Electrons Acceleration in Active Medium

Levi Schächter

Technion - Israel Institute of Technology
Department of Electrical Engineering
Outline

- Overview & Motivation
- PASER: Particle Acceleration by Stimulated Emission of Radiation
- Wake Amplification
- Acceleration in a growing wake
- Acceleration & Saturation
- Summary
Interaction of a single-mode with a bunch of electrons

\[
\frac{d}{d\zeta} a = \alpha \langle e^{-j\chi_i} \rangle
\]

\[
\frac{d}{d\zeta} \gamma_i = -\frac{1}{2} \left[ a e^{j\chi_i} + c.c. \right] \Rightarrow \frac{d}{d\zeta} \left[ \begin{array}{c} \langle \gamma_i \rangle -1 + \frac{|a|^2}{2\alpha} \\ Kinetic Energy \\ EM Energy \end{array} \right] = 0
\]

Energy Conservation
Acceleration & Saturation

Energy conservation in the presence of Active Medium

\[ \frac{d}{d\zeta} \left( \langle \gamma_i \rangle - 1 + \frac{|a|^2}{2\alpha} + \frac{N_{ph} \hbar \omega}{N_e mc^2} \right) = 0 \]
The effect on the population inversion

\[ \frac{d}{d\xi}a = a \langle e^{-j\chi_i} \rangle + \left( \frac{1}{2} \sigma N_{ph} d \right) a \]

\[ \frac{d}{d\xi} \gamma_i = -\frac{1}{2} \left[ a e^{j\chi_i} + c. c. \right] \]

\[ \Rightarrow \quad \frac{d}{d\xi} \left[ \langle \gamma_i \rangle - 1 + \frac{|a|^2}{2\alpha} \right] = \left( \frac{|a|^2}{2\alpha} \right) \left( \sigma N_{ph} d \right) \]

\[ \frac{d}{d\xi} \left[ \langle \gamma_i \rangle - 1 + \frac{|a|^2}{2\alpha} + \frac{N_{ph} \hbar \omega}{N_e mc^2} \right] = 0 \]

\[ \Rightarrow \quad \frac{d}{d\xi} N_{ph} = -\left( \frac{|a|^2}{2\alpha} \right) \left( \sigma d N_e \frac{mc^2}{\hbar \omega} \right) N_{ph} \]
Summary of governing equations

\[
\frac{d}{d\xi} \alpha = \alpha \left( e^{-j\chi_i} \right) + \left( \frac{1}{2} \sigma N_{ph} \right) a
\]

\[
\frac{d}{d\xi} \gamma_i = -\frac{1}{2} \left[ a e^{j\chi_i} + c.c. \right]
\]

\[
\frac{d}{d\xi} \chi_i = \Omega \left( \frac{1}{\beta_i} - \frac{1}{\beta_p} \right)
\]

\[
\frac{d}{d\xi} N_{ph} = -\left( \frac{|a|^2}{2\alpha} \right) \left( \sigma d N_e \frac{mc^2}{\hbar \omega} \right) N_{ph}
\]
**Simulation parameters:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ [µm]</td>
<td>1.06</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$7 \times 10^3$</td>
</tr>
<tr>
<td>$N_e$ [m^{-3}]</td>
<td>$10^5$</td>
</tr>
<tr>
<td>Energy [MeV]</td>
<td>300</td>
</tr>
<tr>
<td>$N_{ph}$ [m^{-3}]</td>
<td>$10^{25}$</td>
</tr>
<tr>
<td>$P_{in}$ [MW]</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma$ [m^2]</td>
<td>$10^{-24}$</td>
</tr>
</tbody>
</table>
Acceleration & Saturation

Saturation

![Graph showing Acceleration & Saturation](image)

- Power [MW]
- Gradient [GV/m]
- $\zeta$

The graph illustrates the relationship between power and gradient as a function of $\zeta$. The saturation point is indicated by the dotted line.
Acceleration & Saturation

Average Energy [MeV]

$\zeta$

$\Delta y/(\langle y \rangle - 1)$ [%]
Acceleration & Saturation

Saturation

Relative Population Inversion [%]

Gradient [GV/m]
Summary

• **PASER:** Point-charge accelerated by energy stored in the medium

• Same energy amplifies a wake-field - Cerenkov

• Eigen-modes move at the speed of the bunch

• Inherent longitudinal e-field: interaction length

• Emittance growth < 0.1\(\pi\) mm-mrad

• Acceleration not affected by medium saturation