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# Enhancement of the allowed gradient in a dielectric-loaded superconducting cavity

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It is shown that by incorporating a nonisotropic conductive thin film in a dielectric loaded super-conductive structure, it is possible to design an accelerator cavity wherein the maximum magnetic field at the surface of the superconductive material is below the critical value, although the gradient on axis may exceed 50 MV/m. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2794432]

Gradient enhancement in the future acceleration structure of the International Linear Collider may have significant implication on its length and thus on its cost. Although superconducting (SC) technology facilitates fabrication of cavities of very high quality factors $Q \simeq 10^9-10^{10}$ (Refs. 1 and 2) compared to room temperature technology, it has an inherent limitation in the form of the *maximum* magnetic field allowed at the structure's surface. Specifically, this must be kept below the critical magnetic field of the superconductor ($H_{c1}$ in case of type II) otherwise, vortices penetrate the SC layer and cause strong rf dissipation. For the case of widely used bulk Nb, this field is of the order of $H_{c1}^{\mathrm{Nb}}=1.35 \times 10^5$ A/m,[3] coinciding with the $Q$-drop magnetic field achieved recently in a Nb cavity.[4] In an attempt to overcome this difficulty, Gurevich[5] at Jefferson National Laboratory proposed to enhance the rf field allowed at the surface of a Nb cavity by coating it with a multilayer structure composed of alternating dielectric and superconductive layers resulting in effectively *higher* $H_c$ at the vacuum-material interface; each layer in this configuration is thinner than the penetration depth.

Dielectric materials are widely used in optics for high-power components due to their inherent low loss. Bragg waveguides or mirrors composed of alternating layers of different dielectrics harness the ability to "rotate" the electromagnetic field components and generate a specified field at a given location, as was demonstrated recently.[6] The concept was adopted at Argonne National Laboratory by Jing *et al.*[7] in the design of a room-temperature structure to be used in a dielectric wake-field accelerator. In this case, for loss reduction, the magnetic field at the metallic wall of a cylindrical waveguide was reduced by employing a $\mathrm{TM}_{03}$ mode in a four-layer dielectric-loaded configuration.

In this letter, we consider the possibility of using a single-layer dielectric-loaded superconducting cavity in order to support high accelerating gradient $E_0$ (the longitudinal electric field amplitude on the $z$ axis) and achieve a high $R/Q$ ($R$ being the shunt impedance and $Q$ the quality factor of the structure). However, in order for such scheme to become feasible, the dielectric layer chosen must satisfy two important conditions: withstand very high electric field (high breakdown threshold) and have ultralow loss tangents, namely, of the order of $10^{-9}-10^{-10}$ (Refs. 8 and 9) in order to avoid degradation of the quality factor of the SC structure.

To demonstrate the concept, consider an azimuthally symmetric waveguide, made of superconductive material, *loaded* with a concentric dielectric layer, as depicted in Fig. 1, $R_{\mathrm{int}}$ and $R_{\mathrm{ext}}$ denoting the internal and external radius of the dielectric layer. Further, it is assumed[6] that an azimuthally symmetric TM mode oscillating at a given angular frequency $\omega$ is propagating with a phase velocity $V_{\mathrm{ph}}=c$ and thus, in the vacuum region, the electromagnetic field is given by

$$E_z = E_0 e^{-j(\omega/c)z},$$

$$E_r = \eta_0 H_\varphi = \frac{j}{2}\frac{\omega}{c} r E_0 e^{-j(\omega/c)z}, \tag{1}$$

where $\eta_0=\sqrt{\mu_0/\varepsilon_0}$ is the impedance of the vacuum. Similarly, in the dielectric layer, the field components are given by

$$E_z = [AH_0^{(2)} + BH_0^{(1)}]e^{-j(\omega/c)z},$$

$$E_r = \frac{Z}{\sqrt{\varepsilon-1}} H_\varphi = \frac{j}{\sqrt{\varepsilon-1}}[AH_1^{(2)} + BH_1^{(1)}]e^{-j(\omega/c)z}, \tag{2}$$

where $Z \equiv \eta_0 \sqrt{\varepsilon-1}/\varepsilon$ is the transverse wave impedance, the argument of the Hankel functions is $kr$, $k \equiv \omega\sqrt{\varepsilon-1}/c$ denoting the transverse wave number, and $A$ and $B$ are the amplitudes of the outgoing and the incoming waves in the dielectric layer, respectively. By further imposing the continuity of the tangential field components $(E_z, H_\varphi)$ at the vacuum-dielectric interface and imposing $E_z$ to be zero at the external

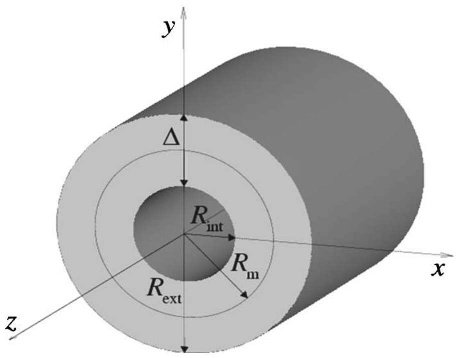

FIG. 1. Schematic cross section of the dielectric loaded cylindrical accelerating structure.

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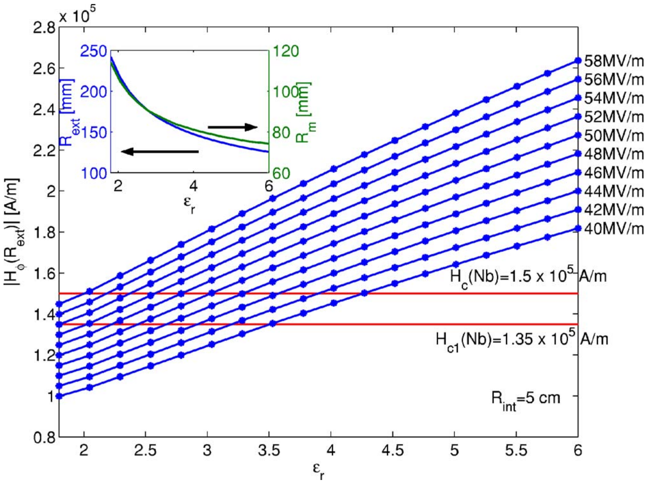

FIG. 2. (Color online) The magnetic field magnitude of $\mathrm{TM}_{02}$ mode at the superconducting boundary as a function of the dielectric constant. The different blue lines correspond to different accelerating gradients whereas the red lines correspond to typical critical magnetic fields of Nb cavity at 2 K. In the inset, the corresponding concentric location of the metallic layer $R_m$ and the superconducting boundary $R_{\mathrm{ext}}$ of the waveguide are also shown as a function of $\varepsilon_r$.

wall ($R_{\mathrm{ext}}$), the dispersion relation is obtained in a closed form.

For $H_\varphi$ of $\mathrm{TM}_{01}$ mode at the boundary with $R_{\mathrm{int}}=5$ cm and a synchronized frequency $f_0=1.3$ GHz as a function of $\varepsilon_r$ (in the range of 1.8–6) yields magnetic fields lower than the critical field as long as the accelerating gradient is kept below 40 MeV—levels which were demonstrated experimentally—in a coupled cavity system.[10,11] In order to enhance the accelerating gradient while keeping $H_\varphi$ below the critical value, it is suggested to harness a $\mathrm{TM}_{02}$ mode (at $f_0=1.3$ GHz) while suppressing the $\mathrm{TM}_{01}$ mode. For this purpose, a very thin $(d \ll \delta \equiv \sqrt{1/\mu_0 \sigma_z \pi f_0})$ anisotropic conductive layer, having a zero transverse conductivity and a nonzero longitudinal conductivity $\sigma_z \neq 0$ is positioned at $r=R_m$—corresponding to the *zero* of the longitudinal component of the electric field ($E_z$) of $\mathrm{TM}_{02}$-mode inside the dielectric filled region. This anisotropic conductivity is transparent to the $\mathrm{TM}_{02}$ mode and it virtually short circuits the $\mathrm{TM}_{01}$ mode. A possible implementation of such a thin anisotropic conductor might be using a structure which effectively confines electrons in *one plane*. As an example, it is possible to conceive a two dimensional electron gas formed at the interface of a heterostructure (e.g., AlGaN/GaN)[12] or a quantum well consisting of a trilayer, where the middle layer is highly doped (e.g., AlGaAs/$n$-GaAs/AlGaAs)[13]—in the former case of GaN, the breakdown field can be significantly higher than 100 MV/m.

Adding normal metal in a high-$Q$ superconducting cavity requires special attention. The added power loss per unit length to $\mathrm{TM}_{01}$ mode is given by $P_l^m \simeq \pi \sigma_z R_m d |E_z(R_m)|^2$, where $d$ is the width of the electron channel. Figure 2 shows the absolute value of the amplitude of the magnetic field $H_\varphi$ of $\mathrm{TM}_{02}$ mode at the outer boundary ($r=R_{\mathrm{ext}}$) as a function of $\varepsilon_r$ in the range of 1.8–6. The different lines correspond to different gradients in the range of 40–58 MV/m and the two red lines correspond to typical critical magnetic fields $H_{c1}=1.35\times10^5$ A/m and $H_c=1.5\times10^5$ A/m of Nb, at 2 K and 1–2 GHz (Refs. 3 and 4). The figure reveals that from the perspective of the surface magnetic field, low dielectric coefficients are more likely to reduce the magnetic field and at the same time support gradients in excess of 50 MV/m on axis. As for the breakdown field limitation, the maximum field occurs at the vacuum-dielectric interface and at a given operating frequency, it depends only on the radius of the vacuum. In our case, $R_{\mathrm{int}}=5$ cm and the peak electric field is $1.2E_0$.

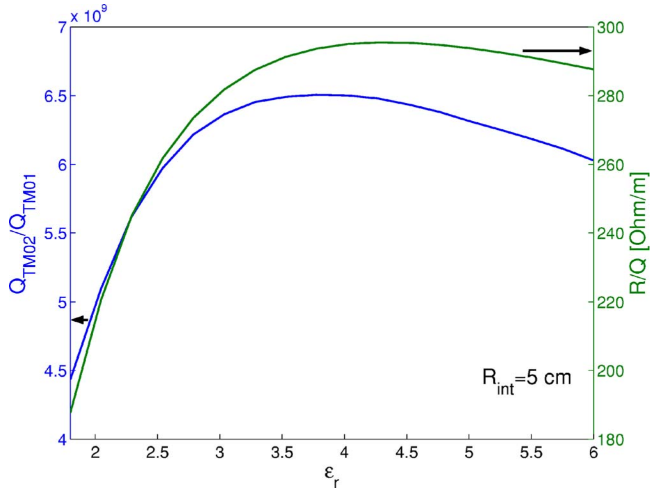

FIG. 3. (Color online) The quality factor ratio between $\mathrm{TM}_{02}$ and $\mathrm{TM}_{01}$ modes is plotted at the left side $y$ axis as a function of the dielectric constant, while at the right side, $R/Q$ parameter of $\mathrm{TM}_{02}$ mode is drawn.

It is now imperative to ensure that in the process of adding this thin layer of normal conductivity, the quality factor $(Q \equiv \omega_0 W_{\mathrm{EM}}/P_l)$ of the structure is not altered. In our simulations, we have assumed that the surface resistance of the superconductor (Nb) is $R_s=10\ \mathrm{n}\Omega$ (Ref. 5) and the dielectric material is characterized by $\tan\delta=10^{-10}$. In Fig. 3, the ratio $Q_{\mathrm{TM02}}/Q_{\mathrm{TM01}}$ (blue line) and the $R/Q$ parameter (green line) are exhibited as a function of $\varepsilon$. The parameters $\sigma_z$ and $d$ employed in the calculation are $3.6\times10^5$ S/m (Ref. 12) and 20 nm, respectively. This figure reveals one of the most important results of this study, namely, it is possible to reduce significantly the surface magnetic field while keeping the accelerating gradient high, by using $\mathrm{TM}_{02}$ and strongly suppressing the $\mathrm{TM}_{01}$. In fact this figure illustrates that the ratio of the quality factors is larger than $10^9$. It is also interesting to note that this ratio has a maximum for $\varepsilon_r=3.5$. Moreover, it is found that the normalized energy velocity $\beta_{\mathrm{EN}} \equiv v_{\mathrm{EN}}/c$ decreases monotonically from about 0.6 to 0.2 upon increasing $\varepsilon_r$ within the same range.

In Fig. 4, the dependence of $|H_\varphi|$ at the outer wall on radius of the vacuum-dielectric interface ($R_{\mathrm{int}}$) has been considered for $\varepsilon_r=2$ and for the gradient range of Fig. 2. While reducing $R_{\mathrm{int}}$ may clearly facilitate higher accelerating gradients, before the critical magnetic field is reached, this benefit is significantly overshadowed by having the beam significantly closer to dielectric materials. In addition, this trend is accompanied by a mild decrease in the quality factor ($Q$) and $\beta_{\mathrm{EN}}$, as shown in the inset.

Incorporating the thin anisotropic metallic layer requires to examine not only its impact on the rf but also its dc effect and, in particular, the temperature increase due to the return currents associated with an electron/positron bunch propagating along the waveguide. For this purpose, we consider a $\Delta t=10$ ps long bunch of about $10^{10}$ electrons inducing a peak return current of about $I=160$ A in the superconducting

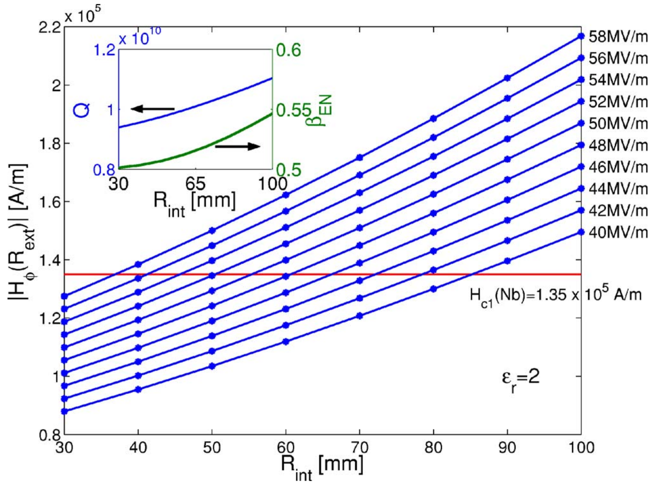

FIG. 4. (Color online) The magnetic field magnitude of $\mathrm{TM}_{02}$ mode at the superconducting boundary as a function of the vacuum core radius $R_{\rm int}$. The plots correspond to different accelerating gradients and a dielectric layer having $\varepsilon_r=2$. In the inset, the corresponding $Q$ and $\beta_{\rm EN}$ of $\mathrm{TM}_{02}$ mode are shown as a function of $R_{\rm int}$.

boundary and the conductive layer. Further assuming a worst case scenario whereby, all the return current $I$ flows in the conductive layer, the dissipated energy per unit length would be roughly given by $E_{\rm diss}\simeq 0.25 I^2 \Delta t/\sigma_z \pi R_m d = 2.8\times 10^{-5}$ J/m where $R_m=0.1$ m is used. On the other hand, by taking the typical heat properties of GaN and assuming that the total thickness $t$ of the quantum structure is about 500 nm, the energy per unit length required to raise the temperature of the conductive layer by 1 K is 0.9 J/K m. Consequently, the expected temperature increase from *one* bunch would be negligibly small $\sim 10^{-5}$ K. Thus, even for the case of $10^3-10^4$ bunches/s, the temperature increase would be $\sim 0.1$ K, though in fact the actual value will be much lower since most of the current is expected to flow in the superconducting boundary and not in the conductive layer as we overestimated earlier.

Before we conclude, one comment is in place: nowadays the only dielectric known to have such an *ultralow-loss* tangent at 2 K is pure sapphire,[14] which has an effective dielectric coefficient of about $\varepsilon_r=10.4$ and a breakdown field of about 48 MV/m (Ref. 3). We do not rule out the possibility that in future, other ultralow-loss, high-breakdown dielectric materials may become available since the loss mechanism in dielectrics depends to a large extent on the fabrication process and on the applied deposition parameters.[15,16]

In conclusion, it is shown that in a dielectric loaded superconductive dielectric structure, the latter may be designed such that the maximum magnetic field at the surface of the superconductive material is below the critical value although the gradient on axis may exceed 50 MV/m. A nonisotropic metallic film is used to suppress the $\mathrm{TM}_{01}$ mode and facilitate operation with the $\mathrm{TM}_{02}$ mode since it is located in a plane where the longitudinal electric field of the latter vanishes.

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[1] P. Kneisel, G. R. Myneni, G. Ciovati, J. Sekutowicz, and T. Carneiro, *Proceedings of the 2005 Particle Accelerator Conference Konxville, Tennessee*, IEEE, New York, pp. 3991–3993.
[2] G. Ciovati, Physica C, **441**, 44 (2006).
[3] A. W. Chao and M. Tigner, *Handbook of Accelerator Physics and Engineering* (World Scientific, Singapore, 1999), pp 359, 527.
[4] G. Ciovati, P. Kneisel, J. Sekutowicz, and W. Singer, Proceedings of the 2005 Particle Accelerator Conference Konxville, Tennessee, IEEE, New York, pp. 3874–3876.
[5] A. Gurevich, Appl. Phys. Lett. **88**, 012511 (2006).
[6] A. Mizrahi and L. Schächter, Phys. Rev. E **70**, 016505 (2004).
[7] C. Jing, W. M. Liu, W. Gai, J. G. Power, and T. Wong, Nucl. Instrum. Methods Phys. Res. A **539**, 445 (2005).
[8] J. Krupka and J. Mazierska, IEEE Trans. Microwave Theory Tech. **48**, 1270 (2000).
[9] J. Krupka, K. Derzakowski, A. Abramowicz, M. E. Tobar, and R. G. Geyer, IEEE Trans. Microwave Theory Tech. **47**, 752 (1999).
[10] L. Lilje, E. Kako, D. Kostin, A. Matheisen, W.-D. Möller, D. Proch, D. Reschke, K. Saito, P. Schmüser, S. Simrock, T. Suzuki, and K. Twarowski, Nucl. Instrum. Methods Phys. Res. A **524**, 1 (2004).
[11] T. Kroc and A. Moretti, Proceedings of the 1990 Linear Accelerafor Conference, Albuquerque, 1991, p. 102; T. Kroc, A. Moretti, and M. Popovic, Proceedings of the 1992 Linear Accelerafor Conference, Ottawa, 1992, p. 187.
[12] J. M. Redwing, M. A. Tischler, J. S. Flynn, S. Elhamri, M. Ahoujja, R. S. Newrock, and W. C. Mitchel, Appl. Phys. Lett. **69**, 963 (1996); M. A. Khan, J. N. Kuznia, J. M. Van Hove, N. Pan, and J. Carter, *ibid.* **60**, 3027 (1992).
[13] R. F. Hummel, *Electronic Properties of Materials*, 2nd ed. (Springer, Berlin, 1993), p. 136–139.
[14] H. N. Louis, U.S. Patent No. 5,089,785 (18 February 1992).
[15] J. M. Martinis, K. B. Cooper, R. McDermott, M. Steffen, M. Ansmann, K. Osborn, K. Cicak, S. Oh, D. P. Pappas, R. W. Simmonds, and C. C. Yu, Phys. Rev. Lett. **95**, 210503 (2005).
[16] J. Y. Kim, M. S. Hwang, Y.-H. Kim, H. J. Kim, and Y. Lee, J. Appl. Phys. **90**, 2469 (2001).