract it. Attraction between macroscopic bodies is known to occur in *scattering* problems where evanescent waves are created. For example, a dielectric sphere may be attracted to a dielectric substrate from which an evanescent wave is incident [6] and trapped with the addition of a probe [7]. Evanescent waves created when light coming from one medium is scattered from a gap with another medium may cause the two bodies to be attracted [8]. The attractive quantum mechanical Casimir force between two metallic plates was shown to be due to contributions from a spectrum of *guided modes*, some generating attractive and some repulsive forces [9].

In the present study, we propose using guided modes between two mirrors as means to control their motion. Contrary to the Casimir force case, the modes here are excited by an external source (laser), resulting in an effect that may be due to a single eigenmode, and a force that is orders of magnitude stronger. We begin by a general analysis of the transverse force exerted by TM and TE modes, either even or odd, on the guiding mirrors. The pressure on the mirrors can be either positive or negative depending on the transverse behavior of the mode. A system of two Bragg mirrors is used to demonstrate the feasibility of the manipulation concept. We present a scheme for controlling the pressure applied on the mirrors by varying the frequency of one guided mode around a central frequency, for which the mirrors are designed. Finally, we propose a way to hold a mirror in a stable equilibrium using two guided modes.

## 2. General analysis

Let us assume that an electromagnetic wave of time-dependence $e^{j\omega t}$ is guided by two mirrors, separated by a distance $2D_{int}$, as illustrated in Fig. 1. The wave propagates along the $z$ axis, and without loss of generality, no variations are assumed along the $y$ axis ($\partial/\partial y \equiv 0$). In the following analysis, we assume nothing about the specific configuration of the mirrors. For instance, each mirror may be made of a metallic plate with an adjacent dielectric slab (forming a dielectric loaded transmission line), or it can be made from only dielectric layers forming a Bragg mirror, or some other photonic band-gap structure. It is, however, assumed that this
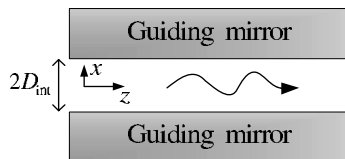


Fig. 1. A general configuration of two mirrors guiding light.

configuration of mirrors supports a specific eigenmode, for which the total force on the mirrors is evaluated.

We shall first examine transverse magnetic (TM) modes propagating within the waveguide. The modes will be denoted by "even" and "odd" according to the transverse behavior of the longitudinal field, and specifically the *even* TM mode reads

$$
\begin{aligned}
E_z &= E_0 \cosh(\Gamma_x x)e^{-jk_z z}, \\
E_x &= E_0 \frac{jk_z}{\Gamma_x} \sinh(\Gamma_x x)e^{-jk_z z}, \\
H_y &= E_0 \frac{j\omega\varepsilon_0}{\Gamma_x} \sinh(\Gamma_x x)e^{-jk_z z},
\end{aligned}
\tag{1}
$$

where $\Gamma_x^2 \triangleq k_z^2 - \left(\frac{\omega}{c}\right)^2$. The *odd* TM mode is given by

$$
\begin{aligned}
E_z &= E_0 \sinh(\Gamma_x x)e^{-jk_z z}, \\
E_x &= E_0 \frac{jk_z}{\Gamma_x} \cosh(\Gamma_x x)e^{-jk_z z}, \\
H_y &= E_0 \frac{j\omega\varepsilon_0}{\Gamma_x} \cosh(\Gamma_x x)e^{-jk_z z},
\end{aligned}
\tag{2}
$$

Evaluation of the field expressions at $\Gamma_x = 0$ requires special attention as the dependence of the amplitude $E_0$ on the frequency is unspecified. Assuming $E_0$ is constant, the even mode has a simple limit for $\Gamma_x \to 0$ which results in the TM field with phase velocity $c$ used for the acceleration of charged particles [10]:

$$
\begin{aligned}
E_z &= E_0 e^{-j\frac{\omega}{c}z}, \\
E_x &= E_0 j\frac{\omega}{c}x e^{-j\frac{\omega}{c}z}, \\
H_y &= \frac{E_0}{\eta_0} j\frac{\omega}{c}x e^{-j\frac{\omega}{c}z}.
\end{aligned}
\tag{3}
$$

However, to obtain the $\Gamma_x \to 0$ limit for the odd mode, one may assume that $E_0$ is proportional to $\Gamma_x$, and observe that the result, at this limit, is a transverse electromagnetic (TEM) mode in the hollow core, which explicitly reads

$$
\begin{aligned}
E_z &= 0, \\
E_x &= E_0 e^{-j\frac{\omega}{c}z}, \\
H_y &= \frac{E_0}{\eta_0} e^{-j\frac{\omega}{c}z}.
\end{aligned}
\tag{4}
$$

For the evaluation of the transverse force exerted on the mirrors, we shall integrate the time-average of the Maxwell stress-tensor [11] over a closed surface. As already indicated, we assume that the mirrors support the guided modes, i.e., the boundary conditions are fulfilled such that the fields vanish beyond the planes $x = \pm x_0$ for some $x_0 > D_{\text{int}}$, including the case $x_0 = \infty$. In the configuration of interest, the time-averaged Maxwell stress-tensor term $\langle T_{xx} \rangle$ for a TM mode is given by:

$$
\langle T_{xx} \rangle = \frac{1}{4}\varepsilon_0|E_x|^2 - \frac{1}{4}\varepsilon_0|E_z|^2 - \frac{1}{4}\mu_0|H_y|^2;
\tag{5}
$$

the other two components are either zero ($T_{xy}$), or have zero contribution due to symmetry ($T_{xz}$). The time-averaged force per unit area exerted on the top mirror of Fig. 1 is therefore given by

Table 1. $\langle T_{xx} \rangle$ and the sign of the time-averaged force between two mirrors guiding light, for even and odd modes (either TE or TM).

| Transverse behavior | Even mode | Odd mode |
|---|---|---|
| Propagating (imaginary $\Gamma_x$) | $-\frac{1}{4}\varepsilon_0|E_0|^2$ (Repulsive) | $-\frac{1}{4}\varepsilon_0|E_0|^2$ (Repulsive) |
| Light-line ($\Gamma_x = 0$) | $-\frac{1}{4}\varepsilon_0|E_0|^2$ (Repulsive) | Zero force |
| Evanescent (real $\Gamma_x$) | $-\frac{1}{4}\varepsilon_0|E_0|^2$ (Repulsive) | $+\frac{1}{4}\varepsilon_0|E_0|^2$ (Attractive) |

$\langle f_x \rangle = -\langle T_{xx} \rangle$, meaning that if, for example, $\langle T_{xx} \rangle$ is negative, the mirrors repel each other. For $\Gamma_x \neq 0$, substituting Eq. (1) and Eq. (2) into Eq. (5), we have for the even mode

$$\langle T_{xx}^{(\text{even})} \rangle = \frac{1}{4}\varepsilon_0|E_0|^2 \frac{\Gamma_x^2 |\sinh(\Gamma_x x)|^2 - |\Gamma_x|^2 |\cosh(\Gamma_x x)|^2}{|\Gamma_x|^2}, \tag{6}$$

whereas for the odd mode

$$\langle T_{xx}^{(\text{odd})} \rangle = \frac{1}{4}\varepsilon_0|E_0|^2 \frac{\Gamma_x^2 |\cosh(\Gamma_x x)|^2 - |\Gamma_x|^2 |\sinh(\Gamma_x x)|^2}{|\Gamma_x|^2}. \tag{7}$$

It is important to point out that in the numerator of Eqs. (6) and (7), only the second occurrence of $\Gamma_x^2$ is in absolute value, meaning that for imaginary values of $\Gamma_x$, the numerator is negative, whereas for real values it is positive. When substituting the $\Gamma_x = 0$ fields into Eq. (5), a negative contribution is obtained for the even mode (Eq. (3)), and zero contribution is obtained for the odd mode (Eq. (4)).

Table 1 summarizes the results for the Maxwell stress-tensor component $\langle T_{xx} \rangle$. This quantity is independent of $x$ in the hollow region between the mirrors, since this volume experiences zero Lorentz force. For nonzero $\Gamma_x$, the force between the mirrors is repulsive for all cases but one: odd transverse evanescent mode. In this case, the mirrors experience an *attractive force*. A similar conclusion was reached in Ref. [9] for the mode contributions to the Casimir force. At the light-line itself ($\Gamma_x = 0$, $k_z = \omega/c$), the mirrors are repelled by the even mode, and experience zero force by the odd mode. It is therefore evident that for the odd mode, when crossing the light-line by increasing the frequency along the mode's dispersion curve, the force changes its nature from attractive to repulsive. It can be verified that Table 1 is valid also for the transverse electric (TE) case. The result of Table 1 is independent of the details of the mirrors, provided that they support the specified eigenmode.

The conclusion that the odd mode and the even mode may differ in the total force they exert on the mirrors can be reached by first recognizing that the first term of $\langle T_{xx} \rangle$ (Eq. (5)) is always attractive, and the other two terms are always repulsive. Choosing the $x = 0$ plane as a convenient plane for the evaluation of $\langle T_{xx} \rangle$, one may readily conclude that the even mode always generates a repulsive force. This is because at $x = 0$ only $E_z$ is nonzero in case of the even TM (Eq. (1)). In case of the odd TM (Eq. (2)), the balance between $E_x$ and $H_y$ determines the sign of the force, since $E_z = 0$ at $x = 0$.

A different perspective on the expression of the stress-tensor component $\langle T_{xx} \rangle$ may be gained by using the equivalence principle [12]. It is possible to represent the fields inside the core by electric and magnetic fictitious surface current densities on the mirror-core interface. By this, we create an *equivalent* problem with surface sources located at $x = \pm D_{\text{int}}$, and zero field outsize the core. The Lorentz force, which would be identical for any type of confining structure, may then be calculated directly on the sources at $x = D_{\text{int}}$ [13]

$$-\langle f_x \rangle = \frac{1}{4}\frac{1}{\varepsilon_0}|\rho_{\text{s,e}}|^2 - \frac{1}{4}\varepsilon_0|J_{\text{s,m}}|^2 - \frac{1}{4}\mu_0|J_{\text{s,e}}|^2. \tag{8}$$

where $\rho_{s,e}$ is the electric surface charge density, $J_{s,m}$ is the magnetic surface current density in the $y$ direction, and $J_{s,e}$ is the electric surface current density in the $z$ direction. As this is simply a different representation of $\langle T_{xx} \rangle$ (Eq. (5)) at $x = D_{int}$, we may interpret the forces on the mirrors as resulting from the interaction of the equivalent sources on the core boundaries. Specifically for the TM case, when the contribution of the surface charge exceeds those of the electric and magnetic surface currents, the force becomes attractive. By virtue of duality [12], the above discussion can be repeated for the TE case with similar conclusions.

## 3.    Manipulation of a Bragg mirror

In the remainder of this study, we shall focus on two implementations of the general analysis presented above. Two metallic mirrors would obviously not support the transverse evanescent modes discussed above. One possibility for having such modes as eigenmodes is considering mirrors with a dielectric layer and a metallic boundary, i.e., a dielectric-loaded transmission line. A more realistic configuration at optical wavelengths is a dielectric photonic band-gap structure, and here we shall consider its one-dimensional version consisting of two Bragg mirrors and a hollow core [14]. The repulsive forces in such a configuration were recently investigated by Povinelli *et al.* [15]. In Ref. [16], it is shown that for a given mode, either odd or even, and a given $\Gamma_x$ and core width $2D_{int}$, the mirrors can be tailored around the mode such that the waveguide will support it, and simultaneously the transverse decay will be optimal in terms of the number of dielectric layers required. This can be done by use of a *matching layer*, which matches between the core mode and a transverse quarter wave Bragg mirror.

Let us consider a mirror made of two materials of refractive indices $\sqrt{\varepsilon_1} = 1.6$ and $\sqrt{\varepsilon_2} = 4.6$. These particular values, which originate in Ref. [17], are chosen for consistency with Ref. [16]. The mirror can be designed for a particular angular frequency $\omega_0$ with a corresponding wavelength $\lambda_0$, for an odd TM mode with $\Gamma_x = 0$. We shall choose the material with the lower refractive index to be adjacent to the core, entailing a matching layer width of $\lambda_0/(2\sqrt{\varepsilon_1 - 1})$, and the remainder of the layers have alternating refractive indices with transverse quarter wavelength width $\lambda_0/(4\sqrt{\varepsilon_v - 1})$ for the $v$'th layer. In fact, the aforementioned mirror design ensures confinement of the TEM ($\Gamma_x = 0$) for any core width $2D_{int}$, as the fields of the TEM are independent of $x$, and therefore, the boundary conditions are fulfilled independently of the location of the mirror. Consequently, the three odd TM dispersion curves for three different values of the core half-width $D_{int} = 0.3\lambda_0$, $0.6\lambda_0$ and $1.0\lambda_0$, that are plotted in Fig. 2 (left), all intersect the light-line at $\omega = \omega_0$. The even TM modes depicted in Fig. 2 intersect the light-line at different frequencies.

In what follows, it is assumed that the distance between the mirrors $2D_{int}$ is fixed, and that the input power $P$, flowing through an area having a width $\Delta_y$ in the $y$ direction, is constant (as would be in a practical scenario). The operating wavelength may be changed along the dispersion curve, implying that the amplitude $E_0$ and the power distribution across the waveguide change with the wavelength, and with it the forces exerted on the mirrors. The quantity $\langle T_{xx} \rangle c\lambda_0 \Delta_y/P$ is dimensionless, and proportional to the transverse pressure exerted on the mirrors. This quantity is plotted in Fig. 3 for both odd and even TM modes, for the three different values of $D_{int}$ considered above. While the even modes always generate a repulsive force on the mirrors, the odd modes cross the zero force line at $\omega = \omega_0$ (for any $D_{int}$), the frequency for which the mirror is designed to support the core TEM. For $\omega > \omega_0$, the odd modes cause the mirrors to be attracted to each other by a force that is increasing with the frequency, reaching a peak, and decreasing to zero at the edge of the band-gap. The attractive pressure at the peak for $D_{int} = 0.3\lambda_0$ is about an order of magnitude smaller than the pressure exerted by a plane wave of intensity $P/\lambda_0\Delta_y$ impinging perpendicularly on a perfect metallic mirror ($2P/c\lambda_0\Delta_y$). For frequencies in the vicinity of the edge of the band-gap, the mode is poorly confined, making
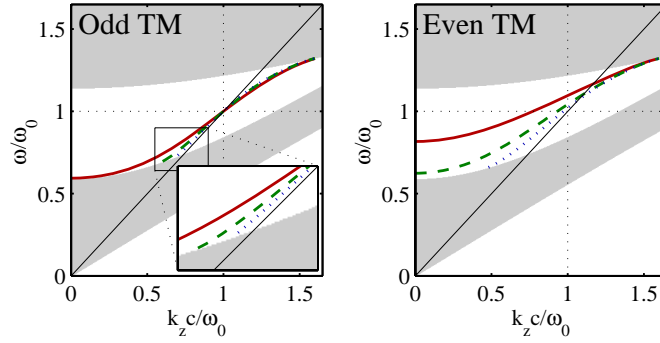
Fig. 2. Dispersion curves for a configuration of two Bragg mirrors: Odd TM (left) and even TM (right). The three curves shown are $D_{int} = 0.3\lambda_0$ (solid red), $D_{int} = 0.6\lambda_0$ (dashed green), and $D_{int} = 1.0\lambda_0$ (dotted blue). The transverse pass-bands of the infinite periodic structures are indicated in gray. The inset shows the odd modes near the lower band-gap edge magnified, emphasizing the difference between the curves.

operation of the mirrors impractical, as a large number of Bragg layers would be required.

The Maxwell stress-tensor was used as a tool for calculation of the transverse total force, knowing only the electromagnetic field propagating in the hollow core. It is important to reemphasize that there is a large variety of structures that may support such a mode. The advantage of using the stress-tensor approach rather than a direct calculation of the Lorentz force, is that in the latter it is necessary to establish the field at any point within the structure. Since we now have a particular design under consideration, the Lorentz force may be calculated directly, taking into account contributions from both polarization currents and polarization surface charges (see for example [18, p. 39] for the force expression). In a similar manner, Planck [13] computed the radiation pressure of a plane wave incident upon a metallic mirror, and recently Mansuripur [19] calculated the Lorentz force in several optical configurations. We have verified that indeed the curves of Fig. 3 may be obtained by the aforementioned method.

Observing the curves of the odd mode pressure on the mirrors shown in Fig. 3, it is possible to suggest a scheme for controlling the transverse motion of the mirrors. Let us suppose that one of the mirrors is free to move along the transverse axis $x$, and its motion is slow enough for the changes in propagating waves to be considered adiabatic. Injecting to the waveguide the odd mode at frequency $\omega_0$, the mirrors experience zero force regardless of the distance between them. Increasing the frequency, the unconstrained mirror will be drawn towards the fixed mirror, and decreasing the frequency will cause the unconstrained mirror to move away from the fixed mirror. The sign of the force is dependent only on whether the frequency is smaller or greater than $\omega_0$. However, the magnitude of the force is dependent, as seen in Fig. 3, also on the distance between the mirrors.

## 4. Mirror trapping

Within the framework of the method described above, where an odd mode attracts or pushes one mirror away from the other by varying the frequency, the mirrors can not be held in a stable equilibrium without an external feedback. This instability is due to the fact that holding the frequency at $\omega_0$, the force is zero regardless of the distance between the mirrors, and the mirror is free to move. To overcome this difficulty, we observe that for $\omega > \omega_0$, an odd mode creates an attractive force, while an even mode creates a repulsive force, suggesting that a superposition of an odd mode and an even mode may create a stable equilibrium state. It can be shown that for
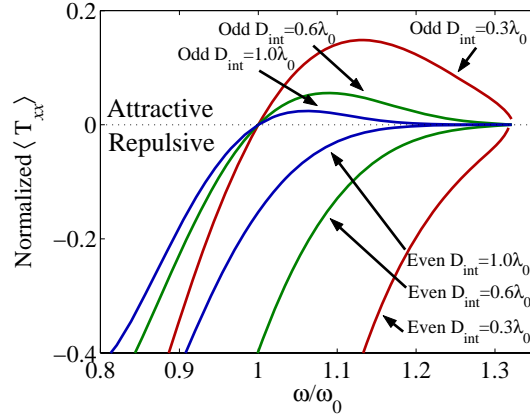
Fig. 3. The normalized time-averaged Maxwell stress-tensor component $\langle T_{xx} \rangle$ as a function of the frequency, for a configuration of two Bragg mirrors.
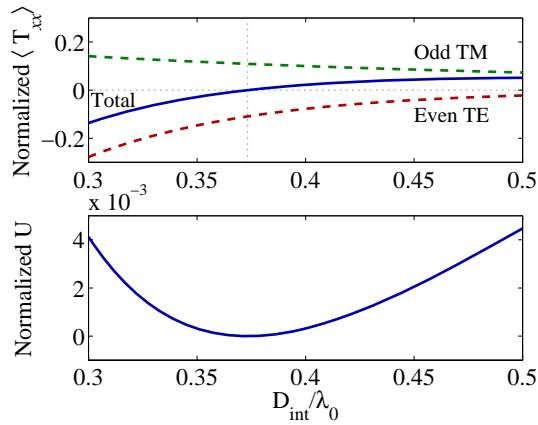


Fig. 4. Trapping of a mirror by a superposition of an even mode and an odd mode. Top: Time-averaged Maxwell stress-tensor component $\langle T_{xx} \rangle$ for the odd TM mode, the even TE mode, and their sum. Bottom: Potential well of the total transverse force.

such a superposition, the stress-tensor component $\langle T_{xx} \rangle$ is the sum of the components of each of the modes, i.e., $\langle T_{xx} \rangle = \langle T_{xx}^{(odd)} \rangle + \langle T_{xx}^{(even)} \rangle$, where either of the modes can be TE or TM.

Considering the same configuration as described above, we shall fix the frequency at $\omega = 1.1\omega_0$, and examine the effect of an odd TM mode and an even TE mode injected into the waveguide. Since the mirrors were designed for the odd TM mode, the even TE mode is weakly confined, implying that its power should be greater than that of the odd TM mode, for their forces to be of the same order of magnitude; in the specific case considered $P_{TE} = 10 P_{TM}$. In Fig. 4 (top), $\langle T_{xx} \rangle$ for the two modes, as well as the total value, is plotted as a function of the core half-width $D_{int}$. The normalized quantity shown is $\langle T_{xx} \rangle c\lambda_0 \Delta_y / P_{TM}$. It can be seen that the sum of the two modes crosses the zero force line, forming a stable equilibrium. Defining the potential by $\langle f_x \rangle \equiv -dU/dx$, the potential well created is shown in Fig. 4 (bottom). If both mirrors are free to move in the transverse direction, then an effect similar to that of optical binding of dielectric particles [20] results.

## 5. Conclusion

A general analysis of the pressure exerted by eigenmodes on the two guiding mirrors was presented. The attractive and repulsive forces generated by the guided modes may be used for the manipulation of the guiding mirrors. A possible application discussed is the control over the motion of a free Bragg mirror by varying the operation wavelength of an odd mode around a central frequency for which the mirror is designed. At the central frequency, the two guiding mirrors experience zero force regardless of the distance between them. Increasing or decreasing the frequency results in attractive or repulsive forces respectively.

Another application presented is a novel way for holding a mirror in a stable equilibrium. A superposition of an odd mode and an even mode are used to create an inherent potential well that traps the mirrors. Within the framework of this method, contrary to other methods that use an external feedback [5], or a Fabry-Perot cavity [3], the light is present only on one side of the mirror. This may have an advantage for operation of such a configuration as an optical spring, as in one of the mirror sides there is virtually no laser power flowing, and it is therefore mechanically free for external pressure to be applied.