Wake-field in an array of metallic posts: Possible application for beam position monitoring

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Received 17 May 2005; received in revised form 13 September 2005; accepted 16 September 2005
Available online 3 October 2005

Abstract

It is proposed to use an array of metallic posts as a beam position monitor (BPM). The structure consists of a set of \( N \times N \) metallic vertical posts bounded from above and below by two horizontal metallic plates. The beam position offset from the symmetry axis is determined by measuring the backward radiation emitted by the beam as it traverses the array. The suggested BPM’s position resolution is dominated by the thermal noise level and the shot noise level of the detectors measuring the radiation whereas the time resolution is dominated by their operation frequencies. In the optical regime and for ultra-relativistic beams the time resolution is of order of 10 ps i.e. at least two orders of magnitude higher than the common time resolution in stripline BPM. Utilizing the state of the art optical detectors the position resolution of such a BPM may reach the level of a few nanometers which is one order of magnitude better than the common position resolution of existing systems.

PACS: 07.62; 11.00; 41.70; 42.10.H

Keywords: Beam position monitoring; Accelerators; Backward radiation

1. Introduction

One of the main considerations to be addressed in the design of any linear accelerator is the stability of the beam’s transverse position, as the latter may affect dramatically the emittance growth of the beam. In particular, the future linear collider has very stringent requirements on the stability of the transverse position of the beam in the accelerator. In such a collider the bunch size may be in the nanometer scale. Therefore, a beam position monitor (BPM) with position resolution better than 100 nm is required in order to control the stability of the beam’s position along the linac [1]. For example, in the Next Linear Collider (NLC) [1] the BPMs are required to have precision of 300 nm, accuracy of 200 µm and stability of 1 µm. In order to enable head-to-head collisions an accurate alignment of the beams along their accelerating path should be achieved; therefore, a high resolution BPM is essential.

A BPM is characterized by two important properties: its position resolution and its time resolution. The position resolution is defined as the smallest spatial deviation of the bunch from the symmetry axis that the BPM is able to detect. Time resolution is the time which a BPM needs to be ready for the next bunch detection i.e. the speed of signal detection. So far, a variety of BPMs were developed in accordance with the requirements and the restrictions dictated by various laboratories all over the world [2–8]. Each BPM is designed to fulfill its particular task under certain conditions [2–8]. However, the most often used are the stripline and cavity BPMs. In what follows, we will introduce a brief overview of both types.

In either stripline or cavity BPMs the monitoring process consists of three stages: in the first stage a “pick-up station” generates the signal according to the displacement of the beam. Then this signal is transferred to the second stage of signal detection electronics. In the latter stage...
information regarding the beam position is extracted from the electromagnetic signal. The third stage consists of the data processing module where the information from the second stage is converted into digital form and is processed with the aid of software. It is the purpose of this study to introduce a new “pick-up station” that may replace either the cavity or the stripline resorting to a structure resembling a photonic crystal.

In a stripline BPM a bunch of electrons propagates along a cylindrical waveguide with stripline electrodes imbedded in the wall—see Fig. 1. The position of the charged beam is deduced from the difference between the electric potentials developing on the electrodes. For ultra-relativistic beams this measured voltage is proportional to the transverse offset of the beam. The magnitude of the voltage signal has a maximum at frequencies where the electrode length is an odd multiple of the quarter wavelength and, therefore, the stripline BPM is usually designed to operate at a given frequency. The position resolution of a stripline BPM is limited by the thermal noise level. In other words, this resolution is chosen to be the beam’s offset for which the voltage signal equals the thermal noise. The second parameter of interest in stripline BPMs is the bandwidth of the voltage signal which determines the time resolution of such a BPM. Typically, the spatial resolution of a stripline BPM is of about 500 nm, whereas its temporal counterpart is of about 1 ns [6]. Stripline BPMs are primarily used in electron storage rings, where the bunch length is small [6,9]. The main drawbacks of this type of BPMs are the complexity of its fabrication and the fact that it detects bunches moving in a predetermined direction while bunches moving in opposite direction produces no signal in output port.

Cavity BPMs are being used at Stanford Linear Accelerator Center (SLAC) since 1967 [10]. Provided that the beam’s current is known, the transverse position of the beam is established by measuring the amplitude of the TM\(_{120}\) mode excited by the bunch as the latter traverses along a passive rectangular cavity. In 1979, it was suggested for the first time by McKeown [2] to use a single cylindrical pill-box cavity as a pick-up BPM. A typical cavity BPM is illustrated in Fig. 2 where a cylindrical cavity is mounted on a beam pipe. As the moving bunch traverses the cavity it excites electromagnetic field modes resonating at certain frequencies. The amplitudes of these modes are determined by the cavity orientation, the bunch charge, the beam position and its energy. Among all the excited modes, dipole modes are mainly used for beam position detection, since their amplitudes depend linearly on the beam position and are zero for a centered beam. Therefore, the larger the offset of the beam, the stronger is the excitation. The TM\(_{110}\) mode is the first dipole mode used for beam position monitoring. In the TM\(_{110}\) mode the electric field is zero on axis and its direction changes sign across the plane of symmetry while the magnetic field has its maximum value on axis and has reversal points where the electric field has maximum. Assuming that the beam current is known, the amplitude of the TM\(_{110}\) mode can be interpreted to give the displacement of the beam from the plane of symmetry while the phase of the mode gives the sign of the displacement [11]. Cavity BPMs are well known due to their high position resolution and the simplicity of their manufacturing. Unfortunately, as the signal revealed by the cavity BPM is a narrow-band signal, the time resolution of such pick-up BPMs is modest. Moreover, the isolation between both transverse directions is unstable [12].

The fabrication of a circular cavity is easier and costs less than the fabrication of rectangular cavity, and indeed cylindrical cavity BPMs are widely used. In addition, the
precision of the cylindrical cavity is higher than the rectangular one. Nowadays, various designs of cavity BPMs have been proposed and developed for a broad frequency range from 1.5GHz (TESLA) up to 33 GHz (CLIC) [11]. Table 1 summarizes some of the existing cavities used for position detection in different laboratories over the world.

The concept employed in the present study does not rely on confining the energy but rather scattering it. Diffraction radiation transmitted by a bunch of electrons as it traverses in the vicinity of array of posts is highly directional in specific direction depending on the posts configuration as well as the relative transverse location of the bunch. The nature of this (diffraction) radiation may be explained as follows: during the motion of a charge in the neighborhood of an ideally conducting body, time-dependent currents are induced at the surface. These currents are the source of this radiation [13–16]. It is shown in this study that any deviation from the symmetry axis reflects into asymmetry in the intensity of radiation lobes as measured by two symmetric detectors. In fact in a reasonable range of parameters the relative change in the intensity of the two symmetric detectors is proportional to the spatial deviation. This array of posts resembles the configuration of photonic band gap (PBG) crystals that have been subject of considerable interest in recent years [17–20]. These quasi-periodic structures may either confine (cavity) or guide radiation (waveguide) at specified frequencies.

The present study is organized as follows: in the next section a general formulation of the electromagnetic (EM) problem is introduced. This is followed by the calculation of the total power emitted by the moving charge as well as its spectrum. In Section 4 simulation results are introduced and discussed. Finally, the feasibility of using an array of metallic posts as a pick-up BPM is demonstrated.

### 2. Formulation of the problem

Consider a photonic crystal structure consisting of an array of $N \times N$ metallic vertical posts that are bounded from above and below by two horizontal metallic plates as illustrated in Fig. 3. The posts are of radius $a$ and the distance between their centers is $L_x$ in the $x$ direction and $L_y$ in the $y$ direction. The distance between the two plates is $g$. A point-charge ($Q$) located at a height $h$ above the lower plate and with an offset $\Delta$ in the $y$ direction is constrained to move at a constant velocity $v$ along the $x$-axis throughout the structure. The motion of such a point-charge generates an electromagnetic wake-field which in turn radiates at the expense of the kinetic energy of the charge. It is the purpose of this study to investigate the characteristics of the total power emitted by the moving charge and employ this information in order to detect the charge's offset from the axis of symmetry.

As a first step in solving the EM problem one should determine the functional behavior of the EM wake-field. To do so, we split the solution into two parts i.e. the primary field (superscript $p$) and the secondary field (superscript $s$). The primary field stands for the solution of the non-homogeneous wave equation in free space e.g. in the absence of the photonic crystal structure, while the secondary field stands for the solution of the homogeneous wave equation in the presence of the structure. Both secondary and primary fields together must satisfy the boundary conditions. Since the moving point-charge is confined to move in one direction ($\chi$) the only component of the magnetic vector potential to be excited is the $x$ component. Accordingly, the primary magnetic vector potential $A_x^p$ in the frequency domain is given by [21]

\[
A_x^p(x, y, z; \omega) = \frac{\mu_0 Q}{2\pi g} \sum_n \frac{1}{\lambda_n} e^{-\lambda_n|y-\Delta|} \sin \left( \frac{\pi n h}{g} \right) \\
\times \sin \left( \frac{\pi n z}{g} \right) e^{-j(\omega/c)x}
\]

wherein

\[
A_n^2 \equiv \left( \frac{\omega}{\gamma \beta c} \right)^2 + \left( \frac{n \pi}{g} \right)^2, \quad \beta \equiv \frac{v}{c} \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}.
\]

Resorting to Maxwell’s equation and utilizing the Lorentz gauge, the different components of the primary EM field may be deduced (see Appendix A).

The secondary EM field is obtained from the homogeneous solution of the problem in the presence of the array of posts forming the photonic crystal structure. Note

### Table 1

<table>
<thead>
<tr>
<th>Lab.</th>
<th>Type</th>
<th>$f_{1/2}$ (GHz)</th>
<th>Resolution (µm)</th>
<th>Excitation current (CW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLAC</td>
<td>Rect.</td>
<td>2.856</td>
<td>10.00</td>
<td>100µA</td>
</tr>
<tr>
<td>CERN</td>
<td>Cyl.</td>
<td>30.00</td>
<td>0.100</td>
<td>—</td>
</tr>
<tr>
<td>DESY</td>
<td>Cyl.</td>
<td>1.517</td>
<td>10.00</td>
<td>5 mA</td>
</tr>
<tr>
<td>KEK</td>
<td>Cyl.</td>
<td>5.712</td>
<td>0.025</td>
<td>—</td>
</tr>
</tbody>
</table>

Fig. 3. Photonic crystal structure—a set of metallic vertical posts bounded by two horizontal metallic plates.
that the electric currents on the metallic posts will have both a z-directed component and an azimuthally directed component. To simplify the solution, the z-directed and the azimuthally directed components of the electric currents on the metallic posts are replaced with electric and magnetic currents that are both z-directed. To this end, the Hertz and Fitzgerald potentials, $P_z$ and $F_z$, are used. These potentials satisfy the following non-homogeneous wave equations

$$\left[ \nabla^2 + \left( \frac{\omega}{c} \right)^2 \right] P_z = \frac{j}{\omega \epsilon_0} \sum_{m,n,r} I_{z,m,n,r}(\rho, \phi) \frac{\delta(\rho - a)}{2\pi} \cos \left( \frac{m \phi}{\rho} \right) e^{-jm\rho} \delta(\rho - a)$$

$$\left[ \nabla^2 + \left( \frac{\omega}{c} \right)^2 \right] F_z = \frac{j}{\omega \mu_0} \sum_{m,n,r} M_{z,m,n,r}(\rho, \phi) \frac{\delta(\rho - a)}{2\pi} \sin \left( \frac{m \phi}{\rho} \right) e^{-jm\rho}$$

where $I_{z,m,n,r}(\rho, \phi)$ and $M_{z,m,n,r}(\rho, \phi)$ are the yet-to-be-determined amplitudes of the $m$th z-directed harmonic and the $n$th azimuthal harmonic of the $z$-directed electric and magnetic currents on the $r$th metallic post, correspondingly. Explicit expressions for these potentials are given by [21]

$$P_z = \frac{1}{2\pi \omega \epsilon_0} \sum_{m,n} I_{z,m,n}(\rho, \phi) \cos \left( \frac{m \phi}{\rho} \right) \sin \left( \frac{m \phi}{\rho} \right)$$

$$F_z = \frac{1}{2\pi \omega \mu_0} \sum_{m,n} M_{z,m,n}(\rho, \phi) \sin \left( \frac{m \phi}{\rho} \right) \sin \left( \frac{m \phi}{\rho} \right)$$

wherein the function $\zeta_{z,m,n}(\rho, \phi)$ reads

$$\zeta_{z,m,n}(\rho, \phi) = e^{-jm\rho} \left\{ \begin{array}{ll} I_m(\zeta_n \rho \phi) K_m(\zeta_n a) & \rho \leq a \\ I_m(\zeta_n \rho \phi) K_m(\zeta_n a) & \rho > a \end{array} \right\}$$

with $\zeta_n = \sqrt{(\kappa \rho)^2 - (\omega / c)^2}$, $\rho_0 = \sqrt{(x-x_i)^2 + (y-y_i)^2}$ and $\phi_0 = \arctan((y-y_i)/(x-x_i))$, and the components of the pair $(x, y)$ represent the x and y coordinates of the center of the $r$th metallic post, respectively. $I_m(\zeta_n)$ and $K_m(\zeta_n)$ are the modified Bessel functions of the first and second type and order $m$, respectively. As the secondary potentials are determined one should be able to derive all the components of the secondary EM field as detailed in Appendix A.

The amplitudes $I_{z,m,n}(\rho, \phi)$ and $M_{z,m,n}(\rho, \phi)$ are obtained by imposing the boundary conditions on the posts i.e. the total azimuthal electric field on each post’s surface vanishes as well as the total z-directed electric field. To do so, we require the electric field to satisfy the boundary conditions on $M$ sample points located on the posts circumference. The index of each sample point is denoted by the pair $(\tilde{\nu}, \tilde{m})$, where $\tilde{\nu}$ is the index of the sampled post and $\tilde{m}$ is the index of the angle of the sample point relative to the post’s center. The x and y coordinates of the $(\tilde{\nu}, \tilde{m})$ sample point are given by the pair

$$(x, \tilde{m}, y, \tilde{m}) = \left[ x_\tilde{m} + a \cos \left( \frac{2\pi \tilde{m}}{M}, y_\tilde{m} + a \sin \left( \frac{2\pi \tilde{m}}{M} \right) \right) \right]$$

where the components of the pair $(x, y)$ represents the x and y coordinates of the center of the $r$th metallic post. In order to simplify the notation we will use the index pair $(\tilde{\nu}, \tilde{m})$ as an abbreviation for the $(x, \tilde{m}, y, \tilde{m})$ pair of coordinates.

The electric field tangential to the circumference of the $r$th post is obtained by using the following simple transformation

$$\tilde{E}_{\phi}(\tilde{\nu}, \tilde{m}) = -E_x(\tilde{\nu}, \tilde{m}) \sin \left( \frac{2\pi \tilde{m}}{M} \right) + E_y(\tilde{\nu}, \tilde{m}) \cos \left( \frac{2\pi \tilde{m}}{M} \right).$$

The primary azimuthally tangential electric field is obtained by applying the latter transformation on the fields that are given in Eqs. (A.2) and (A.3), yielding

$$E_{\phi}^{(s)}(\tilde{\nu}, \tilde{m}) = -\frac{\mu_0 Q}{2\pi \rho} \sum \left\{ \begin{array}{ll} \frac{1}{\rho^2} \sin \left( \frac{2\pi \tilde{m}}{M} \right) + y_\tilde{m} - a \right\}^{\tilde{m}} \sin \left( \frac{2\pi \tilde{m}}{M} \right) \right\}$$

where

$$\zeta(\tilde{\nu}, \tilde{m}, n) = \sin \left( \frac{2\pi \tilde{m}}{M} \right)$$

The secondary azimuthally tangential electric field is obtained by applying the transformation introduced in Eq. (8) on the field’s components that are given in Eqs. (A.9) and (A.10), yielding

$$\tilde{E}_{\phi}^{(s)}(\tilde{\nu}, \tilde{m}) = \sum_{m,n,r} \left\{ \begin{array}{ll} \frac{1}{2\pi \mu_0 g} \left( \zeta_{\tilde{m},n}(\tilde{\nu}, \tilde{m}) \right) R_{\phi,\tilde{m}}(\tilde{\nu}, \tilde{m}) \right\}$$

where

$$R_{\phi,\tilde{m}}(\tilde{\nu}, \tilde{m}) = (y_\tilde{m} - y, \tilde{m}) \sin \left( \frac{2\pi \tilde{m}}{M} \right) + (x_\tilde{m} - x, \tilde{m}) \cos \left( \frac{2\pi \tilde{m}}{M} \right)$$
and

\[ R_{\nu,0}(\tilde{r}, \tilde{m}) = -(x_{\nu} - x) \sin \left( \frac{2\pi \tilde{m}}{M} \right) \]

\[ + (y_{\nu} - y) \cos \left( \frac{2\pi \tilde{m}}{M} \right). \]

In the expressions for the electric field’s components, the orthogonality of the \( z \) dependency of different \( n \) components in the sums enables the separation of the requirement that the boundary conditions are satisfied for the total electric field into separate requirements for each value of \( n \). From Eqs. (A.4), (A.11), (9) and (10), the expressions for the tangential electric field for a given value of \( n \) are obtained, and are given by

\[ \hat{\nu}_{n,\nu}(v, m) = \frac{\varepsilon_{0}Q_{n}}{2\beta g - A_{n}} \cdot e^{-A_{n}[v - A]} \sin \left( \frac{\pi n}{g} \right) e^{-j(\omega/v)\varepsilon} \]

\[ \times \left\{ \begin{array}{ll}
-1 & \quad \text{if } (\nu, m) \\
\xi^{(m,\nu,\nu)}(\tilde{r}, \tilde{m}) & \quad \text{if } (\nu, m) \end{array} \right\} J_{\nu,\nu}(v, m). \]

Therefore, the boundary condition of zero tangential electric field on the surfaces of the metallic structure can be written as

\[ \begin{cases}
\hat{\nu}_{n,\nu}(\tilde{r}, \tilde{m}) &= -\hat{\nu}_{n,\nu}(\tilde{r}, \tilde{m}), \\
\hat{\nu}_{n,\nu}(\tilde{r}, \tilde{m}) &= -\hat{\nu}_{n,\nu}(\tilde{r}, \tilde{m}),
\end{cases} \quad \text{for } n \in \mathbb{Z}. \]

Using Eqs. (11)–(14), Eq. (15) can be rewritten in a matrix form as follows

\[ \begin{bmatrix}
Z_{n,\nu,f} & 0 \\
Z_{n,\nu,f} & Z_{n,\nu,0} & M_{n,\nu}
\end{bmatrix} = \begin{bmatrix}
-\frac{\nu_{n,\nu}}{\xi_{n,\nu}} \left( \tilde{r} \tilde{m} \right) & I_{n,\nu,0}
\end{bmatrix}, \quad \text{for } n \in \mathbb{Z} \]

where the sub-matrices are given by

\[ \begin{align*}
Z_{n,\nu,0} &= \left[ \frac{-\nu_{n,\nu}}{\xi_{n,\nu}} \right] \left( \tilde{r} \tilde{m} \right) \\
Z_{n,\nu,f} &= \left[ \frac{-\nu_{n,\nu}}{\xi_{n,\nu}} \right] \left( \tilde{r} \tilde{m} \right) \\
Z_{n,\nu,f} &= \left[ \frac{-\nu_{n,\nu}}{\xi_{n,\nu}} \right] \left( \tilde{r} \tilde{m} \right)
\end{align*} \]

The sub-vectors are given by

\[ \begin{align*}
\xi_{n,\nu}(\tilde{r}, \tilde{m}) &= \left[ \frac{-\nu_{n,\nu}}{\xi_{n,\nu}} \right] \left( \tilde{r} \tilde{m} \right), \\
\xi_{n,\nu}(\tilde{r}, \tilde{m}) &= \left[ \frac{-\nu_{n,\nu}}{\xi_{n,\nu}} \right] \left( \tilde{r} \tilde{m} \right),
\end{align*} \]

\[ \nu_{n,\nu}(\tilde{r}, \tilde{m}) = \left( \tilde{r} \tilde{m} \right), \quad \text{for } n \in \mathbb{Z} \]

\[ V \]

\[ M \]

\[ \xi_{n,\nu}(\tilde{r}, \tilde{m}) = \left( \tilde{r} \tilde{m} \right), \quad \text{for } n \in \mathbb{Z} \]

3. Power and spectrum

With the explicit expressions for the secondary electromagnetic field it is possible to evaluate the total energy emitted by the moving bunch due to the wake and its
The tangential field’s components can be approximated

\[
E_{\phi}^{(z)}(z) \approx \sum_{m,n,r} \int \frac{1}{2\pi} \sqrt{\frac{\pi}{2\beta_0 R_0}} e^{-j\theta_0 p_{\phi}} J_n(z R_0) M_{z}^{(m,n)}(z R_0) \times e^{-j\omega_0 t_{\phi}} K_m(z R_0) \sin \left( \frac{n\pi z}{g} \right)
\]

(22)

The normalized velocity is given by

\[
P_{n} = \sum_{m,n,r} \int \frac{1}{2\pi} \sqrt{\frac{\pi}{2\beta_0 R_0}} e^{-j\theta_0 p_{\phi}} J_n(z R_0) M_{z}^{(m,n)}(z R_0) \times e^{-j\omega_0 t_{\phi}} K_m(z R_0) \sin \left( \frac{n\pi z}{g} \right)
\]

(23)

The summation over the spectrum of all the plates, \(g\), for \(v = 1, 2, \ldots, V\). On this ring the tangential field’s components can be approximated by

\[
E_{\phi}^{(z)}(z) \approx \sum_{m,n,r} \int \frac{1}{2\pi} \sqrt{\frac{\pi}{2\beta_0 R_0}} e^{-j\theta_0 p_{\phi}} J_n(z R_0) M_{z}^{(m,n)}(z R_0) \times e^{-j\omega_0 t_{\phi}} K_m(z R_0) \sin \left( \frac{n\pi z}{g} \right)
\]

(24)

Hence, the total power radiated is given by

\[
P = \frac{1}{2} \int_{0}^{\infty} \left[ E_{\phi}^{(0)}(R_0) + E_{\phi}^{(1)}(R_0) \right] \, dz
\]

(25)

Utilizing the orthogonality of the trigonometric functions, it can be shown that the spectrum is obtained by summation over the spectrum of all the z-directed harmonics i.e.

\[
P = \sum_{n} P_n
\]

(26)

where the spectrum for each value of \(n\) is given by

\[
P_n = \frac{9[(\omega/c)^2 - (\pi n/g)^2]}{16\omega} \sum_{m,n,r,v} \left[ \frac{1}{\beta_0} M_{z}^{(m,n)}(M_{z}^{(m,n,v)})^* \times K_m(z R_0) \times e^{-j\omega_0 t_{\phi}} K_m(z R_0) \times \sin \left( \frac{n\pi z}{g} \right) \right]
\]

(27)

As a first step we introduce the decay of the normalized power carried by a mode as we increase its z-directed wave number, \(n\). This last quantity versus the z-directed wave number of the mode for three different frequencies (\(\Omega = (\omega L/2\pi c) = 1, 2, 5\) is shown in Fig. 4 frames (a), (b) and (c). The structure is depicted in the insert, the space between posts’ centers, \(L_1\), and \(L_2\), are chosen to be equal to \(L_0\), the radius of each post, \(a\), is 0.2L, the space between the plates, \(g\), is \(L\). The charge is set half way between the plates, \(h = g/2\), has no offset in the y direction, \(A = 0\), and its normalized velocity is given by \(\beta = 0.8\). In all these calculations the highest azimuthal harmonic that is taken into consideration, \(M\), is set to 10. It is evident that as the frequency increases, modes with a higher \(n\) value are excited and therefore, in order to obtain an accurate result the summation over \(n\) should be extended. Note that the power carried by modes with an even \(n\) zero since the charge is located half way between the plates and therefore asymmetric modes cannot be excited.

In order to ensure accurate results, we should check the convergence of the total power as a function of \(M\). Fig. 4 frames (d), (e) and (f) shows the convergence of the total normalized power as the highest azimuthal harmonic that is taken into consideration (\(M\)) is increased for three different frequencies (\(\Omega = 1, 2, 5\)). The structure is depicted in the insert and has the same parameters that were used in Fig. 4 frames (a), (b) and (c)). It is evident that as the frequency increases, modes with a higher \(M\) value are excited and therefore, in order to obtain an accurate result the summation over \(M\) should be extended.

Next, we investigate the power spectrum for different number of posts and different \(\beta\) values. Fig. 5 presents three graphs of the power spectrum for different number of posts, as is depicted in the inserts (2 \times 2, 4 \times 4, 6 \times 6). Each graph shows the power spectrum for two values of \((\beta = 0.99, 0.999)\). All the structure’s parameters remain as before. Fig. 5 allows a few observations to be made. First, there is a cutoff frequency in all the graphs, which implies that the structure does not sustain modes that carry power with frequencies below the cutoff frequency. Second, the radiated power decreases as the frequency increases since for high frequencies the primary field decay rapidly and interact only to a small extent with the metallic posts. Note that as the value of \(\beta\) decreases, the rate of decrease with the frequency is higher. This effect is due to the fact that when the velocity of the point-charge is increased the spectrum of the equivalent current is shifted upward to higher frequencies. Finally, as the number of posts is increased more peaks and dips are emerging in the power spectrum. It is expected that if we continue to increase the number of posts, the structure will resemble a periodic structure and the dips will become the well-known band-gaps that characterize such structures.

In Fig. 6 a magnification of a small region near the highest peak of Fig. 5c is presented. This magnification gives a good approximation of the values of the first two peaks for four different values of \((\beta = 0.9, 0.99, 0.999, 0.9999)\). The radiation pattern of the highest peak, the lowest dip and the next peak are given in the graph. Note that the lobes of the radiation pattern for both peaks imply that it is possible to have backward radiation. The latter will be utilized in the next section in order to demonstrate the feasibility of beam position monitoring. Its main benefits being the possibility to locate the detectors far away from the beam and inherently filter the near field effects.

Now that we have a clear picture of the total emitted power, it is important to determine its angular distribution. In the next section the angular distribution of the emitted...
power for different beam position offset is demonstrated and utilized for beam position monitoring.

4. Photonic crystal structure as a BPM

So far we have established the wake-field generated by a charged particle traversing an array of metallic posts resembling a photonic crystal structure. In this section we will utilize the calculation carried out in order to examine the main characteristics of the angular distribution of the energy emitted by the moving particle and in particular its dependence on the offset. Having in mind a beam position monitor as a potential application of this system, our goal is to determine the correlation between the radiation pattern and the beam position.

The bottom-right frame of Fig. 7 illustrates the radiation pattern of three different beam position offset ($\mathbf{\Delta}/\mathbf{L}$ = 0.0, 0.14, 0.25) when the charged particle traverses an array of 6 × 6 metallic posts with periodicity of ($L_x = L_y = L$), post’s radius of ($a = 0.2L$), particle’s vertical height ($h = g/2$), velocity of $\gamma = 1000$ and normalized frequency of $\Omega = 1.12$. Two facts are evident from these radiation patterns: First, for zero offset the radiation pattern is symmetric, and as the offset value increases, the radiation...
pattern becomes asymmetric. Second, the intensity of the backward radiation increases with the increase of the particle’s offset. Moreover, the radiation pattern is highly directional and spreads over a narrow angular range i.e. the radiation is concentrated in two narrow main lobes. It is important to indicate that for a given frequency, this angular range is fixed for any value of the particle’s offset and its velocity (however, it is energy independent for relativistic particles).

Since for a given frequency, the radiation pattern is limited for a narrow angular range, we may measure the intensity of the emitted radiation by locating two detectors oriented to capture the backward’s radiation main lobes. To this end, we begin by defining the radiation intensity measured on each detector as the integral of the power density spectrum both over the net detection area and over the frequency bandwidth of the detector. As a typical example consider two narrow bandwidth (0.01%) detectors.
of a cross-section area, placed 10 cm from the center of a metallic posts array with all parameters similar to those in Section 3, i.e., \((L_x = L_y = L, a = 0.2L, h = g/2)\). In this case, the radiation intensity impinging upon each detector is calculated by integrating the power density over approximately 6º (which stands for the solid angle covered by the detector’s cross-section area at its given distance from the structure i.e. an angle of about 1.6% of 360º) about each peak (lobe) of the backward radiation.

Next step is to relate between the measured intensities and the particle’s offset from the axis of symmetry. With this purpose in mind, we define the normalized differential intensity

\[
\overline{D}(\Delta) = \frac{|I_1 - I_2|}{(I_1 + I_2)/2} \tag{28}
\]

providing us with a measure of the asymmetry between the intensities of the main lobes as measured by the detectors. Evidently, in case of zero offset, the radiation intensity measured on the detectors will be equal and \(\overline{D}\) will become zero. It is worthy indicating that the suggested BPM configuration may provide us with a measure of the beam offset only in one axis. In order to obtain the beam offset in the other axis, the same configuration, but rotated by 90º, should be used. Conceptually, it is possible to conceive a 3D array of metallic obstacles that will resolve both offsets simultaneously; however, this is beyond the scope of this study. It is expected that the beam in the linac and at the final focus used in the next generation of linear collider will be flat with an aspect ratio between its vertical and horizontal sizes of at least a factor 10 [22,23], and therefore the 3D array to be used is not necessarily symmetric. It is important to indicate that, nowadays, many efforts are directed towards the challenging fabrication of PBG structures at optical wavelengths. It seems to be very promising to have such structures in the near future [24,25].

The normalized differential intensity \(\overline{D}(\Delta)\) is now calculated for different values of beam position offset at different frequencies. Fig. 7 depicts the graph of \(\overline{D}(\Delta)\) versus the normalized frequency \(\Omega = 1.12\) and \(\gamma = 1000\). This frequency was chosen since it stands for a local maximum in the power spectrum graph (see Fig. 6) and since there is a clear split of the radiation pattern into two distinct lobes. Fig. 7 reveals a distinct one-to-one relation between the values of \(\overline{D}\) and \(\Delta\) that can be easily obtained via polynomial fit. Throughout the simulations performed, the offset was assumed to be a fraction of the measured wavelength (\(\lambda\)) i.e. \(\Delta = 0.112\lambda\). Beyond this value, there is still a one-to-one relation between \(\overline{D}\) and \(\Delta\) but it is not necessarily linear. For maximum sensitivity at the wavelength of the detector, we chose the dimensions \((L)\) of the array to be similar to \(\lambda\) i.e. \(L = \lambda\). According to our simulations, the normalized intensity may be approximated by

\[
\overline{D}(\Delta) = 0.107\Delta \tag{29}
\]

for \(\Omega = 1.12\) and \(\gamma = 1000\). We focus here on the absolute value of \(\overline{D}\) but in practice its sign is indicative of the offset direction. Figs. 8 and 9 depict two more graphs of \(\overline{D}(\Delta)\) versus \(\Delta\), one for a slightly lower frequency \(\Omega = 1.0675\) and the other for a slightly higher frequency \(\Omega = 1.135\). Each graph contains curves for different \(\gamma\) values, and it is evident that the curves converge as \(\gamma\) is increased. In these figures, the values of \(\Delta\) were taken directly between 0 and \(\Delta_{\text{max}} = 0.1L\) to show the linear dependency between \(\overline{D}\) and \(\Delta\).

Explicitly, for \(\Omega = 1.0675\) and \(\gamma = 1000\)

\[
\overline{D}(\Delta) = 0.0228\Delta \tag{30}
\]

whereas if \(\Omega = 1.135\) and \(\gamma = 1000\)

\[
\overline{D}(\Delta) = 1.759\Delta. \tag{31}
\]
In the relativistic regime i.e. $\gamma \geq 1000$ it is evident, that the curves of $d\tilde{I}$ versus $\tilde{A}$ overlap and therefore the previous linear relations are valid for any $\gamma \geq 1000$. Although the results in Eqs. (29)–(31) are the outcome of an extensive electromagnetic analysis, in practice the proportionality coefficient may be established experimentally by one or more calibration measurements conducted for known offset and detectors measuring at specified frequency.

So far we have shown that one can deduce the beam position offset from the symmetry axis only by measuring the intensity of the radiation emitted by a charged particle as it traverses an array of metallic posts. In other words, we may monitor the beam position utilizing such a photonic crystal structure. The only question to be addressed now is what are the position and time resolutions of such a pick-up BPM? Actually, the answer to this question depends mainly on the accuracy and the characteristics of the detectors to be used for measuring the intensity of the radiation emitted. Focusing on the optical regime i.e. assuming that the investigated structure is of optical dimensions ($L_x, L_y, a, h, g_{\perp} \approx 1 \pm 10 \mu m$) the resolutions of the BPM are determined by the noise levels in optical detectors. Particularly, the time resolution is dominated by the highest frequency that optical detectors can operate at, while the position resolution is dominated by the noise level of the detectors.

Typically, optical detectors may operate at frequencies up to 100 GHz and therefore the time that takes the detector to be ready for the next radiation detection is of the order of 10 ps ($T = 1/100$ GHz). Accordingly, the time resolution of such a BPM is about 10 ps which is higher by at least two orders of magnitude than the time resolution obtained in stripline and cavity BPMs. Moreover, the position resolution of the suggested BPM is dominated by the accuracy of power measurement at the optical regime. Hence, this accuracy is limited by the noise power level measured in optical detectors. As the noise causes fluctuations in the measurement, it places a lower limit on the smallest amount of power that can be measured. In such detectors the minimum signal power intensity ($I_{\text{lim}}$) that can be measured in the presence of noise is given by $I_{\text{lim}} = (1 + \sqrt{2})I_N$ where $I_N$ is the noise power intensity [26]. The noise power intensity is determined by the so-called shot noise [26] and the thermal noise of the detector and it varies from one detector to another. A practical measure of the magnitude of the noise power level is the noise equivalent power (NEP) of the detector. The NEP is the rms value of a sinusoidal modulated light signal falling on the detector that gives rise to an rms electrical signal equal to the rms noise voltage. Just to illustrate the dependency of the position resolution on the detector’s properties we introduce here two cases: at the first one we will utilize the state of the art optical detector revealing high position resolution of single nanometers and at the second one a simple typical optical detector will be utilized revealing to a modest position resolution of tens of nanometers. To quote an example we consider two different optical detectors manufactured by New-Focus Company. Detector model No. 1011 operates at frequency of 40 GHz (i.e. a time resolution of 25 ps) and its bandwidth is 50 kHz with wavelength range 950–1650 nm. Such a model exhibits NEP of the order of 45 pW/Hz. According to our estimates such a NEP fits to a measurement’s sensitivity of the backward radiation intensity obtained by a deviation of a few tens of nanometers in the particle offset from the axis of symmetry and correspondingly the position resolution is of a few tens of nanometers. In a similar way, if we utilize detector model No. 1411 that operates at frequency of 25 GHz (i.e. a time resolution of 40 ps) and its bandwidth is 50 kHz with wavelength range 950–1650 nm. Such a model exhibit NEP of the order of 30 pW/Hz and therefore the appropriate position resolution is of the order of a few nanometers. Based upon the simulation carried out so far the position resolution depends on the frequency on which we measure the backward radiation i.e. ($\Omega$). An important feature that usually characterizes optical detectors that the higher the operating frequency the higher the NEP consequently, in the context of BPM resolution, there is a trade-off between time resolution and position resolution. It should be indicated that according to some experimental tests of optical diffraction radiation conducted at the KEK accelerator test facilities a non-negligible part of the signal measured was due to transition radiation emitted from beam’s tail when considering flat-beam [27,28]. Hence, in order to ensure accurate performance for the suggested BPM the transverse beam size should be controlled extremely precisely to minimize such an effect.

5. Summary and conclusions

In this study we have investigated the wake-field generated by an electron bunch traversing an array of $N \times N$ metallic vertical posts bounded from above and below by two horizontal metallic plates. The total power emitted by such a bunch was evaluated as well as its spectrum. Moreover, the angular distribution of the emitted energy was demonstrated and it was shown that for zero offset of the beam from the symmetry axis the radiation pattern is symmetric, and as the offset value increases, the radiation pattern becomes asymmetric. In addition, a one-to-one linear relation between the normalized differential intensity ($\tilde{I}$) and the beam offset $\tilde{A}$ was developed. Given the geometrical dimensions of the structure, the characteristics of the bunch and measuring the normalized differential intensity of the backward emitted radiation the beam position offset from the symmetry axis may be determined. The position resolution of such a structure is assumed to be dominated by the thermal noise level and the shot noise level of the detectors while the time resolution is dominated by the operation frequency of the detectors. In optical regime and for ultra-relativistic beams the time resolution is of order of 10 ps i.e. at least two orders of magnitude higher than the common time resolution in stripline BPM.
The position resolution depends mainly on the type of optical detector to be used for the measurement. State of the art optical detectors with high sensitivity may reveal position resolution of single nanometers i.e. about two orders of magnitude higher than the common values achieved so far. The main goal behind the current study was just to prove the feasibility of using photonic crystal structure as a pick-up BPM. A complete design of such a BPM relies on precise investigation of the current generation of optical radiation detectors which is beyond the scope of this study.

Appendix A

As the primary magnetic vector potential is determined by Eq. (1), utilizing the relation

$$\vec{E} = -j\omega \vec{A} - \nabla \phi$$  \hspace{1cm} (A.1)$$

wherein $\phi$ is the electric scalar potential, the primary electric field is obtained, and reads

$$E^{(p)}_x = \frac{-j \mu_0 Q}{2\pi \beta^2 g} \sum_n e^{-A_n (y-A)} \sin \left( \frac{\pi n \beta}{g} \right) \times \sin \left( \frac{\pi n z}{g} \right) e^{-j(\omega/\gamma) x}$$ \hspace{1cm} (A.2)$$

$$E^{(p)}_y = \frac{-c \mu_0 Q}{2\beta g} \sum_n e^{-A_n (y-A)} \sin \left( \frac{\pi n \beta}{g} \right) \times \sin \left( \frac{\pi n z}{g} \right) e^{-j(\omega/\gamma) x}$$ \hspace{1cm} (A.3)$$

$$E^{(p)}_z = \frac{-c \mu_0 Q}{2\beta^2 g} \sum_n A_n \sin \left( \frac{\pi n \beta}{g} \right) \times \cos \left( \frac{\pi n z}{g} \right) e^{-j(\omega/\gamma) x}.$$ \hspace{1cm} (A.4)$$

In a similar way, the secondary field is deduced utilizing the following relations:

$$\vec{E}_{II} = \mu_0 \delta_0 c^2 \vec{P}_z + \nabla \times (\Pi_z \vec{z})$$ \hspace{1cm} (A.5)$$

$$\vec{H}_{II} = j\omega \vec{A} \times (\Pi_z \vec{z})$$ \hspace{1cm} (A.6)$$

$$\vec{E}_{II} = -j \mu_0 \vec{A} \times (F_z \vec{z})$$ \hspace{1cm} (A.7)$$

$$\vec{H}_{II} = \mu_0 \delta_0 c^2 \vec{F}_z + \nabla \times (F_z \vec{z}).$$ \hspace{1cm} (A.8)$$

Accordingly, the secondary electric field is given by

$$E^{(s)}_x = \frac{1}{2\pi} \sum_{m,n} \left( \frac{-j}{\omega_0 g} \right) D^{(m,n,v)}_y I^{(m,n,v)}_z + D^{(m,n,v)}_x M^{(m,n,v)}_z \right) \times \sin \left( \frac{\pi n z}{g} \right)$$ \hspace{1cm} (A.9)$$

$$E^{(s)}_y = \frac{1}{2\pi} \sum_{m,n} \left( \frac{-j}{\omega_0 g} \right) D^{(m,n,v)}_y I^{(m,n,v)}_z - D^{(m,n,v)}_x M^{(m,n,v)}_z \right) \times \sin \left( \frac{\pi n z}{g} \right)$$ \hspace{1cm} (A.10)$$

$$E^{(s)}_z = \frac{j}{2\pi \omega_0} \sum_{m,n} \left[ \left( \frac{\omega_0}{c} \right)^2 - \left( \frac{\pi n z}{\gamma} \right)^2 \right] I^{(m,n,v)}_z \times \cos \left( \frac{\pi n z}{\gamma} \right)$$ \hspace{1cm} (A.11)$$

wherein

$$D^{(m,n,v)}_x(x,y) = \xi^{(m,n,v)}_x(x,y) \frac{(x-x_0)}{\rho_v} + \xi^{(m,n,v)}_x(x,y) \frac{j m (y-y_0)}{\rho_v^2}$$ \hspace{1cm} (A.12)$$

$$D^{(m,n,v)}_y(x,y) = \xi^{(m,n,v)}_y(x,y) \frac{(y-y_0)}{\rho_v} + \xi^{(m,n,v)}_y(x,y) \frac{j m (x-x_0)}{\rho_v^2}$$ \hspace{1cm} (A.13)$$

and $\xi^{(m,n,v)}_x(x,y)$ is given by

$$\xi^{(m,n,v)}_x(x,y) = e^{-j m \rho_v} \left[ I_{m+1} (\rho_v) + \left( \frac{\pi}{\rho_v} \right) I_m (\rho_v) \right] K_m (Y_a a) \quad \rho_v \leq a$$

$$e^{-j m \rho_v} \left[ -I_{m+1} (\rho_v) + \left( \frac{\pi}{\rho_v} \right) I_m (\rho_v) \right] K_m (Y_a a) \quad \rho_v > a.$$ \hspace{1cm} (A.14)$$

References


