Maximum gradients on a charged line moving above a corrugated surface of arbitrary geometry

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Abstract

We present the maximum longitudinal and transverse gradients on a charged line that moves above a grating of arbitrary geometry. These upper limits are evaluated subject to the assumption that the absolute value of each diagonal term of the reflection matrix, that describes the scattering from the surface, is smaller than unity.

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The motion of particles in the vicinity of structures has important applications in advanced accelerators concepts [1–4] and electron microscopy [5–7]. In fact, the wake generated by a bunch of electrons in an acceleration structure or in an acceleration cavity has been investigated thoroughly by several authors [8–13]. In the recent years we have investigated the decelerating force on a point-charge moving in a cylinder of radius \( R \) bored in a dielectric or metallic [14] medium. It was shown analytically that the longitudinal gradient (\( \varepsilon_{||} \)) for a point charge (\( Q \)) representing a bunch of electrons, moving at ultra relativistic velocities (\( \gamma \to \infty \)) is

\[
\varepsilon_{||} = \frac{Q}{4\pi\varepsilon_0 R^2} \times 2
\]  

regardless of whether the material is dielectric or metal. A similar result [15] was demonstrated in the case of a point-charge moving at a height \( h \) above a dielectric half-space (for \( \gamma \to \infty \)) namely

\[
\varepsilon_{||} = \left( \frac{Q}{4\pi\varepsilon_0 (2h)^2} \right) \times 2
\]  

only that in this configuration there is also a transverse gradient that was shown to read

\[
\varepsilon_M = \frac{Q}{4\pi\varepsilon_0 (2h)^2} \times \frac{1}{\gamma}. 
\]  

Both forces were written in terms of the image charge force of a motionless particle above a metallic plane of infinite conductivity. Clearly, the deceleration force is twice this typical force whereas the image force is reduced by a factor \( \gamma \).

In the case of a metallic (\( \sigma \)) half-space the longitudinal gradient for (\( \gamma/\beta \)^3 \( \gg \) \( \sigma_0 \) \( h \)) is

\[
\varepsilon_{||} = \frac{Q}{4\pi\varepsilon_0 (2h)^2} \times 2
\]  

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