Beam-Quality and Guiding Field Effect on a High-Power TWT Operating at 35GHz

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Outline

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- Theoretical Model and Basic Assumptions
- Dynamics of the System
- Simulation Results
- High Order Modes
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Motivation

- For a given accelerating gradient \((E_0)\) and group velocity, the product \(P \times \omega^2 \approx \text{const.}\).
- Explicitly: \(P \propto (E_0 \lambda)^2/V_{\text{gr}}\)
- We reduce the power by a factor of 16 if the frequency is increased by a factor of 4!!
**Definition of the Model**

- **RF (TM\textsubscript{01}):**
  
  \[ E_z(r,z,t) = E_0 I_0(\Gamma r) \cos(\omega t - k z - \psi) \]
  
  \[ E_r(r,z,t) = -\gamma_{ph} E_0 I_1(\Gamma r) \sin(\omega t - k z - \psi) \]
  
  \[ H_\phi(r,z,t) = -\gamma_{ph} \beta_{ph} (E_0 / \eta_0) I_1(\Gamma r) \sin(...) \]

- **DC (beam):**
  
  \[ E_r(r<R_b) = - (en_0/2\varepsilon_0)r \]
  
  \[ \eta_0 H_\phi(r<R_b) = - (v/c^2)(en_0/2\varepsilon_0)r \]

- **Guiding Field - B_0**
Particles’ Dynamics

Azimuthal motion:

\[
\frac{d}{dt} \left[ r_i^2 \left( \gamma_i \frac{d}{dt} \phi_i - \frac{1}{2} \omega_c \right) \right] = 0
\]

\[
\omega_c = \frac{eB_0}{m}
\]

\[
\Rightarrow \frac{d}{dt} \phi_i = \frac{\omega_c}{2 \gamma_i r_i^2} \left( r_i^2 - r_{i, \text{in}}^2 \right)
\]

\[
\left. \frac{d}{dt} \phi_i \right|_{r_i = r_{i, \text{in}}} = 0
\]
Particles’ Dynamics

Radial motion:

\[
\frac{d}{dt} \left( \gamma_i \frac{d}{dt} r_i \right) + \frac{\omega_c^2}{4 \gamma_i r_i^3} \left( r_i^4 - r_{i,in}^4 \right) - \frac{\omega_p^2}{2 \gamma_i^2} r_i = -\frac{e}{m} \left( E_r + V_z \mu_0 H_\phi \right)_i
\]
Particles’ Dynamics

\section*{Longitudinal motion:}

\[ \frac{d}{dt} (\gamma_i \beta_{z,i}) = - \frac{e}{mc} (E_z + V_r \mu_0 H_{\phi}) \]

\approx - \frac{e}{mc} E_z

\approx - \frac{e}{mc} E_0 I_0(\Gamma r_i) \cos[\omega t - k z_i(t) - \psi]
Field Dynamics

- Poynting theorem:
  \[ \frac{d}{dz} \langle P(z) \rangle_t = -\int_{cs} da \langle \mathbf{J} \cdot \mathbf{E} \rangle \]

- Interaction impedance:
  \[ \langle P(z) \rangle_t = \frac{1}{2} \frac{E_0^2 (\pi R^2_{int})}{Z_{int}} \]

- Neglect contribution of the radial motion to energy exchange
  \[ | \beta_{r,i} | \gamma_i \ll | \beta_{z,i} | \]
Field Dynamics

\section*{Amplitude equation:}
\[
\frac{d\overline{E}_0}{dz} = \frac{IZ_{\text{int}}}{\pi R_{\text{int}}^2} \left\langle I_0 \left[ \Gamma r_i(z) \right] e^{-j\chi_i} \right\rangle_i
\]

\section*{Phase equation:}
\[
\frac{d\chi_i}{dz} = \frac{\omega}{c} \frac{1}{\beta_{z,i}} - k
\]

\[\gamma_{z,i} = [1 - \beta_{z,i}^2]^{-1/2} = \gamma_i \left[ 1 + (\gamma r_i) \right] + \left( r_i^2 - r_{i,\text{in}}^2 \right)^2 \left( \frac{\omega_c}{2cr_i} \right)^2 \right]^{-1/2}\]
Comparison of 35GHz & 8.75GHz

- **1D motion**
- **Identical beams (500kV, 200A, 3mm radius)**
- **Energy conservation** \( \Rightarrow \) **same input (20kW)**
- **At the input** \( \beta_{ph} = \langle \beta_i \rangle \)
- **Same gain:** \( \text{Im}(kd) \Rightarrow Z_{int} \times \left( \frac{\omega}{c}d \right) = \text{const} . \)
- **Phase advance per cell** \( 120^\circ \)
Comparison of 35GHz & 8.75GHz

Parameters:

\[ R_{\text{int}} = 6.0, 6.5, 7.0, 7.5 \text{[mm]} \Rightarrow \]
\[ Z_{\text{int}} (35\text{GHz}) = 73, 52, 37, 26 \text{[Ω]} \]
\[ Z_{\text{int}} (8.75\text{GHz}) = 2700, 2193, 1812, 1519 \text{[Ω]} \]

At this stage it is assumed that if the phase velocity varies, the change in the interaction impedance is negligible.
Comparison of 35GHz & 8.75GHz

- Growth-rate at 35GHz sensitive to beam quality.
- Not critical!!

In practice, energy spread of 1% is achievable.
Comparison of 35GHz & 8.75GHz

- Energy spread at the output as a function of the energy spread at the input for the same parameters.
- Note the dramatic effect of high interaction impedance.
Guiding Field Effect

- Energy spread at the input 1%.
- Tapered structure:
  \[
  \frac{1}{\beta_{ph}} = \left\langle \frac{1}{\beta_i} \right\rangle
  \]
- Simulation terminates if: particles hit the wall or are reflected.
Beam-Quality Effect

- $R_{int} = 6\text{mm}$
- $B = 0.5\text{T}$
Beam-Quality Effect

\( R_{\text{int}} = 6 \text{mm} \)

\( B = 0.5 \text{T} \)
The dependence of the interaction impedance on the phase velocity was calculated analytically at 35GHz:

\[
Z_{\text{int}} \left( R_{\text{int}} = 6\text{mm} \right) = 302 \beta_{ph}^{8.76} [\text{Ohm}]
\]

\[
Z_{\text{int}} \left( R_{\text{int}} = 7\text{mm} \right) = 219 \beta_{ph}^{11.28} [\text{Ohm}]
\]

\[
Z_{\text{int}} \left( R_{\text{int}} = 8\text{mm} \right) = 166 \beta_{ph}^{14.02} [\text{Ohm}]
\]
High-Efficiency Interaction

- Varying interaction impedance
- Three tapering criteria
- For a good density modulated beam 65% efficiency is achievable
High Order Mode Suppression

- Preserving a high internal radius (>6mm) we solved one problem but we generated another: high order modes.
- Solution open structure
Summary

- For less than 5% energy spread at the input, a KA system should operate as X-band one.
- $B=0.5\,\text{T}$ should suffice to confine a 3mm radius beam which produces radiation with 30% efficiency in a 6mm internal radius.
- Allowing the interaction impedance to vary in space facilitates increase of efficiency of up to 65% ($R_b=2\,\text{mm}$).