Analytic method for evaluation of the field of a charge traversing a geometric discontinuity

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An analytic time-domain method is developed for the electromagnetic field generated by a charge traversing a geometric discontinuity. The essence of the method employed here is to use the linear independence of the exponential functions, that control the temporal behavior of the field. As a result, we avoid the large (“infinite”) matrix inversion necessary for a frequency-domain solution. This method was utilized for the investigation of the wake generated by electrons in an optical accelerator as well as evaluation of the emittance growth and energy spread. © 2002 American Institute of Physics. [DOI: 10.1063/1.1472477]

Analytic solutions of electromagnetic problems in the time domain are quite rare. A Gaussian wave or a point charge, for example, may have their field components described analytically when propagating in free-space, however, even these two simple examples become complex if a geometric discontinuity is involved. In fact, books1–3 have been written about various ways to investigate the scattering of an electromagnetic wave by a geometric discontinuity and an abundant literature is available on transition4 or diffraction5 radiation which accounts for the electromagnetic field that develops when a moving charge encounters a medium discontinuity. The essence of the method employed here is to use the linear independence of the exponential functions, that control the temporal behavior of the field. As a result, we avoid the large (“infinite”) matrix inversion necessary for a frequency-domain solution. This current density excites the longitudinal component of the magnetic vector potential that satisfies the nonhomogeneous wave equation. Its solution has two components: the primary field, generated by the moving charge as it traverses along the axis of each one of the sections—disregarding the presence of the discontinuity. And the secondary field, linked to the discontinuity at \(z=0\). The former is a solution of the nonhomogeneous wave equation whereas the latter is a solution of the homogeneous one.

The primary field (superscript \(p\)) may be derived from the magnetic vector potential that is given in the time domain by\(^3\)

\[
A^p_z(t) = \sum_{r=1}^{\infty} \alpha_{r,1} J_0 \left( \frac{r}{\Omega r_1} \right) e^{-\Omega r_1 |t-z(0)|} \quad t<0, \quad z>0 \]

\[
A^p_z(t) = \sum_{r=1}^{\infty} \alpha_{r,2} J_0 \left( \frac{r}{\Omega r_2} \right) e^{-\Omega r_2 |t-z(0)|} \quad t>0, \quad z<0 \]

wherein \(\Omega_{r,i} = (p_i \gamma \beta c / R_j)\), \(\alpha_{r,i} = (-q/2\pi v_0 R_j^2) \times (\gamma \beta^2 [J_0(p_i R_j / R_2) / \Omega r_2 J_2(p_i)] ; i=s,v, j=1,2\), with \(\beta = v_0/c\), \(\gamma = 1/\sqrt{1-\beta^2}\), \(v_0\) is the free-space permittivity, \(J_0(\xi)\) is the zero order of Bessel function of the first kind.

Contrary to scattering of an electromagnetic pulse, where a homogeneous solution of the wave equation needs to be determined, in the case of a wake generated by a moving charge the solution is nonhomogeneous and to some extent more complex. Moreover, analytic solutions of wakes are basically nonexistent and the best we usually may hope for, is a quasianalytic solution in the frequency domain.10–12 In this letter, we develop an analytic solution for the field generated by a charge approaching or leaving an azimuthally symmetric discontinuity.

Consider a ring of radius \(R_b\) carrying a charge \(q\) and moving at a constant velocity \(v_0\) along the axis of a cylindrical waveguide consisting of two uniform sections of radii \(R_1\) and \(R_2\), respectively, see Fig. 1. By choosing a cylindrical coordinate system whose reference point for the \(z\) axis \((z=0)\) coincides with the discontinuity, we may represent the current density linked to the moving charge by

\[J_z(q, z, t) = -q v_0 \delta(z - v_0 t) \delta(r - R_j) / 2\pi r\]

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FIG. 1. The schematics of the system under consideration.