Wake field of an electron bunch moving parallel to a dielectric cylinder

D. Schieber and L. Schächter

Department of Electrical Engineering, Technion-Israel Institute of Technology, Haifa 32000, Israel

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The wake field of an electron bunch moving parallel to the axis of a dielectric cylinder is being considered. It is shown that for a relativistic bunch ($\gamma \gg 1$) the circular harmonic of order zero contributes a decelerating force inversely proportional to $\gamma$, whereas the circular harmonics of nonzero order contribute a $\gamma$-independent force. Moreover, the wake linked to the circular harmonic of order zero may grow in space in case the dielectric cylinder consists of an active medium; however, this growth rate does not depend on the value of $\gamma$. On the other hand, no growth is anticipated for the case of circular harmonics of nonzero order.

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I. INTRODUCTION

Acceleration of electrons by radiation at optical wavelengths is one of the most promising alternatives for future electron acceleration. Generally speaking, optical schemes may be divided into two main groups: in the case of plasma-based schemes, a laser pulse is injected into a plasma where it excites a space-charge wake that, in turn, may accelerate a trailing bunch of electrons [1–4]. Another group corresponds to various inverse radiation processes such as inverse Cherenkov [5–7], inverse free-electron laser (IFEL) [8,9], and inverse transition radiation [10,11]. In the case of an inverse radiation process, the laser pulse is injected at identical conditions as when the radiation is emitted by electrons propagating within the structure. For example, in an IFEL, the laser pulse exhibits a polarization and a wavelength such that in the presence of the wiggler, the motion of the electron is synchronous with the wave, the phase corresponding to an accelerating force. In all these laser-driven systems, energy stored in an active medium is transformed into radiation inside the laser cavity being further used for acceleration in various structures.

It was suggested in Refs. [12,13] to directly use energy stored in an active medium in order to accelerate electrons. Specifically, it was demonstrated that a Cherenkov wake, generated by a small trigger bunch, may be amplified by the medium. A second bunch trailing behind may be accelerated by the amplified wake. The concept was demonstrated within the framework of a linear theory when the charged-particle moves in the active medium. A possible practical experiment is to launch a bunch of electrons parallel to a dielectric cylinder that may be active, e.g., a Nd:YAG rod, and examine the acceleration of electrons by the amplified wake. Since the transverse dimension of the bunch (100 $\mu$m diameter) is significantly smaller than that of the dielectric rod (6 mm), many nonsymmetric modes may be excited.

It is the purpose of this study to determine the radiation characteristics generated by a relativistic bunch of electrons moving parallel to a dielectric cylinder.

II. MODEL FORMULATION

Consider a cylinder of radius $R$ consisting of dielectric material ($\varepsilon$). The axis of a cylindrical coordinate system ($r, \phi, z$) coincides with that of the cylinder. Parallel to this axis, at a radius $r=h>R$ and at an angle $\phi=\phi_0$, a point charge is moving at a velocity $V$—see Fig. 1. In its motion, the point charge generates a current density

$$J_z(r, \phi, z; t) = -qV \frac{1}{r} \delta(r-h) \delta(\phi-\phi_0) \delta(z-Vt)$$

(1)

whose time-Fourier transform reads

$$J_z(r, \phi, z; \omega) = -q \delta(r-h) \delta(\phi-\phi_0) \frac{1}{2\pi} e^{-j\omega(Vz)}.$$

(2)

In the absence of the cylinder this current density excites a primary (superscript $p$) magnetic vector potential $A_z^{(p)}(r, \phi, z; \omega)$ which is a solution of the equation

$$\left[ \nabla^2 + \frac{\omega^2}{c^2} \right] A_z^{(p)}(r, \phi, z; \omega) = -\mu_0 J_z(r, \phi, z; \omega).$$

(3)

In the cylindrical coordinate system resorted to, this solution reads

$$A_z^{(p)}(r, \phi, z; \omega) = -\frac{q\mu_0}{(2\pi)^2} e^{-j\omega(Vz)} \sum_{n=-\infty}^{\infty} e^{jn(\phi-\phi_0)} \left\{ I_h(\Gamma h)K_h(\Gamma r) \quad r>h, \right.$$  

$$K_h(\Gamma h)I_h(\Gamma r) \quad r<h, \right.$$  

(4)

FIG. 1. Basic setup of the system under consideration; a dielectric cylinder (e.g., Nd:YAG) of radius $R$ and a bunch of electrons injected parallel to the axis at a radius $r=h$ and an angle $\phi=\phi_0$.  

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