Enhanced Cherenkov-Wake Amplification by an Active Medium

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A Cherenkov wake confined by perfectly reflecting transverse walls is amplified if the dielectric medium is active. Because of the multiple-reflections process, the effective gain of the wake is enhanced compared to a ray propagating in a straight line. Higher enhancement occurs when the electron velocity is close to the Cherenkov velocity. This Cherenkov wake can then accelerate a second bunch of electrons trailing the first. Gradients larger than 1 GV/m are predicted before saturation becomes a major impediment

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In the category of structureless acceleration schemes, there are two main conceptual mechanisms which may be divided according to the origin of the energy: in one case, the energy source is an intense laser pulse [1] injected in a plasma and the particles are accelerated by the trailing space-charge wake. In the second paradigm, the laser pulse is replaced by an intense electron beam [2] as the energy source—the acceleration itself is again facilitated by the emerging space-charge wake. In a third alternative, a similar trigger bunch generates a Cherenkov wake in a dielectric-loaded waveguide—this is a structure-based acceleration scheme and is called dielectric wakefield acceleration [3–5]. Before describing our novel paradigm, let us briefly recapitulate the amplification process in a laser since, as we shall shortly assert, a similar amplification may be designed for a Cherenkov wake.

In its most simplistic description, the amplification process in a laser (light amplification by stimulated emission of radiation) may be conceived in terms of the interaction of a (stimulating) photon with an atom in a resonant excited state. By that, we mean that the outmost external electron of the atom, during the interaction process, is not in the ground state but rather in an excited state, such that the energy difference between the two energy states corresponds to the energy of the impinging photon. After the collision, the bounded electron returns to the ground state and two identical photons emerge. In what follows, we refer to an ensemble of excited atoms as an active medium since the process described above may occur a multiple number of times—becoming a collective rather than a single particle process. As such, the amplification process, within the framework of a linear and macroscopic theory, may be conceived in terms that the change in the radiation intensity in an infinitesimal layer (Δz) is proportional to the density of excited atoms (population inversion density Δn), the cross section for stimulated emission ($\sigma_{\rm st}$), and the local radiation intensity. Subject to these assumptions, an electromagnetic pulse grows exponentially with a gain parameter $\alpha \propto \Delta n \sigma_{\rm st}$. In other words, the energy of the electromagnetic pulse has increased at the expense of the energy stored in the active medium.

In the past, we suggested [6,7] to employ the energy stored in an active medium to amplify a Cherenkov wake initiated by a trigger bunch. In the 1999 study [6], we examined the boundless case and, later in 2001 [7], we considered a confined interaction (Fig. 1) in a parametric way by postulating (i) an effective impedance that relates the guided power with the accelerating field on axis and (ii) that the gain of the Cherenkov wake is identical to that of an electromagnetic pulse injected in the same structure. Experimentally [8], in 2006, we demonstrated that a single train of bunches (Ref. [9]) may directly gain weak amounts of energy from a boundless active medium [10].

In this Letter, we demonstrate a new paradigm that has the potential of elevating the accelerating gradient by several orders of magnitude. Contrary to the scheme mentioned above where the system operated below Cherenkov velocity ($\beta < \beta_C \equiv 1/\sqrt{\varepsilon_r}$) and the wake propagated in a virtually boundless environment, in what follows, the beam velocity is above the Cherenkov velocity and the radiation is transversally confined. Its essence is energy transfer from an active medium to a Cherenkov wake, which experiences multiple reflections from the confining walls. Because of the multiple-reflections process, the net gain is enhanced compared to that of a plane wave propagating in a straight line.

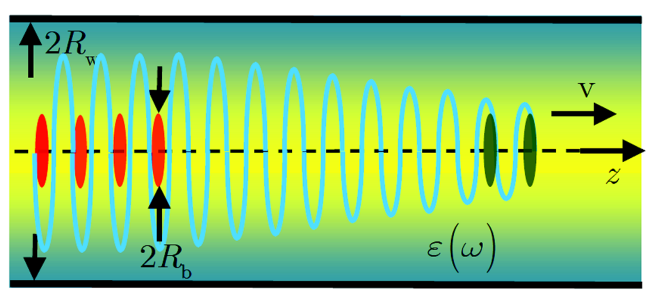

FIG. 1 (color online). Schematic of the conceived concept. A trigger bunch generates a weak wake that is amplified by the medium which in turn accelerates a trailing bunch. The shown amplification is greatly downsized compared to the estimated values.

For a conceptual description, we consider the wake generated by a thin charged loop (Q_b) of radius R_σ, located at $t = 0$ at z_σ as it moves with a velocity v in a waveguide of radius R_w uniformly filled with a medium characterized by a relative dielectric coefficient ε_r. Provided the latter is frequency independent, all three components of the electromagnetic field (E_r, E_z, H_φ) can be derived from the magnetic vector potential

$$A_z = \frac{Q_b \mu_0 c^2}{2\pi} \sum_s \frac{G_s(r, R_\sigma) \exp(j\omega\tau_\sigma)}{-(p_s/R_w)^2 - (\omega/v)^2 + \varepsilon_r(\omega/c)^2}$$
$$= -\frac{Q_b \mu_0 c^2}{2\pi} \sum_s G_s(r, R_\sigma) \begin{cases} \frac{-\exp(-\omega_{C,s}|\tau_\sigma|)}{2\bar{\varepsilon}\omega_{C,s}} & \bar{\varepsilon} < 0 \\ \frac{\sin(\omega_{C,s}\tau_\sigma)h(\tau_\sigma)}{\bar{\varepsilon}\omega_{C,s}} & \bar{\varepsilon} > 0; \end{cases} \tag{1}$$

here, $G_s(r, r') \equiv J_0(p_s(r/R_w))J_0(p_s(r'/R_w))[(1/2)R_w^2 J_1^2(p_s)]^{-1}$, $\tau_\sigma \equiv t - (z - z_\sigma)/v$, μ_0 is the free-space permeability, $J_0(u)$ and $J_1(u)$ are the Bessel functions of the zero and first kinds, respectively, and p_s are the 0s of $J_0(u)$. In this time-domain representation, the field is a superposition of a discrete spectrum of evanescent waves, if the charge moves slower than the Cherenkov velocity ($\bar{\varepsilon} \equiv \varepsilon_r - \beta^{-2} < 0$), and of propagating waves, if the velocity exceeds the Cherenkov velocity ($\bar{\varepsilon} > 0$). The Cherenkov eigenfrequencies composing the wake are $\omega_{C,s}^2 = p_s^2 c^2 / R_w^2 |\bar{\varepsilon}|$, and $h(u)$ is the Heaviside step function.

In the framework of our analysis, we rely on the fact that an ensemble of charges, moving in a resonant medium described by

$$\varepsilon(\omega) = \varepsilon_r + \frac{\omega_p^2}{\omega_0^2 + j\omega\Delta\omega - \omega^2}, \tag{2}$$

generates a more complex spectrum and, under certain conditions, may lead to a growing wake. In Eq. (2), ω_0 is the resonance of the medium, $\Delta\omega$ represents the characteristic bandwidth, and ω_p, the "plasma frequency," will be determined based on standard laser theory.

In the case of a frequency-dependent dielectric, the electromagnetic field is derived from the magnetic vector potential given by the same expression as at the top of Eq. (1), except that the frequency-independent dielectric (ε_r) should replaced by $\varepsilon(\omega)$. As a result, rather than the eigenfrequencies being a solution of $-(p_s/R_w)^2 - (\omega/v)^2 + \varepsilon_r(\omega/c)^2 = 0$, which is a second order polynomial, they are a solution of $-(p_s/R_w)^2 - (\omega/v)^2 + \varepsilon(\omega)(\omega/c)^2 = 0$, or explicitly,

$$(j\omega)^4 + \Delta\omega(j\omega)^3 + (\omega_0^2 + \omega_{C,s}^2 + \omega_{C,p}^2)(j\omega)^2 + \Delta\omega\omega_{C,s}^2(j\omega) + \omega_0^2\omega_{C,s}^2 = 0, \tag{3}$$

which is a fourth order polynomial; here, we defined the Cherenkov-plasma frequency as $\omega_{C,p}^2 \equiv \omega_p^2/\bar{\varepsilon}$. One of the four complex solutions for each radial index (s) may correspond to a growing mode, provided that the Cherenkov condition is satisfied $\bar{\varepsilon} > 0$. We focus our attention on this growing mode.

The most important result of our analysis reveals itself when one of the Cherenkov eigenfrequencies ($s = s_0$) equals the resonance of the medium ($\omega_{C,s_0} \simeq \omega_0$). Examining the imaginary component of the pole, namely, $\Delta\omega_d \simeq \Delta\omega/4 - \sqrt{(\Delta\omega/4)^2 - \omega_p^2/4\bar{\varepsilon}}$, we conclude that the energy transfer from the medium to the wake can differ from the energy transfer to a straight ray by a factor of $1/\bar{\varepsilon}$. When operating very close to the Cherenkov condition, the slippage parameter ($\bar{\varepsilon}$) may be significantly smaller than unity ($\bar{\varepsilon} \ll 1$), leading to enhanced energy exchange. In order to clarify the last statement, it is necessary to quantify the parameters defining the medium ($\omega_p^2, \Delta\omega, \varepsilon_r$).

For realistic estimates of the parameters, we adopt the values for proven CO_2 laser systems as reported by the BNL [11] and UCLA [12] groups. For example, in the regenerative amplifier of the system described in Ref. [11], the small-signal gain with respect to the intensity (at 10 atm) is reported as $2\alpha \sim 1$ to 2 m^{-1}. Other sources provide similar results, e.g., Ref. [13] $p = 10$ atm, $2\alpha \simeq 3.5$ m^{-1}, Ref. [14] $p = 8$ atm, $2\alpha \simeq 2.7$ m^{-1}.

We are now in a position to correlate this quantity with the plasma frequency in Eq. (2) by imposing that at resonance and for a plane wave, the gain parameter α, which is well known from laser theory [15], equals the gain associated with the imaginary component of the wave vector, or explicitly, $(\omega_0/c)\mathrm{Im}[\sqrt{\varepsilon(\omega_0)}] = \alpha$; thus, $\omega_p^2 = -2c\alpha\Delta\omega$. For what follows, we consider $\alpha = 1$ m^{-1}, and the corresponding effective bandwidth (at about 10 atm) is $\Delta\omega \simeq 2\pi \times 37$ GHz; thus, $|\omega_p|/\omega_0 \sim 6.6 \times 10^{-5}$.

The relative dielectric coefficient (ε_r) for carbon dioxide, nitrogen, and helium may be found from the data provided by Bideau-Mehu *et al.* [16], Peck and Khanna [17], and Mansfield and Peck [18], respectively. At a wavelength of 10.6 μm, the $\varepsilon_r - 1$ values at standard pressure and temperature are 9.4×10^{-4}, 5.9×10^{-4}, and 6.9×10^{-5}, respectively. We average the $\varepsilon_r - 1$ values for the gases weighed by their partial pressure and temperature of 300° K in accordance with the Lorentz-Lorenz equation [19]. In the simulations that follow, we use $\varepsilon_r - 1 \simeq 1.42 \times 10^{-3}$. Note that Pantell and co-workers [20] demonstrated emission of Cherenkov radiation in a gas at atmospheric pressure from a relativistic beam (500 MeV) within the range of our parameters.

At this point, we can better quantify the wake-amplification process. In the absence of reflections, the amplification of a plane wave in our active medium is given by $\exp(\alpha z)$, whereas in the case of reflections, the wake amplification is determined by

$$\exp\left[-\left(t-\frac{z}{v}\right)\Delta\omega_d\right]h\left(t-\frac{z}{v}\right), \tag{4}$$

and for small slippage $\bar{\varepsilon} \ll 1$, the gain is much higher than in the former $c\alpha \ll |\Delta\omega_d| \simeq \sqrt{c\alpha\Delta\omega/2\bar{\varepsilon}}$.

In order to envision the process, consider first a laser pulse propagating in a straight line in an active medium (α) at a distance L; its gain is αL. Once the beam propagates at angle θ (relative to the z axis), it bounces back and forth between the two reflecting walls, and since the effective interaction length is extended to $L/\cos\theta$, then the effective gain is elevated to $\alpha L/\cos\theta$. In the case of Cherenkov radiation, the wake is forced to bounce between the two reflecting walls at the Cherenkov angle $\cos\theta_{\rm Cher} = 1/\beta\sqrt{\varepsilon_r}$; therefore, the gain is $\alpha' L/\sin\theta_{\rm Cher} = \alpha' L\sqrt{\varepsilon_r/\bar{\varepsilon}}$. Clearly, for a dilute gas $\varepsilon_r \simeq 1$, the gain is elevated by the Cherenkov slippage term $1/\sqrt{\bar{\varepsilon}}$, namely, $\alpha' L/\sin\theta_{\rm Cher} \simeq \alpha' L/\sqrt{\bar{\varepsilon}}$.

Next, we consider the entire manifold of modes. In the simulation that follows, we assume a density-modulated trigger beam of total current I_0 and modulation depth $M_{\rm tr}$ of the form $J_z(r,\tau) \propto I_0[1 + M_{\rm tr}\cos(\omega_0\tau)]$; the values of all other parameters are listed in Table I. The wakefield is expressed in terms of the longitudinal electric field that is related to $A_z(\tau) = \int_{-\infty}^{\infty} d\omega \mathcal{A}_z(\omega)\exp(j\omega\tau)$ via the Lorentz gauge $E_z(\tau) = -\int_{-\infty}^{\infty} d\omega\mathcal{A}_z(\omega)j\omega[1 - 1/\beta^2\varepsilon(\omega)]\exp(j\omega\tau)$.

Figure 2 shows the contribution of the various modes at a distance $L = 6000\lambda_0$ behind the trigger bunch. Only one or two modes in the vicinity of resonance have a significant contribution, and the remainder are more than 5 orders of magnitude weaker. The spectrum of E_z in the vicinity of resonance with a total of 1000 modes was considered. Near resonance $\omega_{C,s_0} \simeq \omega_0$, this peak is several orders of magnitude above off-resonance modes, regardless of whether the geometry is properly selected, such that $p_{s_0=360} = \omega_0 R_w\sqrt{\bar{\varepsilon}}/c$ is exactly satisfied or only approximately satisfied $p_{360}/ \leq \omega_0 R_w\sqrt{\bar{\varepsilon}}/c \leq p_{361}$.

TABLE I. Simulation parameters.

Parameter	Designation	Value
Medium resonance wave length (CO_2)	λ_0	10.6 μm
Medium resonance wave frequency	ω_0	1.78×10^{14} rad/s
Medium resonance bandwidth (at ∼10 atm)	$\Delta\omega$	2.32×10^{11} rad/s
Medium growth rate coefficient	α	1 m^{-1}
Medium plasma frequency	$\lvert\omega_p\rvert$	1.18×10^{10} rad/s
Medium relative permittivity	ε_r	1.00142
Waveguide radius	R_w	50.674 mm
Resonance mode	s_0	360
e-beam Lorentz factor	γ	600
e-beam total charge	$Q_{\rm total}$	10^9 *e*
e-beam length	$L_{\rm tr}$	$150\lambda_0$
e-beam modulation	$M_{\rm tr}$	20%
e-beam radius	R_b	4 mm

The enhanced exponential gain is clearly revealed in Fig. 3, where the gradient is illustrated for three cases: $s = s_0 = 360$, $|s - s_0| \leq 160$, and $1 \leq s \leq 1000$; the asymptotic behavior in all three cases is determined by the resonant mode (s_0), while all the others determine the near field adjacent to the trigger bunch. For comparison, if the Cherenkov factor were not present, then at a distance $L = 6000\lambda_0$, the gain would have been negligible $\exp(\alpha L) \simeq 1.07$ and not $\exp(|\Delta\omega_d|L/c) \sim 10^{10}$ as the present analysis predicts.

Whether the resonance condition is exactly satisfied or only approximately makes a difference of up to 5 orders of magnitude reduced gradient in the latter case, implying that saturation occurs further away from the trigger train. In addition, we have assessed the typical signal-to-noise ratio by comparing the gradient for the case of a 20% current modulation to a zero-modulated beam—a signal-to-noise ratio of 40 dB was found.

Being analyzed in the framework of a linear theory, the gain does not account for the medium depletion. A rough assessment of this saturation effect was developed based on energy conservation and two assumptions: the population inversion is uniform and only the resonant mode interacts. The details of this analysis are beyond the scope of this Letter nevertheless, we present next the main result. Denoting by $\bar{w}$ the average electromagnetic energy density associated with the resonant mode normalized to the average energy density stored in the medium, we found that the saturation field is $E_z^{(\rm sat)}/E_0 \simeq 1/\sqrt{\bar{w}}$, wherein E_0 is the amplitude of the growing wave immediately after the

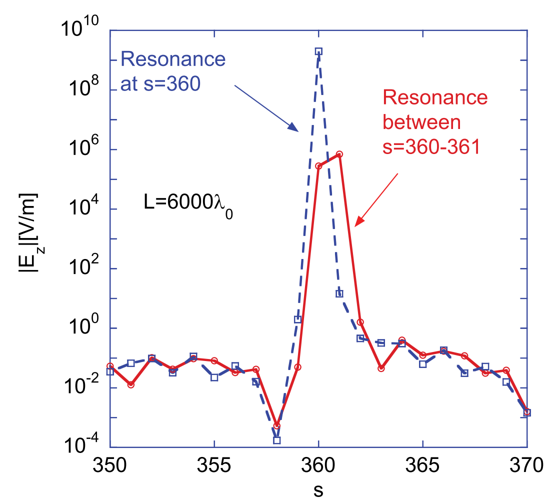

FIG. 2 (color online). Contribution of the various modes to E_z. The off-resonance modes have a contribution which is more than 5 orders of magnitude weaker. Squares (blue) correspond to the exact resonance condition ($\omega_{C,s_0} = \omega_0$); circles (red) correspond to a situation whereby the resonance is between two modes and neither is exactly at resonance ($\omega_{C,s_0} \simeq \omega_0$).

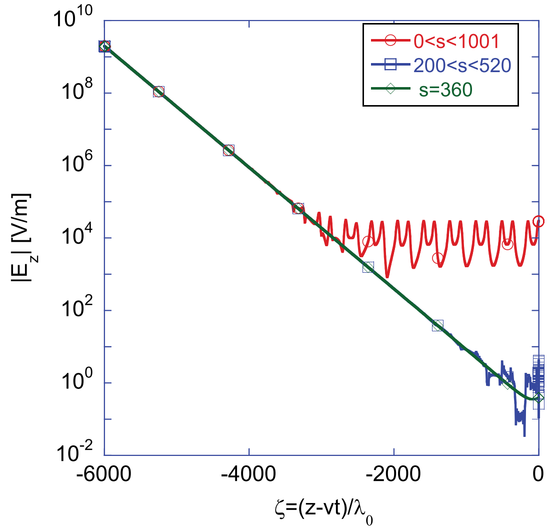

FIG. 3 (color online). Amplitude of E_z behind the trigger bunch (time domain) for three sets of Bessel harmonics: 1000 modes (red circles) and 320 ($200 \le s \le 520$) modes around the resonant harmonic (blue squares); finally, the sole resonant harmonic is represented by green diamonds. Clearly, sufficiently far away from the trigger bunch ($\zeta < -4000$), the resonant mode is dominant.

trigger bunch. Considering $\Delta n = 1.3 \times 10^{23}$ (m^{-3}), $E_{z,0} = 1$ (V/m), and $\lambda_0 = 10.6$ (μm), we have $\bar{w} \simeq 3.1 \times 10^{-18}$ and thus $E_z^{(\mathrm{sat})} \simeq 0.57$ (GV/m) for a single spectral line. It should be pointed out that for a less conservative set of parameters, saturation may occur at gradients somewhat less than 10 (GV/m). At these field intensities, ionization may become a major obstacle since it may convert energy from the wake into heat. Or, even worse, the optical properties of the ions are in general different than those of the neutral atom or molecule. However, there are experimental indications (Ref. [21]) that gas (e.g., nitrogen) can sustain, at several atmospheres, laser (0.69 μm) intensities of the order of 50 ($\mathrm{GW/cm^2}$)—which corresponds to an electric field of the order of 2 (GV/m).

In conclusion, the energy transfer from an active medium to a Cherenkov wake undergoing multiple reflections is enhanced by the latter. As the Cherenkov condition is approached, more reflections from the transverse walls occur. As a result, the effective gain the wake experiences is enhanced. A rough assessment of the saturation indicates that a gradient between 1 and 10 GV/m may be expected before breakdown and saturation become major impediments.

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