Geometric effects on blackbody radiation

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Planck’s formula for blackbody radiation was formulated subject to the assumption that the radiating body is much larger than the emitted wavelength. We demonstrate that thermal radiation exceeding Planck’s law may occur in a narrow spectral range when the local radius of curvature is comparable with the wavelength of the emitted radiation. Although locally the spectral enhancement may be of several orders of magnitude, the deviation from the Stefan-Boltzmann law is less than one order of magnitude. The fluctuation-dissipation theorem needs to be employed for adequate assessment of the spectrum in this regime. Several simple examples are presented as well as experimental results demonstrating the effect. For each configuration a geometric form factor needs to be incorporated into Planck’s formula in order to properly describe the emitted radiation.

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I. INTRODUCTION

From the early days of quantum mechanics via astrophysical measurements to today’s nanostructures, blackbody radiation (BBR) is playing a pivotal role in physics. As the emitting bodies were always significantly larger than the wavelength of interest, Planck’s formula (PF) described adequately the general trend of the emerging radiation and any deviations were described in terms of the so-called emissivity. Conceptually, the emissivity of a passive body was assumed to be always smaller than unity, explicitly assuming that PF provides the upper limit of what a body can emit [1–5]. For quite some time, manufacturing techniques have facilitated the implementation of minute structures of a size smaller than or of the same order of magnitude as the radiation wavelength, leading to a new regime of operation in which PF no longer describes adequately the BBR. Assuming PF as an absolute law of physics is a misconception which has been criticized even in textbooks (e.g., Ref. [6], p. 126).

In the remainder of this Introduction, we highlight several of the BBR investigations relevant to the ideas we intend to convey in this study. By no means have we intended this to be a comprehensive review of the field. First we describe in detail Planck’s derivation of the radiation within an ideal cavity. This we do in some detail in order to emphasize the source of its limitations. In addition, we do not distinguish here between BBR that is generally attributed to closed structures (cavities) and thermal radiation (TR) that describes radiation emitted by a body of nonzero temperature into free-space.

Planck’s [7] original argument consists of three steps. In the first one he considered an ensemble of oscillators in thermal equilibrium and he established, using the classical Maxwell-Boltzmann statistics and using elementary quantum notions, that the energy of a system consisting of $N_{osc}$ oscillators at a given frequency is $E = N_{osc} \Theta(T, \omega)$, wherein\(\Theta(T, \omega) = \hbar \omega [\exp(\hbar \omega/k_B T) - 1]^{-1}\) denotes the mean energy of a single oscillator.

The second step was to count the number of modes ($\Delta N_{cavity}$) within a frequency interval—that is to say, the density of states (DOS)—in a cavity of perfectly reflecting walls of volume $V_{cavity}$. Subject to the tacit assumption that the wavelength is much shorter than the typical dimension of the cavity $\sqrt{V_{cavity}}$, the number of modes in a range of frequencies $\Delta \omega$ starting at $\omega$ is

$$\Delta N_{cavity} = V_{cavity} \frac{\omega^2 \Delta \omega}{\pi^2 c^3},$$

accounting for both possible polarizations.

His third step was to correlate the statistics of oscillators with the DOS in a cavity, which is a delicate matter. Essentially, there must be an equilibrium between the radiation in vacuum and its source in matter, which comes about when a wave impinging upon the walls is absorbed, causing another wave to be radiated so that the walls can be conceived as perfect reflectors; in other words, $N_{osc} = \Delta N_{cavity}$. With this assumption, the energy spectral density ($u = U/\Delta \omega$) is

$$\frac{u}{V_{cavity}} = \frac{\omega^2}{\pi^2 c^3} \frac{\hbar \omega}{\exp(\hbar \omega/k_B T) - 1}.$$

What is unique about Planck’s formula is the fact that the right-hand side is independent of the geometry or the properties of the body. As such, many consider it as a fundamental law in this regard it as an upper limit to what a body can emit.

In the framework of Planck’s formulation, there is a distinction between the number of oscillations $N_{osc}$ which is derived from geometrical considerations, and their mean energy $\Theta$ which is derived from statistical considerations and is therefore independent of the geometry of the problem. While $\Theta$ is correct because there is a large number of possible energy states in a harmonic oscillator, and $\Theta = N_{osc} \Theta$ is almost always correct since the number of atoms (microscopic emitters) is very large, one can question the validity of the calculation of the DOS. The latter is a good approximation only for a cavity of “infinite” volume in respect to the wavelengths of interest. A formal mathematical proof for the validity of (2) given this assumption is given by Courant and Hilbert [8].

Planck himself, when determining the thermal energy density within a cavity, states that “No matter how small the frequency interval $\Delta \nu$ may be assumed to be, we can nevertheless choose $l$ sufficiently great,” where $l$ is the cavity’s dimension [7], p. 273. Later, Rytov [9], Eq. (5.5) indicates that Planck’s law is applicable only if $l \gg \Delta \lambda/\nu \gg (\lambda/l)^2$, where $\Delta \lambda = \lambda \Delta \nu/\nu$ is the frequency interval measured in wavelengths, “thus, the conditions for the validity of PF are first, a not too large mono-chromaticity of the spectral interval,